Zeal Education Society's

ZEAL POLYTECHNIC, PUNE.

NARHE | PUNE -41 | INDIA

FIRST YEAR (FY)

DIPLOMA IN CIVIL ENGINEERING

SCHEME: I SEMESTER: II

NAME OF SUBJECT: Applied Mathematics

Subject Code: 22201

MSBTE QUESTION PAPERS & MODEL ANSWERS

1. MSBTE SUMMER-18 EXAMINATION
2. MSBTE WINTER-18 EXAMINATION
3. MSBTE SUMMER-19 EXAMINATION
4. MSBTE WINTER-19 EXAMINATION

22201

11920

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any FIVE of the following : **10**

a) If $f(x) = 16^x - \log_2 x$ find $f\left(\frac{1}{4}\right)$

b) If $f(x) = ax^2 - bx - 1$, $f(2) = 5$, $f(-2) = 10$ find a and b.

c) Find $\frac{dy}{dx}$, if $y = x \sin^{-1} x$

d) Evaluate : $\int \frac{dx}{3x^2 + 4}$

e) Evaluate : $\int \sin^3 x \, dx$

f) Find the volume obtained by revolving the area under the curve $9x^2 - 4y^2 = 36$ in the interval from $x = 2$ to $x = 4$ about x-axis.

g) Find order and degree of the differential equation $\frac{d^2y}{dx^2} = \left(y + \frac{dy}{dx}\right)^{3/2}$

P.T.O.

Marks

2. Attempt any THREE of the following: **12**

a) If $x^p y^q = (x+y)^{p+q}$ show that $\frac{dy}{dx} = \frac{y}{x}$

b) If $y = 3\sin\theta - 2\sin^3\theta$ and $x = 3\cos\theta - 2\cos^3\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

c) Find the radius of curvature of the curve $xy = c$ at point (c, c)

d) Discuss maxima and minima of the function "$\tan x - 2x$".

3. Attempt any THREE of the following: **12**

a) Find the equation of tangent and normal to the curve $y = x(2-x)$ at point (2, 0).

b) Find $\frac{dy}{dx}$, $y = (\sin^{-1}x)^x + (\cos x)^{\sin x}$

c) If $y = \tan^{-1}\left[\frac{5x-4}{5+4x}\right]$ find $\frac{dy}{dx}$

d) Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}\, dx$

4. Attempt any THREE of the following: **12**

a) Evaluate : $\int \frac{dx}{2x^2+3x+1}$

b) Evaluate : $\int \frac{dx}{1+\sin x+\cos x}$

c) Evaluate : $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\, dx$

d) Evaluate : $\int_0^{\pi/2} \frac{\tan x}{1+\tan x}\, dx$

e) Evaluate : $\int \frac{x}{(x^2+4)(x^2+9)}\, dx$

Marks

5. Attempt any TWO of the following: **12**

a) Find the area between the curves $y^2 - 2x = 0$ and $y^2 + 4x - 12 = 0$

b) Attempt the following:

(i) Form the differential equation if

$y = A\cos(\log x) + B\sin(\log x)$

(ii) Solve

$x \log x \dfrac{dy}{dx} + y = 2\log x$

c) A circular column of radius 'x' and having depth y support a load. The equation of equilibrium is $2\dfrac{dy}{dx} - kx = 0$ where 'k' is constant. Find the relation between x and y.

6. Attempt any TWO of the following: **12**

a) Using Simpson's $1/3$rd rule, evaluate $\int_0^2 \dfrac{1}{1+x^3}\,dx$ with $n = 4$.

b) Using Simpson's $3/8$th rule, evaluate $\int_0^{\pi/2} \cos x \; dx$ with $n = 8$

c) Attempt the following:

(i) Using Trapezoidal rule, evaluate $\int_{-1}^{1} (1 + x + x^2 + x^3)\,dx$ by taking $n = 2$.

(ii) Using Simpson's $1/3$rd rule evaluate $\int_1^3 \dfrac{dx}{x}$, taking $h = 0.5$.

WINTER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of following:**	**10**
	a)	If $f(x)=16^x-\log_2 x$ find $f\left(\frac{1}{4}\right)$	**02**
	Ans	$f(x)=16^x-\log_2 x$	
		$f\left(\frac{1}{4}\right)=16^{\frac{1}{4}}-\log_2\left(\frac{1}{4}\right)$	1
		$=2$	1
		--	
	b)	If $f(x)=ax^2-bx-1, f(2)=5, f(-2)=10$ find a and b.	**02**
	Ans	$f(x)=ax^2-bx-1$	
		$f(2)=5$	
		$4a-2b-1=5$	
		$4a-2b=6$ -----------------(1)	½
		$f(-2)=10$	
		$4a+2b-1=10$	
		$4a+2b=11$ ----------------(2)	
		From (1) and (2)	½
		$a=\frac{17}{8}$	½
		$b=\frac{5}{4}$	½

WINTER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	b)	Find $\frac{dy}{dx}$, if $y = x\sin^{-1}x$	**02**
	Ans	$y = x\sin^{-1}x$	
		$\frac{dy}{dx} = x\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x.1$	2
		$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x$	
		--	
	c)	Evaluate: $\int \frac{dx}{3x^2+4}$	**02**
	Ans	$\int \frac{dx}{3x^2+4}$	
		$= \int \frac{dx}{\left(\sqrt{3}x\right)^2 + 2^2}$	½
		$= \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right)\frac{1}{\sqrt{3}} + c$	1
		$= \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + c$	½
		--	
	e)	Evaluate $\int \sin^3 x\, dx$	**02**
	Ans	$\int \sin^3 x\, dx$	
		since $\sin 3x = 3\sin x - 4\sin^3 x$ $\therefore \sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$	
		$\therefore \int \frac{1}{4}(3\sin x - \sin 3x)\, dx$	1
		$= \frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + c$	1
		OR	
		$\int \sin^3 x\, dx$	
		$= \int \sin^2 x \, \sin x\, dx$	
		$= \int \left(1-\cos^2 x\right)\sin x\, dx$	½
		Put $\cos x = t$	
		$\therefore -\sin x dx = dt$	
		$\therefore \sin x dx = -dt$	½

WINTER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	e)	$\therefore \int (1-t^2)(-dt)$	
		$= -\int (1-t^2)dt$	
		$= -\left(t - \frac{t^3}{3}\right) + c$	½
		$= -\left(\cos x - \frac{\cos^3 x}{3}\right) + c$	½
		--	
	f)	Find the volume obtained by revolving the area under the curve $9x^2 - 4y^2 = 36$ in the interval from $x = 2$ to , $x = 4$ about $x -$axis	**02**
	Ans	$9x^2 - 4y^2 = 36$	
		$y^2 = \frac{9}{4}(x^2 - 4)$	
		$\text{volume} = \pi \int_a^b y^2 dx$	
		$= \pi \int_2^4 \frac{9}{4}(x^2 - 4)dx$	
		$= \frac{9\pi}{4}\left[\frac{x^3}{3} - 4x\right]_2^4$	½
		$= \frac{9\pi}{4}\left[\left(\frac{4^3}{3} - 4(4)\right) - \left(\frac{2^3}{3} - 4(2)\right)\right]$	1
		$= 24\pi$	½
		--	
	g)	Find order and degree of the differential equation $\frac{d^2y}{dx^2} = \left(y + \frac{dy}{dx}\right)^{\frac{3}{2}}$	**02**
	Ans	$\frac{d^2y}{dx^2} = \left(y + \frac{dy}{dx}\right)^{\frac{3}{2}}$	
		Squaring on both sides	
		$\left(\frac{d^2y}{dx^2}\right)^2 = \left(y + \frac{dy}{dx}\right)^3$	
		$\therefore Order = 2$	1
		$\therefore Degree = 2$	1
		--	
2.		**Attempt any THREE of the following:**	**12**

WINTER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)	If $x^p y^q = (x+y)^{p+q}$ show that $\frac{dy}{dx} = \frac{y}{x}$	**04**
	Ans	$x^p y^q = (x+y)^{p+q}$	
		$\log\left(x^p y^q\right) = \log(x+y)^{p+q}$	
		$\log x^p + \log y^q = (p+q)\log(x+y)$	½
		$p\log x + q\log y = (p+q)\log(x+y)$	½
		$p\frac{1}{x} + q\frac{1}{y}\frac{dy}{dx} = (p+q)\left(\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)\right)$	1
		$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y} + \frac{p+q}{x+y}\frac{dy}{dx}$	
		$\frac{q}{y}\frac{dy}{dx} - \frac{p+q}{x+y}\frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$	½
		$\frac{dy}{dx}\left(\frac{q}{y} - \frac{p+q}{x+y}\right) = \frac{p+q}{x+y} - \frac{p}{x}$	
		$\frac{dy}{dx}\left(\frac{qx+qy-py-qy}{y(x+y)}\right) = \frac{px+qx-px-py}{x(x+y)}$	½
		$\frac{dy}{dx}\left(\frac{qx-py}{y}\right) = \frac{qx-py}{x}$	
		$\frac{dy}{dx} = \frac{y}{x}$	1
		--	
	b)	If $y = 3\sin\theta - 2\sin^3\theta,\ x = 3\cos\theta - 2\cos^3\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$	**04**
	Ans	$y = 3\sin\theta - 2\sin^3\theta$	
		$\therefore \frac{dy}{d\theta} = 3\cos\theta - 6\sin^2\theta\cdot\cos\theta$	1
		$= 3\cos\theta\left(1-2\sin^2\theta\right)$	½
		$x = 3\cos\theta - 2\cos^3\theta$	
		$\frac{dx}{d\theta} = -3\sin\theta + 6\cos^2\theta\cdot\sin\theta$	1
		$= -3\sin\theta\left(1-2\cos^2\theta\right)$	½
		$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\cos\theta\left(1-2\sin^2\theta\right)}{-3\sin\theta\left(1-2\cos^2\theta\right)}$	
		$= \frac{3\cos\theta(\cos 2\theta)}{-3\sin\theta(-\cos 2\theta)}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	b)	$=\cot\theta$	
		$\therefore at\ \theta=\frac{\pi}{4}$	
		$\therefore \frac{dy}{dx}=\cot\frac{\pi}{4}$	
		$=1$	½
		--	
	c)	Find the radius of curvature of the curve $xy=c$ at point (c,c)	**04**
	Ans	$xy=c$	
		$x\frac{dy}{dx}+y.1=0$	
		$\frac{dy}{dx}=-\frac{y}{x}$	1
		$\frac{d^2y}{dx^2}=-\frac{\left[x\frac{dy}{dx}-y\right]}{x^2}$	1
		at point (c,c)	
		$\frac{dy}{dx}=-\frac{c}{c}=-1$	½
		$\frac{d^2y}{dx^2}=-\frac{\left[c(-1)-c\right]}{c^2}$	½
		$=\frac{2}{c}$	
		$\therefore$ radius of curvature $=\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$=\frac{\left(1+(-1)^2\right)^{\frac{3}{2}}}{\frac{2}{c}}$	
		$=2^{\frac{1}{2}}c$ or $\sqrt{2}c$	1
		--	
	d)	Discuss the maxima and minima of the function "$\tan x-2x$"	**04**
	Ans	Let $y=\tan x-2x$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$\therefore \frac{dy}{dx} = \sec^2 x - 2$	½
		$\therefore \frac{d^2y}{dx^2} = 2\sec x \sec x \tan x = 2\sec^2 x \tan x$	½
		consider $\frac{dy}{dx} = 0$	
		$\therefore \sec^2 x - 2 = 0$	
		$\therefore \sec^2 x = 2$	
		$\therefore \sec x = \sqrt{2}, -\sqrt{2}$	
		$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$	1
		at $x = \frac{\pi}{4}$	
		$\therefore \frac{d^2y}{dx^2} = 2\sec^2 \frac{\pi}{4} \tan \frac{\pi}{4} = 2(2)(1) = 4 > 0$	½
		$\therefore$ function is minimum at $x = \frac{\pi}{4}$	
		$\therefore y_{\min} = \tan\frac{\pi}{4} - 2\left(\frac{\pi}{4}\right) = 1 - \frac{\pi}{2}$	½
		at $x = \frac{3\pi}{4}$	
		$\therefore \frac{d^2y}{dx^2} = 2\sec^2\left(\frac{3\pi}{4}\right)\tan\left(\frac{3\pi}{4}\right) = 2(2)(-1) = -4 < 0$	½
		$\therefore$ function is maximum at $x = \frac{3\pi}{4}$	
		$\therefore y_{\max} = \tan\left(\frac{3\pi}{4}\right) - 2\left(\frac{3\pi}{4}\right) = -1 - \frac{3\pi}{2}$	½
		--	
3.		**Attempt any THREE of the following:**	**12**
	a)	Find the equation of tangent and normal to the curve $y = x(2-x)$ at point $(2,0)$	**04**
	Ans	$y = x(2-x)$	
		$\therefore y = 2x - x^2$	
		$\therefore \frac{dy}{dx} = 2 - 2x$	1
		at $(2,0)$	
		$\therefore \frac{dy}{dx} = 2 - 2(2)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	a)	$\therefore \frac{dy}{dx} = -2$	½
		$\therefore$ slope of tangent , $m = -2$	
		Equation of tangent at $(2,0)$ is	
		$y - 0 = -2(x-2)$	
		$\therefore y = -2x + 4$	
		$\therefore 2x + y - 4 = 0$	1
		$\therefore$ slope of normal , $m' = \frac{-1}{m} = \frac{1}{2}$	½
		Equation of normal at $(2,0)$ is	
		$y - 0 = \frac{1}{2}(x-2)$	
		$\therefore 2y = x - 2$	
		$\therefore x - 2y - 2 = 0$	1
		--	
	b)	Find $\frac{dy}{dx}$, $y = (\sin^{-1} x)^x + (\cos x)^{\sin x}$	**04**
	Ans	Let $u = (\sin^{-1} x)^x$	
		$\therefore \log u = \log(\sin^{-1} x)^x$	
		$\therefore \log u = x \log(\sin^{-1} x)$	½
		$\therefore \frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{\sin^{-1} x}\frac{1}{\sqrt{1-x^2}} + \log(\sin^{-1} x).1$	1
		$\therefore \frac{1}{u}\frac{du}{dx} = \frac{x}{\sqrt{1-x^2}\sin^{-1} x} + \log(\sin^{-1} x)$	
		$\therefore \frac{du}{dx} = u\left(\frac{x}{\sqrt{1-x^2}\sin^{-1} x} + \log(\sin^{-1} x)\right)$	
		$\therefore \frac{du}{dx} = (\sin^{-1} x)^x\left(\frac{x}{\sqrt{1-x^2}\sin^{-1} x} + \log(\sin^{-1} x)\right)$	½
		Let $v = (\cos x)^{\sin x}$	
		$\therefore \log v = \log(\cos x)^{\sin x}$	
		$\therefore \log v = \sin x \log(\cos x)$	½
		$\therefore \frac{1}{v}\frac{dv}{dx} = \sin x \cdot \frac{1}{\cos x}.(-\sin x) + \log(\cos x) \cdot \cos x$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$\therefore \frac{1}{v}\frac{dv}{dx} = -\sin x \tan x + \log(\cos x)\cdot \cos x$	
		$\therefore \frac{dv}{dx} = v\left[-\sin x \tan x + \log(\cos x)\cdot \cos x\right]$	
		$\therefore \frac{dv}{dx} = (\cos x)^{\sin x}\left[-\sin x \tan x + \log(\cos x)\cdot \cos x\right]$	½
		$\therefore \frac{dy}{dx} = (\sin^{-1} x)^{x}\left(\frac{x}{\sqrt{1-x^2}\sin^{-1} x} + \log(\sin^{-1} x)\right) + (\cos x)^{\sin x}\left[-\sin x \tan x + \log(\cos x)\cdot \cos x\right]$	½
	c)	If $y = \tan^{-1}\left[\frac{5x-4}{5+4x}\right]$ find $\frac{dy}{dx}$	**04**
	Ans	$y = \tan^{-1}\left[\frac{5x-4}{5+4x}\right]$	
		$y = \tan^{-1}\left[\frac{x-\frac{4}{5}}{1+\frac{4}{5}x}\right]$	1
		$\text{y} = \tan^{-1} x - \tan^{-1}\frac{4}{5}$	2
		$\frac{dy}{dx} = \frac{1}{1+x^2} - 0$	1
		$\frac{dy}{dx} = \frac{1}{1+x^2}$	
	d)	Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx$	**04**
	Ans	$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}dx$	
		Let $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	½
		$= \int \frac{1}{(1+t)(2+t)}dt$	
		Consider	
		$\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
	d)	$\therefore 1 = A(2+t) + B(1+t)$	
		Put $t=-1 \quad \therefore A=1$	1
		Put $t=-2 \quad \therefore B=-1$	1
		$\therefore \frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$	
		$\therefore \int \frac{1}{(1+t)(2+t)} dt = \int \left(\frac{1}{1+t} + \frac{-1}{2+t}\right) dt$	
		$= 1\log(1+t) - 1\log(2+t) + c$	1
		$= \log\left(\frac{1+t}{2+t}\right) + c$	
		$= \log\left(\frac{1+\tan x}{2+\tan x}\right) + c$	½
		OR	
		$\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$	
		Put $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	½
		$\int \frac{1}{(1+t)(2+t)} dt$	
		$= \int \frac{1}{t^2+3t+2} dt$	½
		Third Term $= \frac{3^2}{4} = \frac{9}{4}$	
		$= \int \frac{1}{t^2+4t+\frac{9}{4}-\frac{9}{4}+2} dt$	½
		$= \int \frac{1}{\left(t+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dt$	½
		$= \frac{1}{2.\frac{1}{2}} \log\left\|\frac{t+\frac{3}{2}-\frac{1}{2}}{t+\frac{3}{2}+\frac{1}{2}}\right\| + c$	1
		$= \log\left\|\frac{t+1}{t+2}\right\| + c$	
		$= \log\left\|\frac{\tan x+1}{\tan x+2}\right\| + c$	1
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate : $\int \frac{1}{2x^2+3x+1} dx$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)Ans	$\int \frac{1}{2x^2+3x+1}\,dx = \frac{1}{2}\int \frac{1}{x^2+\frac{3}{2}x+\frac{1}{2}}\,dx$	
		Third term $= \left(\frac{1}{2}\times\frac{3}{2}\right)^2 = \frac{9}{16}$	
		$= \frac{1}{2}\int \frac{1}{x^2+\frac{3}{2}x+\frac{9}{16}-\frac{9}{16}+\frac{1}{2}}dx$	1
		$= \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$	1
		$= \frac{1}{2}\left[\frac{1}{2\left(\frac{1}{4}\right)}\log\left(\frac{x+\frac{3}{4}-\frac{1}{4}}{x+\frac{3}{4}+\frac{1}{4}}\right)\right]+c$	1
		$= \log\left(\frac{2x+1}{2x+2}\right)+c$	1
		OR	
		$\int \frac{1}{2x^2+3x+1}dx = \int \frac{1}{(2x+1)(x+1)}dx$	½
		Let $\frac{1}{(2x+1)(x+1)} = \frac{A}{2x+1}+\frac{B}{x+1}$	
		$1 = A(x+1)+B(2x+1)$	
		Put $x = \frac{-1}{2}$	
		$\therefore A = 2$	1
		Put $x = -1$	1
		$\therefore B = -1$	
		$\frac{1}{(2x+1)(x+1)} = \frac{2}{2x+1}+\frac{-1}{x+1}$	½
		$\int \frac{1}{(2x+1)(x+1)}dx = \int\left(\frac{2}{2x+1}+\frac{-1}{x+1}\right)dx$	1
		$= \frac{2\log(2x+1)}{2}-\log(x+1)+c$	
		$= \log(2x+1)-\log(x+1)+c$	
		OR	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	$\int \frac{1}{2x^2+3x+1}dx$	
	Ans	Third term $=\frac{(M.T.)^2}{4(F.T.)}=\frac{9}{8}$	
		$=\int \frac{1}{2x^2+3x+\frac{9}{8}-\frac{9}{8}+1}dx$	1
		$=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\frac{1}{8}}dx$	
		$=\int \frac{1}{\left(\sqrt{2}x+\frac{3}{\sqrt{8}}\right)^2-\left(\frac{1}{\sqrt{8}}\right)^2}dx$	1
		$=\frac{1}{\sqrt{2}}\left[\frac{1}{2\left(\frac{1}{\sqrt{8}}\right)}\log\left(\frac{\sqrt{2}x+\frac{3}{\sqrt{8}}-\frac{1}{\sqrt{8}}}{\sqrt{2}x+\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{8}}}\right)\right]+c$	1
		$=\log\left(\frac{2x+1}{2x+2}\right)+c$	1
		--	
	b)	Evaluate $\int \frac{dx}{1+\sin x+\cos x}$	**04**
	Ans	$\int \frac{dx}{1+\sin x+\cos x}$	
		Put $\tan\frac{x}{2}=t$ $\therefore \sin x=\frac{2t}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$, $dx=\frac{2dt}{1+t^2}$	
		$\therefore \int \frac{dx}{1+\sin x+\cos x}=\int \frac{1}{1+\frac{2t}{1+t^2}+\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	1
		$=\int \frac{2\,dt}{1+t^2+2t+1-t^2}dt$	
		$=2\int \frac{1}{2t+2}dt$	
		$=\int \frac{dt}{t+1}$	1
		$=\log(t+1)+c$	1
		$=\log\left(\tan\frac{x}{2}+1\right)+c$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	Evaluate : $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$	**04**
	Ans	$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$	
		put $\sin^{-1}x = t \quad \therefore x = \sin t$	
		$\frac{1}{\sqrt{1-x^2}}dx = dt$	1
		$\int t\sin t\ dt$	
		$= t\int \sin t dt - \int\left(\int \sin t dt\right)\frac{d}{dt}(t)dt$	1
		$= t(-\cos t) - \int -\cos t \cdot 1 \cdot dt$	½
		$= -t\cos t + \sin t + c$	½
		$= -\sin^{-1}x\cos\left(\sin^{-1}x\right) + \sin\left(\sin^{-1}x\right) + c$	1
		---	
	d)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\tan x}{1+\tan x}dx$	**04**
	Ans	Let $I = \int_0^{\frac{\pi}{2}} \frac{\tan x}{1+\tan x}dx$	
		$= \int_0^{\frac{\pi}{2}} \frac{\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}dx$	½
		$\therefore I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}dx$ ---------------------(1)	
		$I = \int_0^{\pi/2} \frac{\sin\left(\pi/2 - x\right)}{\sin\left(\pi/2 - x\right) + \cos\left(\pi/2 - x\right)}dx$	1
		$\therefore I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}dx$ ---------------------(2)	½
		add (1) and (2)	
		$I + I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}dx + \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)	$2I=\int_0^{\pi/2}\frac{\sin x+\cos x}{\sin x+\cos x}\,dx$	
		$2I=\int_0^{\pi/2}1\,dx$	½
		$2I=\left[x\right]_0^{\pi/2}$	½
		$2I=\frac{\pi}{2}-0$	
		$I=\frac{\pi}{4}$	½
		--	
	e)	Evaluate: $\int\frac{x}{\left(x^2+4\right)\left(x^2+9\right)}dx$	**04**
	Ans	$\int\frac{x}{\left(x^2+4\right)\left(x^2+9\right)}dx$	
		Put $x^2=t$	
		$2xdx=dt$	
		$xdx=\frac{dt}{2}$	½
		$\int\frac{\frac{dt}{2}}{(t+4)(t+9)}$	
		$=\frac{1}{2}\int\frac{dt}{(t+4)(t+9)}$	
		$\frac{1}{(t+4)(t+9)}=\frac{A}{t+4}+\frac{B}{t+9}$	
		$1=A(t+9)+B(t+4)$	
		Put $t=-4\quad\therefore A=\frac{1}{5}$	1
		Put $t=-9\quad\therefore B=-\frac{1}{5}$	1
		$\frac{1}{(t+4)(t+9)}=\frac{\frac{1}{5}}{t+4}+\frac{-\frac{1}{5}}{t+9}$	
		$\int\frac{dt}{(t+4)(t+9)}=\int\left(\frac{\frac{1}{5}}{t+4}+\frac{-\frac{1}{5}}{t+9}\right)dt$	½

WINTER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
	e)	$=\frac{1}{5}\log(t+4)-\frac{1}{5}\log(t+9)+c$	½
		$=\frac{1}{5}\log(x^2+4)-\frac{1}{5}\log(x^2+9)+c$	½
5.		**Attempt any TWO of the following:**	**12**
	a)	Find area of between the curve $y^2-2x=0$ and $y^2+4x-12=0$	**06**
	Ans	$y^2=2x$ ---------(1)	
		$y^2=12-4x$	
		$\therefore 2x=12-4x$	
		$\therefore 6x=12$	
		$\therefore x=2\ ,\ y=\pm 2$	1
		$\therefore x=\frac{y^2}{2}\quad ,\quad x=\frac{12-y^2}{4}$	
		Area $A=\int_a^b (x_1-x_2)\,dy$	
		$\therefore A=\int_{-2}^{2}\left(\frac{12-y^2}{4}-\frac{y^2}{2}\right)dy$	1
		$\therefore A=\frac{3}{4}\int_{-2}^{2}(4-y^2)\,dy$	1
		$\therefore A=\frac{3}{4}\left(4y-\frac{y^3}{3}\right)_{-2}^{2}$	1
		$\therefore A=\frac{3}{4}\left(4(2)-\frac{(2)^3}{3}-\left(4(-2)-\frac{(-2)^3}{3}\right)\right)$	1
		$\therefore A=8$	1
	b)	Attempt the following:	**06**
	(i)	Form the differential equation If $y=A\cos(\log x)+B\sin(\log x)$	**03**
	Ans	$y=A\cos(\log x)+B\sin(\log x)$	
		$\therefore \frac{dy}{dx}=-\frac{A\sin(\log x)}{x}+\frac{B\cos(\log x)}{x}$	1
		$\therefore x\frac{dy}{dx}=-A\sin(\log x)+B\cos(\log x)$	½
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-\frac{A\cos(\log x)}{x}-\frac{B\sin(\log x)}{x}$	1
		$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-A\cos(\log x)-B\sin(\log x)$	

WINTER – 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22201**

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)i)	$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-y$	
		$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$	½
		--	
	b)ii)	Solve $x\log x\frac{dy}{dx}+y=2\log x$	**03**
	Ans	$x\log x\frac{dy}{dx}+y=2\log x$	
		$\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2\log x}{x\log x}$	
		$\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$	½
		$I.F=e^{\int\frac{1}{x\log x}dx}$	½
		$=e^{\log(\log x)}=\log x$	
		Solution is,	½
		$y.\log x=\int\frac{2}{x}.\log xdx$	
		Let $\text{I}_1=\int\frac{2}{x}.\log xdx$	
		Put $\log x=t$	½
		$\frac{1}{x}dx=dt$	
		$\therefore \text{I}_1=2\int tdt$	
		$=2\frac{t^2}{2}+c$	½
		$=(\log x)^2+c$	
		$y.\log x=(\log x)^2+c$	½
		--	
	c)	A circular column of radius 'x' and having depth y support a load.The equation of equilibrium is $2\frac{dy}{dx}-kx=0$ where 'k' is constant.Find the relation between x and y.	**06**
	Ans	$2\frac{dy}{dx}-kx=0$	
		$2\frac{dy}{dx}=kx$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$\frac{dy}{dx}=\frac{k}{2}x$	½
		$dy=\frac{k}{2}xdx$	1
		$\int dy=\int\frac{k}{2}xdx$	1
		$y=\frac{k}{2}\frac{x^2}{2}+c_1$	2
		$4y=kx^2+4c$	1
		$4y=kx^2+c$	½
		--	
6.		**Attempt any TWO of the following:**	**12**
	a)	Using Simpson's $1/3^{rd}$ rule evaluate $\int_0^2\frac{1}{1+x^3}dx$ with $n=4$.	**06**
	Ans	Let $y=\frac{1}{1+x^3}$ $a=0,b=2$ and $n=4$	
		$\therefore h=\frac{b-a}{n}=\frac{2-0}{4}=\frac{1}{2}$	1

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y=\frac{1}{1+x^3}$	1	$\frac{8}{9}$	$\frac{1}{2}$	$\frac{8}{35}$	$\frac{1}{9}$

(Marking: 2)

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

$$\therefore\int_0^1 f(x)dx=\frac{1/2}{3}\left[\left(1+\frac{1}{9}\right)+4\left(\frac{8}{9}+\frac{8}{35}\right)+2\left(\frac{1}{2}\right)\right]$$ 2

$$\therefore\int_0^1\frac{1}{1+x^3}dx=1.0968$$ 1

OR

Let $y=\frac{1}{1+x^3}$ $a=0,b=2$ and $n=4$

$\therefore h=\frac{b-a}{n}=\frac{2-0}{4}=\frac{1}{2}=0.5$ 1

Q. No.	Sub Q. N.	Answer	Marking Scheme

6. a)

x	0	0.5	1	1.5	2
$y=\frac{1}{1+x^3}$	1	0.8889	0.5	0.2286	0.1111

Marks: 2

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)\,dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

$$\therefore\int_0^1 f(x)\,dx=\frac{0.5}{3}\left[(1+0.1111)+4(0.8889+0.2286)+2(0.5)\right]$$

Marks: 2

$$\int_0^1\frac{1}{1+x^2}\,dx=1.0969$$

Marks: 1

---

b) Using Simpson's $3/8^{th}$ rule,evaluate $\int_0^{\frac{\pi}{2}}\cos x\,dx$ with $n=8$ **06**

Ans

Here $n=8$

$y=\cos x \quad a=0,\quad b=\frac{\pi}{2}$

$$\therefore h=\frac{b-a}{n}=\frac{\frac{\pi}{2}-0}{8}=\frac{\pi}{16}$$

Marks: 1

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$	$\frac{5\pi}{16}$	$\frac{3\pi}{8}$	$\frac{7\pi}{16}$	$\frac{\pi}{2}$
$y=\cos x$	1	0.9808	0.9239	0.8315	0.7071	0.5556	0.3827	0.1951	0

Marks: 2

Using Simpson's $3/8^{th}$ rule.

$$\int_a^b f(x)\,dx=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4...+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$$

$$\therefore\int_0^{\frac{\pi}{2}}\cos x\,dx=\frac{3\left(\frac{\pi}{16}\right)}{8}\begin{bmatrix}(1+0)+3(0.9808+0.9239+0.7071+0.5556+0.1951)\\+2(0.8315+0.3827)\end{bmatrix}$$

Marks: 2

$$\therefore\int_0^{\frac{\pi}{2}}\cos x\,dx=0.9952$$

Marks: 1

---

c) Attempt the following: **06**

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)(i)	Using Trapezoidal rule, evaluate $\int_{-1}^{1}\left(1+x+x^2+x^3\right)dx$, by taking $n=2$.	03
	Ans	$y=1+x+x^2+x^3 \quad a=-1, \quad b=1$	
		$\therefore h=\frac{b-a}{n}=\frac{1+1}{2}=1$	½
		<table><tr><td>x</td><td>-1</td><td>0</td><td>1</td></tr><tr><td>$y=1+x+x^2+x^3$</td><td>0</td><td>1</td><td>4</td></tr></table>	1
		$\int_a^b f(x)dx=\frac{h}{2}\left[\left(y_0+y_n\right)+2\left(y_1+y_2+...+y_{n-1}\right)\right]$	
		$a=-1, b=1$ and $h=1$	
		$\therefore \int_{-1}^{1}\left(1+x+x^2+x^3\right)dx=\frac{1}{2}\left[(0+4)+2(1)\right]$	½
		$=3$	1
	ii)	Using Simpson's $1/3^{rd}$ rule evaluate $\int_1^3 \frac{dx}{x}$ taking $h=0.5$.	
	Ans	Let $y=\frac{1}{x}$, $\text{h}=0.5, a=1, b=3$	
		<table><tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td></tr><tr><td>$y=\frac{1}{x}$</td><td>1</td><td>$\frac{2}{3}$</td><td>$\frac{1}{2}$</td><td>$\frac{2}{5}$</td><td>$\frac{1}{3}$</td></tr></table>	1
		Using Simpson's $1/3^{rd}$ rule	
		$\int_a^b f(x)dx=\frac{h}{3}\left[\left(y_0+y_n\right)+4\left(y_1+y_3+...+y_{n-1}\right)+2\left(y_2+y_4+...+y_{n-2}\right)\right]$	
		$\therefore \int_1^3 \frac{dx}{x}=\frac{0.5}{3}\left[\left(1+\frac{1}{3}\right)+4\left(\frac{2}{3}+\frac{2}{5}\right)+2\left(\frac{1}{2}\right)\right]$	1
		$\int_1^3 \frac{dx}{x}=1.1$	1
		OR	
		Let $y=\frac{1}{x}$, $\text{h}=0.5, a=1, b=3$	
		<table><tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td></tr><tr><td>$y=\frac{1}{x}$</td><td>1</td><td>0.6667</td><td>0.5</td><td>0.4</td><td>0.3333</td></tr></table>	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)(ii)	Using Simpson's $1/3^{rd}$ rule	
		$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$	
		$\therefore \int_1^3 \frac{dx}{x} = \frac{0.5}{3}\left[(1+0.3333)+4(0.6667+0.4)+2(0.5)\right]$	1
		$\int_1^3 \frac{dx}{x} = 1.1$	1

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.

22201

21819

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. **Attempt any FIVE of the following:** **10**

a) Define Implicit function with suitable example.

b) State whether the function $f(x) = \frac{e^{x} - e^{-x}}{2}$, is even or odd.

c) Find $\frac{dy}{dx}$; if $y = \left(x^{4} + 2x\right) \cdot \sin 3x$

d) Evaluate $\int x \cdot \cos x \, dx$

e) Evaluate $\int \left[e^{2\log x} + e^{x\log 2}\right] dx$

f) Find the area under the curve $y = x^{2}$ from $x = 0$ to $x = 3$ with x–axis.

g) State Simpson's one third rule of numerical integration.

Marks

2. Attempt any <u>THREE</u> of the following: **12**

a) If $y = f(x) = \frac{x-5}{5x-1}$, show that $f(y) = x$.

b) Find $\frac{dy}{dx}$, if $13x^2 + 2x^2 y + y^3 = 1$

c) A metal wire 40 cm long is bent to form a rectangle. Find its dimensions when area is maximum.

d) Show that radius of curvature at any point on the curve $y = a\log\left(\sec\frac{x}{a}\right)$, Where a is constant is $a\sec\frac{x}{a}$.

3. Attempt any <u>THREE</u> of the following: **12**

a) The slope of the curve $2y^3 = ax^2 + b$ at point (1, –1) is same as the slope of $x + y = 0$. Find a and b.

b) Find $\frac{dy}{dx}$, if $y = \sec^{-1}\left[\frac{1}{4x^3 - 3x}\right]$

c) If $x = a\cos^3\theta, y = a\sin^3\theta$ *find* $\frac{dy}{dx}$ *at* $\theta = \frac{\pi}{4}$

d) Evaluate $\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx$

4. Attempt any <u>THREE</u> of the following: **12**

a) Evaluate $\int \frac{\sec^2 x\,dx}{3\tan^2 x - 2\tan x - 5}$

b) Evaluate $\int \frac{dx}{1 + \sin x + \cos x}$

c) Evaluate $\int x^2 \cos 2x\,dx$

Marks

d) Evaluate $\int_{5}^{10} \frac{dx}{(x-1)(x-2)}$

e) Evaluate $\int_{3}^{7} \frac{(10-x)^2}{x^2+(10-x)^2}\,dx$

5. Attempt any TWO of the following: **12**

a) Find the area of the region included between parabola $y=x^2$ and $y=4$.

b) Attempt the following:

(i) Verify that $y = \log x$ is a solution of differential equation

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$$

(ii) Solve: $\frac{dy}{dx}=e^{2x-y}+x^2e^{-y}$

c) A particle starting with velocity 6 m/s has an acceleration $(1 - t^2)$ m/s^2. When does it first comes to rest? How far has it then travelled?

Marks

6. Attempt any TWO of the following: **12**

a) Attempt the following:

(i) Using Trapezoidal rule, calculate approximate value of $\int_3^8 \log_e x \, dx$

by using following table.

x	3	4	5	6	7	8
$y = \log_e x$	1.098	1.3863	1.6094	1.7918	1.9458	2.0794

(ii) Using Simpson's $\frac{1}{3}^{rd}$ rule, calculate the approximate value of $\int_0^4 e^x \, dx$ by using following data:

x	0	1	2	3	4
$y = e^x$	1	2.72	7.39	20.09	54.60

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Trapezoidal rule, taking $n = 4$.

Hence, obtain approximate value of π.

c) Evaluate $\int_0^{\pi/2} \sqrt{\cos x} \, dx$ using Simpson's $\frac{3}{8}$ rule with $n = 8$

SUMMER– 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22201**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	Define Implicit function with suitable example.	**02**
	Ans	Implicit Function: If the variables x and y are not separated from each other from the function $f(x,y)=0$ then function is called implicit function. e.g. $x^2+xy+y^2=0$	2
	b)	State whether the function $f(x)=\dfrac{e^x-e^{-x}}{2}$, is even or odd.	**02**
	Ans	$f(x)=\dfrac{e^x-e^{-x}}{2}$ $\therefore f(-x)=\dfrac{e^{-x}-e^{-(-x)}}{2}$	
		$\therefore f(-x)=\dfrac{e^{-x}-e^{x}}{2}=\dfrac{-\left(e^x-e^{-x}\right)}{2}$	1
		$\therefore f(-x)=-f(x)$	1
		$\therefore$ function is odd.	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	Find $\frac{dy}{dx}$; if $y=\left(x^4+2x\right)\cdot\sin 3x$	**02**
	Ans	$y=\left(x^4+2x\right)\cdot\sin 3x$	
		$\therefore \frac{dy}{dx}=\left(x^4+2x\right)\frac{d}{dx}\left(\sin 3x\right)+\sin 3x\frac{d}{dx}\left(x^4+2x\right)$	
		$=\left(x^4+2x\right)\cos 3x\left(3\right)+\sin 3x\left(4x^3+2\right)$	2
		$=3\left(x^4+2x\right)\cos 3x+\sin 3x\left(4x^3+2\right)$	
	d)	Evaluate $\int x\cdot\cos x\,dx$	**02**
	Ans	$\int x\cdot\cos x\,dx$	
		$=x\int\cos x\,dx-\int\left[\int\cos x\,dx\frac{d}{dx}(x)\right]dx$	½
		$=x\sin x-\int\sin x.1\,dx$	1
		$=x\sin x-\left(-\cos x\right)+c$	½
		$=x\sin x+\cos x+c$	
	e)	Evaluate $\int\left[e^{2\log x}+e^{x\log 2}\right]dx$	**02**
	Ans	$\int\left[e^{2\log x}+e^{x\log 2}\right]dx$	
		$=\int\left[e^{\log x^2}+e^{\log 2^x}\right]dx$	½
		$=\int\left[x^2+2^x\right]dx$	½
		$=\frac{x^3}{3}+\frac{2^x}{\log 2}+c$	1
	f)	Find the area under the curve $y=x^2$ from $x=0$ to $x=3$ with x-axis.	**02**
	Ans	Area $A=\int_a^b y\,dx$	
		$=\int_0^3 x^2dx$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	f)	$\therefore A=\left[\frac{x^3}{3}\right]_0^3$	½
		$\therefore A=\left[\frac{3^3}{3}-\frac{0}{0}\right]$	
		$\therefore A=9$	½
	g)	State Simpson\`s one third rule of numerical integration.	**02**
	Ans	Simpson\`s one third rule:	
		$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...)+2(y_2+y_4+.....)\right]$	2
		where $h=\frac{b-a}{n}$	
2.		**Attempt any THREE of the following:**	**12**
	a)	If $y=f(x)=\frac{x-5}{5x-1}$, show that $f(y)=x$.	**04**
	Ans	$f(x)=\frac{x-5}{5x-1}$	
		$\therefore f(y)=\frac{y-5}{5y-1}$	
		$=\frac{\left(\frac{x-5}{5x-1}\right)-5}{5\left(\frac{x-5}{5x-1}\right)-1}$	1
		$=\frac{(x-5)-5(5x-1)}{5(x-5)-(5x-1)}$	1
		$=\frac{x-5-25x+5}{5x-25-5x+1}$	
		$=\frac{-24x}{-24}$	1
		$=x$	1
	b)	Find $\frac{dy}{dx}$, if $13x^2+2x^2y+y^3=1$	**04**

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	b)	$13x^2 + 2x^2 y + y^3 = 1$	
	Ans		
		$\therefore 13(2x) + 2\left(x^2 \dfrac{dy}{dx} + y \cdot 2x\right) + 3y^2 \dfrac{dy}{dx} = 0$	1
		$\therefore 26x + 2x^2 \dfrac{dy}{dx} + 4xy + 3y^2 \dfrac{dy}{dx} = 0$	1
		$\therefore \dfrac{dy}{dx}\left(2x^2 + 3y^2\right) = -26x - 4xy$	1
		$\therefore \dfrac{dy}{dx} = \dfrac{-26x - 4xy}{2x^2 + 3y^2}$	1
		--	
	c)	A metal wire 40 cm long is bent to form a rectangle. Find its dimensions when area is maximum.	**04**
	Ans	Let length of rectangle = x , breadth = y	
		$\therefore 2x + 2y = 40$	
		$\therefore y = 20 - x$	
		Area $A = x \times y$	
		$A = x(20 - x)$	1
		$\therefore A = 20x - x^2$	
		$\therefore \dfrac{dA}{dx} = 20 - 2x$	½
		$\therefore \dfrac{d^2A}{dx^2} = -2$	½
		Let $\dfrac{dA}{dx} = 0$	
		$\therefore 20 - 2x = 0$	½
		$\therefore x = 10$	
		at $x = 10$	½
		$\dfrac{d^2A}{dx^2} = -2 < 0$	½
		Area is maximum at $x = 10$	
		Length $= 10$; breadth $= 10$	½
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	Show that radius of curvature at any point on the curve $y = a\log\left(\sec\frac{x}{a}\right)$, Where a is constant is $a\sec\frac{x}{a}$.	04
	Ans	$y = a\log\left(\sec\left(\frac{x}{a}\right)\right)$	
		$\frac{dy}{dx} = a\frac{1}{\sec\left(\frac{x}{a}\right)}\sec\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)$	1
		$\frac{dy}{dx} = \tan\left(\frac{x}{a}\right)$	
		$\frac{d^2y}{dx^2} = \sec^2\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)$	1
		$\therefore$ Radius of curvature is $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho = \frac{\left[1+\tan^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)}$	1
		$\therefore \rho = \frac{a\left[\sec^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)}$	
		$\therefore \rho = \frac{a\sec^3\left(\frac{x}{a}\right)}{\sec^2\left(\frac{x}{a}\right)}$	
		$\therefore \rho = a\sec\left(\frac{x}{a}\right)$	1
3.		**Attempt any <u>THREE</u> of the following:**	**12**

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	a)	The slope of the curve $2y^3 = ax^2 + b$ at point $(1,-1)$ is same as the slope of $x + y = 0$ Find *a* and *b*.	**04**
	Ans	$2y^3 = ax^2 + b \quad -----(1)$	
		$\therefore 6y^2 \frac{dy}{dx} = 2ax$	
		$\therefore \frac{dy}{dx} = \frac{2ax}{6y^2}$	
		$\therefore \frac{dy}{dx} = \frac{ax}{3y^2}$	1
		at $(1,-1)$	
		$\frac{dy}{dx} = \frac{a(1)}{3(-1)^2} = \frac{a}{3} \quad ----(2)$	½
		$\because x + y = 0$	
		$\therefore 1 + \frac{dy}{dx} = 0$	
		$\therefore \frac{dy}{dx} = -1 \quad ----(3)$	1
		from (2) and (3)	
		$\frac{a}{3} = -1$	
		$\therefore a = -3$	1
		$\therefore$ From (1)	
		$2(-1)^3 = a(1)^2 + b$	
		$\therefore -2 = -3 + b$	
		$\therefore b = 1$	½
	b)	Find $\frac{dy}{dx}$, if $y = \sec^{-1}\left[\frac{1}{4x^3 - 3x}\right]$	**04**
	Ans	$y = \sec^{-1}\left[\frac{1}{4x^3 - 3x}\right]$	
		Put $x = \cos\theta \ \therefore \ \theta = \cos^{-1} x$	1
		$\therefore y = \sec^{-1}\left[\frac{1}{4\cos^3\theta - 3\cos\theta}\right]$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	$y=\sec^{-1}\left[\frac{1}{\cos 3\theta}\right]$	½
		$\therefore y=\sec^{-1}\left[\sec 3\theta\right]$	½
		$\therefore y=3\theta$	½
		$\therefore y=3\cos^{-1}x$	½
		$\therefore \frac{dy}{dx}=\frac{-3}{\sqrt{1-x^2}}$	1
	c)	If $x=a\cos^3\theta$, $y=a\sin^3\theta$ find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$	**04**
	Ans	$\because x=a\cos^3\theta$,	
		$\therefore \frac{dx}{d\theta}=3a\cos^2\theta\cdot(-\sin\theta)$	1
		$\because y=a\sin^3\theta$	
		$\therefore \frac{dy}{d\theta}=3a\sin^2\theta\cdot\cos\theta$	1
		$\therefore \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$	
		$\therefore \frac{dy}{dx}=\frac{3a\sin^2\theta\cdot\cos\theta}{3a\cos^2\theta\cdot(-\sin\theta)}$	1
		$\therefore \frac{dy}{dx}=-\tan\theta$	
		at $\theta=\pi/4$	
		$\therefore \frac{dy}{dx}=-\tan\pi/4$	
		$\therefore \frac{dy}{dx}=-1$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	Evaluate $\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx$	**04**
	Ans	$\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx$	
		Put $xe^x = t$	½
		$\therefore e^x(x+1)dx = dt$	
		$= \int \frac{1}{\cos^2 t}dt$	1
		$= \int \sec^2 \text{t}\, dt$	1
		$= \tan t + c$	1
		$= \tan(xe^x) + c$	½
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate $\int \frac{\sec^2 x\, dx}{3\tan^2 x - 2\tan x - 5}$	**04**
	Ans	$I = \int \frac{\sec^2 x\, dx}{3\tan^2 x - 2\tan x - 5}$	
		Put $\tan x = t$	1
		$\therefore \sec^2 x dx = dt$	
		$= \int \frac{dt}{3t^2 - 2t - 5}$	
		$= \frac{1}{3}\int \frac{dt}{t^2 - \frac{2}{3}t - \frac{5}{3}} dt$	½
		Third term $= \left(\frac{1}{2} \times \frac{-2}{3}\right)^2 = \frac{1}{9}$	
		$= \frac{1}{3}\int \frac{dt}{t^2 - \frac{2}{3}t + \frac{1}{9} - \frac{1}{9} - \frac{5}{3}}$	
		$= \frac{1}{3}\int \frac{dt}{\left(t - \frac{1}{3}\right)^2 - \left(\frac{4}{3}\right)^2}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	a)	$=\frac{1}{3}\cdot\frac{1}{2\times\frac{4}{3}}\log\left(\frac{t-\frac{1}{3}-\frac{4}{3}}{t-\frac{1}{3}+\frac{4}{3}}\right)+c$	1
		$=\frac{1}{8}\log\left(\frac{3t-5}{3t+3}\right)+c$	
		$I=\frac{1}{8}\log\left(\frac{3\tan x-5}{3\tan x+3}\right)+c$	½
	b)	Evaluate $\int\frac{dx}{1+\sin x+\cos x}$	**04**
	Ans	$\int\frac{dx}{1+\sin x+\cos x}$	
		Put $\tan\frac{x}{2}=t$ $\therefore \sin x=\frac{2t}{1+t^2}$ $,\cos x=\frac{1-t^2}{1+t^2},$ $dx=\frac{2dt}{1+t^2}$	1
		$\therefore\int\frac{dx}{1+\sin x+\cos x}=\int\frac{1}{1+\frac{2t}{1+t^2}+\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	
		$=\int\frac{2\,dt}{1+t^2+2t+1-t^2}dt$	½
		$=2\int\frac{1}{2t+2}dt$	
		$=\int\frac{dt}{t+1}$	1
		$=\log(t+1)+c$	1
		$=\log\left(\tan\frac{x}{2}+1\right)+c$	½
	c)	Evaluate $\int x^2\cos 2x\,dx$	**04**
	Ans	$I=\int x^2\cos 2x\,dx$	
		$=x^2\cdot\int\cos 2x\,dx-\int\left[\int\cos 2x\,dx\frac{d}{dx}\left(x^2\right)\right]dx$	½
		$=x^2\frac{\sin 2x}{2}-\int\frac{\sin 2x}{2}(2x)\,dx$	1
		$=2x^2\frac{\sin 2x}{2}-\int x\sin 2x\,dx$	

SUMMER– 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22201**

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	$=x^2(\sin 2x)-\left[x\int(\sin 2x)\,dx-\int\left(\int(\sin 2x)\,dx\frac{d}{dx}(x)\right)\right]dx$	½
		$=x^2\sin 2x-x\left(-\frac{\cos 2x}{2}\right)+\int\left(-\frac{\cos 2x}{2}\right)1dx$	½
			½
		$=x^2\sin 2x+\frac{x}{2}\cos 2x-\frac{1}{2}\int\cos 2x\,dx$	½
		$=x^2\sin 2x+\frac{x}{2}\cos 2x-\frac{1}{4}\sin 2x+c$	½
	d)	Evaluate $\int_5^{10}\frac{dx}{(x-1)(x-2)}$	**04**
	Ans	$I=\int_5^{10}\frac{dx}{(x-1)(x-2)}$	
		Let $\frac{1}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2}$	
		$1=A(x-2)+B(x-1)$	
		Put $x=1$ $\therefore A=\frac{1}{-1}=-1$	½
		Put $x=2$ $\therefore B=\frac{1}{1}=1$	½
		$\frac{1}{(x-1)(x-2)}=\frac{-1}{x-1}+\frac{1}{x-2}$	
		$\int_5^{10}\frac{dx}{(x-1)(x-2)}=\int_5^{10}\left[\frac{-1}{x-1}+\frac{1}{x-2}\right]dx$	
		$=\left[-\log(x-1)+\log(x-2)\right]_5^{10}$	2
		$=\left[-\log(10-1)+\log(10-2)\right]-\left[-\log(5-1)+\log(5-2)\right]$	1
		$=\left[-\log 9+\log 8\right]-\left[-\log 4+\log 3\right]=\log\frac{8}{9}-\log\frac{3}{4}$	
		$=\log\left[\frac{8}{9}\times\frac{4}{3}\right]=\log\frac{32}{27}$	
	e)	Evaluate $\int_3^{7}\frac{(10-x)^2}{x^2+(10-x)^2}\,dx$	**04**

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)Ans	$I=\int_3^7 \frac{(10-x)^2}{x^2+(10-x)^2}\ dx --------(1)$	
		$I=\int_3^7 \frac{[10-(10-x)]^2}{(10-x)^2+[10-(10-x)]^2}\ dx$	1
		$I=\int_3^7 \frac{x^2}{(10-x)^2+x^2}\ dx --------(2)$	½
		Adding (1) and (2)	
		$2I=\int_3^7 \frac{(10-x)^2+x^2}{(10-x)^2+x^2}dx$	1
		$2I=\int_3^7 1\,dx$	½
		$2I=[x]_3^7=7-3=4$	½
		$I=2$	½
5.		**Attempt any TWO of the following:**	**12**
	a)	Find the area of the region included between parabola $y=x^2$ and $y=4$	
	Ans	We have $y=x^2$ and $y=4$	**06**
		$\therefore x^2-4=0$ $\therefore x^2=4$ $\therefore x=\pm 2$	1
		Area $=\int_a^b (y_1-y_2)\,dx$	
		$=\int_{-2}^{2}(x^2-4)\,dx$	1
		$=\left[\frac{x^3}{3}-4x\right]_{-2}^{2}$	2
		$=\left[\frac{2^3}{3}-4(2)-\left(\frac{(-2)^3}{3}-4(-2)\right)\right]$	1
		$=\frac{-32}{3}$ Area is always $+ve$	
		$\therefore A=\frac{32}{3}$ or 10.667	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b)	Attempt the following:	**06**
	(i)	Verify that $y=\log x$ is a solution of differential equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	**03**
	Ans	$y=\log x$	
		$\therefore \frac{dy}{dx}=\frac{1}{x}$	1
		$\therefore x\frac{dy}{dx}=1$	1
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	1
		OR	
		$y=\log x$	
		$\therefore \frac{dy}{dx}=\frac{1}{x}$	1
		$\therefore \frac{d^2y}{dx^2}=-\frac{1}{x^2}$	1
		$L.H.S.=x\frac{d^2y}{dx^2}+\frac{dy}{dx}=x\left(-\frac{1}{x^2}\right)+\frac{1}{x}$	
		$=-\frac{1}{x}+\frac{1}{x}$	
		$=0=R.H.S.$	1
	ii)	Solve: $\frac{dy}{dx}=e^{2x-y}+x^2e^{-y}$	**03**
	Ans	$\frac{dy}{dx}=e^{2x-y}+x^2e^{-y}$	
		$\therefore \frac{dy}{dx}=\left(e^{2x}\cdot e^{-y}+x^2e^{-y}\right)$	
		$\therefore \frac{dy}{dx}=e^{-y}\left(e^{2x}+x^2\right)$	
		$\therefore \frac{dy}{e^{-y}}=\left(e^{2x}+x^2\right)dx$	1
		$\therefore \int e^{y}dy=\int\left(e^{2x}+x^2\right)dx$	1
		$\therefore e^{y}=\frac{e^{2x}}{2}+\frac{x^3}{3}+c$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	A particle starting with velocity 6 m/s has an acceleration $\left(1-t^2\right)m/s^2$. When does it first comes to rest? How far has it then travelled?	**06**
	Ans	We have acceleration $=\frac{dv}{dt}=\left(1-t^2\right)$ $\therefore dv=\left(1-t^2\right)dt$	
		$\therefore \int dv=\int\left(1-t^2\right)dt$	½
		$\therefore v=t-\frac{t^3}{3}+c_1$	1
		At rest $t=0, v=6$ $\therefore c_1=6$	½
		$\therefore v=t-\frac{t^3}{3}+6$ When particle first comes to rest $, v=0$ $\therefore 0=t-\frac{t^3}{3}+6$ $\therefore 0=3t-t^3+18$	
		$\therefore t=3$, $t^2+3t+6=0$	1
		$\therefore$ Particle first time comes to rest at $t=3$ $v=\frac{dx}{dt}=t-\frac{t^3}{3}+6$ $\therefore dx=\left(t-\frac{t^3}{3}+6\right)dt$	
		$\therefore \int dx=\int\left(t-\frac{t^3}{3}+6\right)dt$	½
		$\therefore x=\frac{t^2}{2}-\frac{t^4}{12}+6t+c_2$	1
		at $t=0, x=0$ $\therefore c_2=0$	½
		$\therefore x=\frac{t^2}{2}-\frac{t^4}{12}+6t$ at $t=3$	
		$\therefore x=\frac{(3)^2}{2}-\frac{(3)^4}{12}+6(3)$	½
		$\therefore x=15.75\ m$	½
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(i)	**Attempt any <u>TWO</u> of the following:**	**12**
		Attempt the following:	**06**
		Using Trapezoidal rule , calculate the approximate value of $\int_3^8 \log_e x$ by using following table.	**03**
	Ans	(see table below)	

x	3	4	5	6	7	8
$y=\log_e x$	1.098	1.3863	1.6094	1.7918	1.9458	2.0794

Trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$$

where $h=\frac{b-a}{n}=\frac{8-3}{5}=1$ — 1

$$\int_3^8 \log_e x = \frac{1}{2}\left[(1.098+2.0794)+2(1.3863+1.6094+1.7918+1.9458)\right]$$ — 1

$$=\frac{1}{2}\left[3.1774+2(6.7333)\right]$$

$$=8.322$$ — 1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(ii)	Using Simpson`s $\frac{1}{3}^{rd}$ rule ,calculate the approximate value of $\int_0^4 e^x dx$ by using following data:	03

x	0	1	2	3	4
$y=e^x$	1	2.72	7.39	20.09	54.60

Ans

Let $y=e^x$ $a=0$, $b=4$ and $n=4$

$\therefore h=\frac{b-a}{n}=\frac{4-0}{4}=1$ — 1

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...)+2(y_2+y_4+...)\right]$$

$$\int_0^4 e^x dx=\frac{1}{3}\left[(1+54.60)+4(2.72+20.09)+2(7.39)\right]$$ — 1

$$=53.8733$$ — 1

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b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Trapezoidal rule,taking n = 4.Hence,obtain approximate value of π. — 06

Ans

Trapezoidal rule

$$\int_a^b f(x)dx=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$$ — 1

where $h=\frac{b-a}{n}=\frac{1-0}{4}=\frac{1}{4}$ or 0.25 — 1

x	0	0.25	0.5	0.75	1
$y=f(x)$	1	0.9412	0.8	0.64	0.5

$$\int_0^1 \frac{dx}{1+x^2}=\frac{0.25}{2}\left[(1+0.5)+2(0.9412+0.8+0.64)\right]$$ — 1

$$\therefore \int_0^1 \frac{dx}{1+x^2}=0.7828-----(1)$$ — 1

$$\int_0^1 \frac{dx}{1+x^2}=\left[\tan^{-1}x\right]_0^1=\tan^{-1}(1)-\tan^{-1}(0)=\frac{\pi}{4}-----(2)$$ — 1

From (1) and (2)

$$\frac{\pi}{4}=0.7828$$ — 1

$$\therefore \pi=3.1312$$

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	Evaluate $\int_0^{\pi/2} \sqrt{\cos x}dx$ using Simpson's $\frac{3}{8}$ rule with $n=8$	**06**
	Ans	$h=\frac{b-a}{n}=\frac{\frac{\pi}{2}-0}{8}=\frac{\pi}{16}$	1

x	0	$\pi/16$	$\pi/8$	$3\pi/16$	$\pi/4$	$5\pi/16$	$3\pi/8$	$7\pi/16$	$\pi/2$
y	1	0.9903	09612	0.9118	0.8409	0.7454	0.6186	0.4417	0

2

Using Simpson's $3/8^{th}$ rule.

$$\int_a^b f(x)dx=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+...)+2(y_3+y_6+...)\right]$$

$$\therefore \int_0^{\pi/2}\sqrt{\cos x}\,dx=\frac{3\left(\pi/16\right)}{8}\left[(1+0)+3(0.9903+0.9612+0.8409+0.7454+0.4417)+2(0.9118+0.6186)\right]$$ 1

$$\therefore \int_0^{\pi/2}\sqrt{\cos x}\,dx=0.3749\pi=1.178$$ 2

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.

22201

11819

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory.*

(2) Illustrate your answers with neat sketches wherever necessary.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any FIVE of the following : 10

(a) Define odd and even function with suitable example.

(b) If $f(x) = \frac{x^2 + 9}{\sqrt{x - 3}}$, find f(4) + f(5).

(c) Find $\frac{dy}{dx}$ if $y = (3a)^x + x^{(\log 3)} + x^a + a^a$

(d) Evaluate $\int x^2 \cdot \log x \, dx$

(e) Evaluate $\int \frac{dx}{x^2 + 4x + 5}$

(f) Find the area bounded by the curve $y = \sin x$, x axis and the ordinate $x = 0$, $x = \frac{\pi}{2}$.

(g) State the trapezoidal rule of numerical integration.

2. Attempt any THREE of the following : 12

(a) Find $\frac{dy}{dx}$ if $x^2 + y^2 + xy - y = 0$ at $(1, 2)$

(b) If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

(c) The rate of working of an engine is given by the expression $10\ V + \frac{4000}{V}$, where 'V' is the speed of the engine. Find the speed at which the rate of working is the least.

(d) A telegraph wire hangs in the form of a curve $y = a \cdot \log\left[\sec\left(\frac{x}{a}\right)\right]$. Where 'a' is constant. Show that the curvature at any point is $\frac{1}{a}\cos\left(\frac{x}{a}\right)$.

3. Attempt any THREE of the following : 12

(a) Find equation of tangent to curve $x = \frac{1}{t}$, $y = 1 - \frac{1}{t}$ when $t = 2$.

(b) Find $\frac{dy}{dx}$ if $y = x^x + x^{\sqrt{x}}$

(c) Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left[\frac{x}{\sqrt{1 - x^2}}\right]$

(d) Evaluate $\int \frac{\sec^2 x}{(1 + \tan x)(3 + \tan x)}\, dx$.

4. Attempt any THREE of the following : 12

(a) Evaluate $\int \frac{1}{x[9 + (\log_e x)^2]} dx$

(b) Evaluate $\int \frac{1}{2 \sin x + 3 \cos x} dx$

(c) Evaluate $\int \sec^3 x \, dx$

(d) Evaluate $\int \frac{2x^2 + 5}{(x - 1)(x + 2)(x + 3)} dx$

(e) Evaluate $\int_0^{\pi/2} \frac{1}{1 + \sqrt{\cot x}} dx$

5. Attempt any TWO of the following : 12

(a) Find area of the region by the parabolas.

$y^2 = 9x$ and $x^2 = 9y$

(b) Attempt the following :

(i) Form a differential equation by eliminating arbitrary constant. If $y = A \sin x + B \cos x$.

(ii) Solve $(1 + x^3)dy - x^2y \, dx = 0$

(c) An electrical circuit containing an inductance L henries resistance R in series with an electromotive force. E sin ωt satisfies the equation $L\frac{di}{dt} + Ri = E \sin \omega t$.

Find the value of the current at any time t, if initially there is no current.

6. Attempt any TWO of the following : 12

(a) (i) Using trapezoidal rule, calculate the approximate value of $\int_0^4 \sqrt{x}\,dx$, given by

x	0	1	2	3	4
$y = \sqrt{x}$	0	1	1.4142	1.7321	2

(ii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule by using following data :

x	0	1	2	3	4	5	6
$y = \frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.588	0.0385	0.027

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}^{rd}$ rule by taking 6 sub intervals.

(c) Using Simpson's $\frac{3}{8}^{th}$ rule to find $\int_0^{0.6} e^{-x^2}\,dx$ by taking seven ordinates.

WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of following:**	**10**
	a)	Define odd and even function with suitable example.	**02**
	Ans	If $f(-x)=-f(x)$ then the function is an odd function	½
		e.g. $f(x)=x^3+x$	
		$\therefore f(-x)=(-x)^3+(-x)$	
		$=-(x^3+x)$	
		$=-f(x)$	½
		If $f(-x)=f(x)$ then the function is an even function	½
		e.g. $f(x)=x^2+1$	
		$\therefore f(-x)=(-x)^2+1$	
		$=x^2+1$	
		$=f(x)$	½
		--	
	b)	If $f(x)=\dfrac{x^2+9}{\sqrt{x-3}}$, find $f(4)+f(5)$.	**02**
	Ans	$f(4)+f(5)=\left(\dfrac{4^2+9}{\sqrt{4-3}}\right)+\left(\dfrac{5^2+9}{\sqrt{5-3}}\right)$	½+½
		$=25+\dfrac{34}{\sqrt{2}}=49.042$	1
		OR	
		$f(4)=\dfrac{4^2+9}{\sqrt{4-3}}=25$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	b)	$f(5)=\frac{5^2+9}{\sqrt{5-3}}=\frac{34}{\sqrt{2}}$	½
		$\therefore f(4)+f(5)$	
		$=25+\frac{34}{\sqrt{2}}$	1
		$=49.042$	
		--	
	c)	Find $\frac{dy}{dx}$ if $y=(3a)^x+x^{(\log 3)}+x^a+a^a$	**02**
	Ans	$\therefore \frac{dy}{dx}=(3a)^x\log 3a+\log 3\cdot x^{(\log 3)-1}+a\cdot x^{a-1}+0$	½+½+½+½
		$=(3a)^x\log 3a+\log 3\cdot x^{(\log 3)-1}+a\cdot x^{a-1}$	
		OR	
		$\because y=(3a)^x+x^{(\log 3)}+x^a+a^a$	
		$\because y=3^x a^x+x^{(\log 3)}+x^a+a^a$	
		$\therefore \frac{dy}{dx}=3^x a^x\log a+a^x 3^x\log 3+\log 3\cdot x^{(\log 3)-1}+a\cdot x^{a-1}+0$	½+½+½+½
		$=3^x a^x(\log a+\log 3)+\log 3\cdot x^{(\log 3)-1}+a\cdot x^{a-1}$	
		--	
	d)	Evaluate $\int x^2\cdot\log x dx$	**02**
	Ans	$\int x^2\cdot\log x dx=\log x\int x^2 dx-\int\left[\int x^2 dx\cdot\frac{d}{dx}\log x\right]dx$	½
		$=\log x\cdot\frac{x^3}{3}-\int\frac{x^3}{3}\cdot\frac{1}{x}dx$	½
		$=\log x\cdot\frac{x^3}{3}-\frac{1}{3}\int x^2 dx$	
		$=\log x\cdot\frac{x^3}{3}-\frac{1}{3}\cdot\frac{x^3}{3}+c$	
		$=\log x\cdot\frac{x^3}{3}-\frac{x^3}{9}+c$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	e)	Evaluate $\int \frac{dx}{x^2+4x+5}$	**02**
	Ans	$\int \frac{dx}{x^2+4x+5}$	
		Third term $=\frac{(4x)^2}{4\times x^2}=4$	½
		$=\int \frac{dx}{x^2+4x+4-4+5}$	½
		$=\int \frac{dx}{(x+2)^2+1}$	½
		$=\frac{1}{1}\tan^{-1}\left(\frac{x+2}{1}\right)+c$	
		$=\tan^{-1}(x+2)+c$	½
	f)	Find the area bounded by the curve $y=\sin x, x-$axis and the ordinate $x=0, x=\frac{\pi}{2}$	**02**
	Ans	Area $A=\int_a^b ydx$	
		$=\int_0^{\frac{\pi}{2}} \sin xdx$	1
		$=[-\cos x]_0^{\pi/2}$	½
		$=-[0-1]$	
		$=1$	½
	g)	State the trapezoidal rule of numerical integration.	**02**
	Ans	Trapezoidal rule	
		$\int_a^b f(x)dx=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	2
		where $h=\frac{b-a}{n}$	
2.		**Attempt any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $x^2+y^2+xy-y=0$ at $(1,2)$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)Ans	$\because x^2+y^2+xy-y=0$	
		$\therefore 2x+2y\frac{dy}{dx}+x\frac{dy}{dx}+y-\frac{dy}{dx}=0$	1
		$\therefore 2x+y+(2y+x-1)\frac{dy}{dx}=0$	
		$\therefore (2y+x-1)\frac{dy}{dx}=-(2x+y)$	1
		$\therefore \frac{dy}{dx}=\frac{-(2x+y)}{2y+x-1}$	1
		at $(1,2)$	
		$\frac{dy}{dx}=\frac{-(2(1)+2)}{2(2)+1-1}=-1$	1
		--	
	b)	If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$	**04**
	Ans	$\because x=a(\cos t+t\sin t)$	
		$\therefore \frac{dx}{dt}=a(-\sin t+t\cos t+\sin t)$	
		$=at\cos t$	1
		$\because y=a(\sin t-t\cos t)$	
		$\therefore \frac{dy}{dt}=a(\cos t+t\sin t-\cos t)$	
		$=at\sin t$	1
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	
		$\therefore \frac{dy}{dx}=\frac{at\sin t}{at\cos t}$	
		$=\tan t$	1
		at $t=\frac{\pi}{4}$	
		$\frac{dy}{dx}=\tan\frac{\pi}{4}$	
		$=1$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	The rate of working of an engine is given by the expression $10V+\frac{4000}{V}$, where 'V' is the speed of the engine.Find the speed at which the rate of working is the least.	**04**
	Ans	The rate of working is, $W=10V+\frac{4000}{V}$	
		$\therefore \frac{dW}{dV}=10-\frac{4000}{V^2}$	½
		$\therefore \frac{d^2W}{dV^2}=\frac{8000}{V^3}$	½
		Consider $\frac{dW}{dV}=0$	
		$\therefore 10-\frac{4000}{V^2}=0$	½
		$\therefore 10=\frac{4000}{V^2}$	
		$\therefore V^2=400$	
		$\therefore V=20,-20$	1
		at $V=20$	
		$\therefore \frac{d^2W}{dV^2}=\frac{8000}{(20)^3}=1>0$	1
		$\therefore$ The speed is $V=20$ at which the rate of working is least	½
		--	
	d)	A telegraph wire hangs in the form of a curve $y=a\cdot\log\left[\sec\left(\frac{x}{a}\right)\right]$. Where '$a$' is constant.Show that the curvature at any point is $\frac{1}{a}\cos\left(\frac{x}{a}\right)$.	**04**
	Ans	$y=a\log\left(\sec\left(\frac{x}{a}\right)\right)$	
		$\therefore \frac{dy}{dx}=a\frac{1}{\sec\left(\frac{x}{a}\right)}\sec\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)$	
		$\therefore \frac{dy}{dx}=\tan\left(\frac{x}{a}\right)$	1
		$\therefore \frac{d^2y}{dx^2}=\sec^2\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)$	1
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	d)	$\therefore \rho = \dfrac{\left[1+\tan^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)\left(\frac{1}{a}\right)}$	1
		$\therefore \rho = \dfrac{a\left[\sec^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right)}$	
		$\therefore \rho = \dfrac{a\sec^3\left(\frac{x}{a}\right)}{\sec^2\left(\frac{x}{a}\right)}$	
		$\therefore \rho = a\sec\left(\frac{x}{a}\right)$	
		$\therefore \text{curvature} = \dfrac{1}{\rho} = \dfrac{1}{a}\cos\left(\frac{x}{a}\right)$	1
		--	
3.		**Attempt any THREE of the following:**	**12**
	a)	Find equation of tangent to curve $x=\frac{1}{t}, y=1-\frac{1}{t}$ when $t=2$.	**04**
	Ans	$x=\frac{1}{t}$ $\quad y=1-\frac{1}{t}$	
		$\therefore \frac{dx}{dt}=\frac{-1}{t^2}$ $\quad \frac{dy}{dt}=\frac{1}{t^2}$	½+½
		$\therefore \dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$	
		$=\dfrac{\frac{1}{t^2}}{\frac{-1}{t^2}}$	
		$=-1$	
		$\therefore$ slope of tangent $=m=-1$	1
		when $t=2$	
		$x=\frac{1}{t}=\frac{1}{2}$ $\quad y=1-\frac{1}{t}=1-\frac{1}{2}=\frac{1}{2}$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	a)	$\therefore$ Point is $\left(\frac{1}{2},\frac{1}{2}\right)$	1
		$\therefore$ Equation of tangent is,	
		$y-\frac{1}{2}=-1\left(x-\frac{1}{2}\right)$	
		$\therefore y-\frac{1}{2}=-x+\frac{1}{2}$	
		$\therefore x+y-1=0$	1
		--	
	b)	Find $\frac{dy}{dx}$ if $y=x^x+x^{\sqrt{x}}$	**04**
	Ans	Let $u=x^x$	
		$\therefore \log u=\log x^x$	½
		$=x\log x$	
		$\therefore \frac{1}{u}\frac{du}{dx}=x\cdot\frac{1}{x}+\log x$	
		$=1+\log x$	
		$\therefore \frac{du}{dx}=u\left(1+\log x\right)$	
		$=x^x\left(1+\log x\right)$	1
		Let $v=x^{\sqrt{x}}$	
		$\therefore \log v=\log x^{\sqrt{x}}$	½
		$=\sqrt{x}\log x$	
		$\therefore \frac{1}{v}\frac{dv}{dx}=\sqrt{x}\cdot\frac{1}{x}+\log x\cdot\frac{1}{2\sqrt{x}}$	
		$=\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}$	
		$\therefore \frac{dv}{dx}=v\left(\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}\right)$	
		$=x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}\right)$	1
		$\because y=u+v$	
		$\therefore \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$	
		$=x^x\left(1+\log x\right)+x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}\right)$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right]$	**04**
	Ans	$y=\tan^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right]$	
		Let $x=\sin\theta \quad \therefore \theta=\sin^{-1}x$	1
		$\therefore y=\tan^{-1}\left[\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right]$	
		$=\tan^{-1}(\tan\theta)$	1
		$=\theta$	1
		$=\sin^{-1}x$	
		$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$	1
		OR	
		Let $x=\cos\theta \quad \therefore \theta=\cos^{-1}x$	1
		$\therefore y=\tan^{-1}\left[\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}\right]$	
		$=\tan^{-1}(\cot\theta)$	1
		$=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\theta\right)\right)$	½
		$=\frac{\pi}{2}-\theta$	½
		$=\frac{\pi}{2}-\cos^{-1}x$	
		$\therefore \frac{dy}{dx}=-\frac{-1}{\sqrt{1-x^2}}$	
		$=\frac{1}{\sqrt{1-x^2}}$	1
		OR	
		$y=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$	
		$\therefore \frac{dy}{dx}=\frac{1}{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2}\cdot\frac{\sqrt{1-x^2}(1)-x\cdot\frac{1}{2\sqrt{1-x^2}}(-2x)}{1-x^2}$	2

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	c)	$= \frac{1-x^2}{1-x^2+x^2} \cdot \frac{\sqrt{1-x^2}+\frac{x^2}{\sqrt{1-x^2}}}{1-x^2}$	
		$= \sqrt{1-x^2}+\frac{x^2}{\sqrt{1-x^2}}$	1
		$= \frac{1-x^2+x^2}{\sqrt{1-x^2}}$	
		$= \frac{1}{\sqrt{1-x^2}}$	1
		--	
	d)	Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$	**04**
	Ans	$\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$	
		Let $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	1
		$= \int \frac{1}{(1+t)(3+t)} dt$	
		Consider	
		$\frac{1}{(1+t)(3+t)} = \frac{A}{1+t}+\frac{B}{3+t}$	½
		$\therefore 1 = A(3+t)+B(1+t)$	
		Put $t=-1 \quad \therefore A = \frac{1}{2}$	½
		Put $t=-3 \quad \therefore B = \frac{-1}{2}$	½
		$\therefore \frac{1}{(1+t)(3+t)} = \frac{\frac{1}{2}}{1+t}+\frac{-\frac{1}{2}}{3+t}$	
		$\therefore \int \frac{1}{(1+t)(3+t)} dt = \int \left(\frac{\frac{1}{2}}{1+t}+\frac{-\frac{1}{2}}{3+t} \right) dt$	
		$= \frac{1}{2}\log(1+t)-\frac{1}{2}\log(3+t)+c$	½+½
		$= \frac{1}{2}\log\left(\frac{1+t}{3+t}\right)+c$	
		$= \frac{1}{2}\log\left(\frac{1+\tan x}{3+\tan x}\right)+c$	½

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate : $\int \frac{1}{x\left[9+\left(\log_e x\right)^2\right]}dx$	**04**
	Ans	$\int \frac{1}{x\left[9+\left(\log_e x\right)^2\right]}dx$	
		Let $\log_e x = t$	
		$\therefore \frac{1}{x}dx = dt$	1
		$= \int \frac{1}{9+t^2}dt$	
		$= \int \frac{1}{3^2+t^2}dt$	1
		$= \frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$	1
		$= \frac{1}{3}\tan^{-1}\left(\frac{\log_e x}{3}\right)+c$	1
		--	
	b)	Evaluate: $\int \frac{1}{2\sin x+3\cos x}dx$	**04**
	Ans	$\int \frac{1}{2\sin x+3\cos x}dx$	
		Let $\tan\frac{x}{2}=t$	
		$\therefore \sin x=\frac{2t}{1+t^2}, \cos x=\frac{1-t^2}{1+t^2}, dx=\frac{2dt}{1+t^2}$	1
		$= \int \frac{1}{2\left(\frac{2t}{1+t^2}\right)+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	
		$= \int \frac{1}{4t+3-3t^2}\cdot 2dt$	
		$= \int \frac{1}{-\left(3t^2-4t-3\right)}\cdot 2dt$	½
		Third term $= \frac{(-4t)^2}{4\times 3t^2} = \frac{4}{3}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$=-2\int\frac{1}{3t^2-4t+\frac{4}{3}-\frac{4}{3}-3}dt$	
		$=-2\int\frac{1}{\left(\sqrt{3}t-\frac{2}{\sqrt{3}}\right)^2-\left(\sqrt{\frac{13}{3}}\right)^2}dt$	½
		$=-2\cdot\frac{1}{2\sqrt{\frac{13}{3}}}\log\left(\frac{\sqrt{3}t-\frac{2}{\sqrt{3}}-\sqrt{\frac{13}{3}}}{\sqrt{3}t-\frac{2}{\sqrt{3}}+\sqrt{\frac{13}{3}}}\right)\cdot\frac{1}{\sqrt{3}}+c$	1
		$=\frac{-1}{\sqrt{13}}\log\left(\frac{\sqrt{3}\tan\frac{x}{2}-\frac{2}{\sqrt{3}}-\sqrt{\frac{13}{3}}}{\sqrt{3}\tan\frac{x}{2}-\frac{2}{\sqrt{3}}+\sqrt{\frac{13}{3}}}\right)+c$	½
		$=\frac{-1}{\sqrt{13}}\log\left(\frac{3\tan\frac{x}{2}-2-\sqrt{13}}{3\tan\frac{x}{2}-2+\sqrt{13}}\right)+c$	
		OR	
		Let $\tan\frac{x}{2}=t$	
		$\therefore\sin x=\frac{2t}{1+t^2},\cos x=\frac{1-t^2}{1+t^2},dx=\frac{2dt}{1+t^2}$	1
		$=\int\frac{1}{2\left(\frac{2t}{1+t^2}\right)+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	
		$=\int\frac{1}{4t+3-3t^2}\cdot 2dt$	
		$=\int\frac{1}{-3\left(t^2-\frac{4}{3}t-1\right)}\cdot 2dt$	½
		$=\frac{-2}{3}\int\frac{1}{t^2-\frac{4}{3}t-1}dt$	
		Third term $=\frac{\left(\frac{-4}{3}t\right)^2}{4t^2}=\frac{4}{9}$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	$=\frac{-2}{3}\int\frac{1}{t^2-\frac{4}{3}t+\frac{4}{9}-\frac{4}{9}-1}dt$	
		$=\frac{-2}{3}\int\frac{1}{\left(t-\frac{2}{3}\right)^2-\left(\frac{\sqrt{13}}{3}\right)^2}dt$	½
		$=\frac{-2}{3}\cdot\frac{1}{2\frac{\sqrt{13}}{3}}\log\left(\frac{t-\frac{2}{3}-\frac{\sqrt{13}}{3}}{t-\frac{2}{3}+\frac{\sqrt{13}}{3}}\right)+c$	1
		$=\frac{-1}{\sqrt{13}}\log\left(\frac{\tan\frac{x}{2}-\frac{2}{3}-\frac{\sqrt{13}}{3}}{\tan\frac{x}{2}-\frac{2}{3}+\frac{\sqrt{13}}{3}}\right)+c$	½
		$=\frac{-1}{\sqrt{13}}\log\left(\frac{3\tan\frac{x}{2}-2-\sqrt{13}}{3\tan\frac{x}{2}-2+\sqrt{13}}\right)+c$	
	c)	Evaluate : $\int\sec^3 x\, dx$	**04**
	Ans	Let $I=\int\sec^3 x\, dx$	
		$=\int\sec^2 x\cdot\sec x\, dx$	½
		$=\sec x\int\sec^2 x\, dx-\int\left[\int\sec^2 x\, dx\cdot\frac{d}{dx}\sec x\right]dx$	½
		$=\sec x\tan x-\int[\tan x\cdot\sec x\cdot\tan x]dx$	½
		$=\sec x\tan x-\int\tan^2 x\cdot\sec x dx$	
		$=\sec x\tan x-\int(\sec^2 x-1)\cdot\sec x dx$	½
		$=\sec x\tan x-\int(\sec^3 x-\sec x)dx$	
		$=\sec x\tan x-\int\sec^3 x dx+\int\sec x dx$	½
		$I=\sec x\tan x-I+\log(\sec x+\tan x)+c$	1
		$\therefore 2I=\sec x\tan x+\log(\sec x+\tan x)+c$	
		$\therefore I=\frac{1}{2}(\sec x\tan x+\log(\sec x+\tan x))+c$	½
	d)	Evaluate $\int\frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$	**04**

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	d)Ans	$\int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$	
		Consider $\frac{2x^2+5}{(x-1)(x+2)(x+3)} = \frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x+3}$	½
		$\therefore 2x^2+5 = A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2)$	
		Put $x=1 \Rightarrow$ $2(1)^2+5 = A(1+2)(1+3)$ $\therefore A = \frac{7}{12}$	½
		Put $x=-2 \Rightarrow$ $2(-2)^2+5 = B(-2-1)(-2+3)$ $\therefore B = \frac{-13}{3}$	½
		Put $x=-3 \Rightarrow$ $2(-3)^2+5 = C(-3-1)(-3+2)$ $\therefore C = \frac{23}{4}$	½
		$\therefore \frac{2x^2+5}{(x-1)(x+2)(x+3)} = \frac{\frac{7}{12}}{x-1}+\frac{\frac{-13}{3}}{x+2}+\frac{\frac{23}{4}}{x+3}$	
		$\therefore \int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx = \int \left(\frac{\frac{7}{12}}{x-1}+\frac{\frac{-13}{3}}{x+2}+\frac{\frac{23}{4}}{x+3} \right) dx$	½
		$= \frac{7}{12}\log(x-1) - \frac{13}{3}\log(x+2) + \frac{23}{4}\log(x+3) + c$	½+½+½
		--	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}}\,dx$	**04**
	Ans	Let $I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}}\,dx$	
		$= \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\frac{\cos x}{\sin x}}}dx$	½
		$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ -------------------(1)	½
		$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$	1
		$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ -------------------(2)	½
		Add (1) and (2)	
		$I + I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$	
		$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½
		$2I = \int_0^{\frac{\pi}{2}} 1dx$	
		$2I = \left[x\right]_0^{\frac{\pi}{2}}$	½
		$2I = \frac{\pi}{2} - 0$	
		$I = \frac{\pi}{4}$	½
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.		**Attempt any TWO of the following:**	**12**
	a)	Find area of the region by the parabolas, $y^2 = 9x$ and $x^2 = 9y$	**06**
	Ans	$y^2 = 9x$ ---------(1) $x^2 = 9y$ $\therefore y = \frac{x^2}{9}$ $\therefore$ equ. (1) $\Rightarrow \left(\frac{x^2}{9}\right)^2 = 9x$ $\therefore \frac{x^4}{81} = 9x$ $\therefore x^4 = 729x$ $\therefore x^4 - 729x = 0$ $\therefore x\left(x^3 - 9^3\right) = 0$ $\therefore x = 0, 9$	1
		Area $A = \int_a^b \left(y_1 - y_2\right) dx$ $\therefore A = \int_0^9 \left(3\sqrt{x} - \frac{x^2}{9}\right) dx$	1
		$\therefore A = \left(\frac{3x^{3/2}}{\frac{3}{2}} - \frac{x^3}{27}\right)_0^9$	2
		$= \left(\frac{3(9)^{3/2}}{\frac{3}{2}} - \frac{9^3}{27}\right) - 0$	1
		$\therefore A = 27$	1
		--	
	b)	Attempt the following:	**06**
	(i)	Form a differential equation by eliminating arbitrary constant.If $y = A\sin x + B\cos x$	**03**
	Ans	$y = A\sin x + B\cos x$ $\therefore \frac{dy}{dx} = A\cos x - B\sin x$	1
		$\therefore \frac{d^2y}{dx^2} = A\left(-\sin x\right) - B\cos x$ $= -\left(A\sin x + B\cos x\right) = -y$	1
		$\therefore \frac{d^2y}{dx^2} + y = 0$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)(ii)	Solve $\left(1+x^3\right)dy-x^2ydx=0$	**03**
	Ans	$\because \left(1+x^3\right)dy-x^2ydx=0$	
		$\therefore \left(1+x^3\right)dy=x^2ydx$	
		$\therefore \frac{dy}{y}=\frac{x^2dx}{1+x^3}$	1
		$\therefore$ Solution is,	
		$\int\frac{dy}{y}=\int\frac{x^2}{1+x^3}dx$	
		$=\frac{1}{3}\int\frac{3x^2}{1+x^3}dx$	1
		$\therefore \log y=\frac{1}{3}\log\left(1+x^3\right)+c$	1
		--	
	c)	An electrical circuit containing an inductance L henries resistance R in series with an electromotive force. $E\sin\omega t$ satisfies the equation $L\frac{di}{dt}+Ri=E\sin\omega t$. Find the value of the current at any time t, if initially there is no current.	**06**
	Ans	$L\frac{di}{dt}+Ri=E\sin\omega t$	
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}\sin\omega t$	
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{R}{L}$ and $Q=\frac{E}{L}\sin\omega t$	
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is	
		$i\cdot IF=\int Q\cdot IFdt+c$	
		$i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}\sin\omega te^{\frac{R}{L}t}dt+c$	1
		$=\frac{E}{L}\int\sin\omega te^{\frac{R}{L}t}dt+c----(1)$	
		Let I= $\int\sin\omega te^{\frac{R}{L}t}dt$	
		$=\sin\omega t\int e^{\frac{R}{L}t}dt-\int\left[\int e^{\frac{R}{L}t}dt\cdot\frac{d}{dt}\sin\omega t\right]dt$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$=\sin\omega t\cdot\dfrac{e^{\frac{R}{L}t}}{\frac{R}{L}}-\int\dfrac{e^{\frac{R}{L}t}}{\frac{R}{L}}\cdot\cos\omega t\cdot\omega dt$	
		$=\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{L\omega}{R}\int e^{\frac{R}{L}t}\cos\omega tdt$	
		$=\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{L\omega}{R}\left[\cos\omega t\cdot\int e^{\frac{R}{L}t}dt-\int\left[\int e^{\frac{R}{L}t}dt\cdot\dfrac{d}{dt}\cos\omega t\right]dt\right]$	½
		$=\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{L\omega}{R}\left[\cos\omega t\cdot\dfrac{e^{\frac{R}{L}t}}{\frac{R}{L}}-\int\left[\dfrac{e^{\frac{R}{L}t}}{\frac{R}{L}}\cdot(-\sin\omega t)\omega\right]dt\right]$	
		$I=\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}-\dfrac{\omega^2L^2}{R^2}\int e^{\frac{R}{L}t}\sin\omega tdt$	
		$I=\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}-\dfrac{\omega^2L^2}{R^2}I$	
		$\left(1+\dfrac{\omega^2L^2}{R^2}\right)I=\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}$	
		$I=\dfrac{R^2}{R^2+\omega^2L^2}\left[\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}\right]$	
		$\therefore$ equation (1) becomes	
		$ie^{\frac{R}{L}t}=\dfrac{E}{L}\dfrac{R^2}{R^2+\omega^2L^2}\left[\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}\right]+c$	1
		initially $i=0$	
		$\therefore$ when $t=0,i=0$	
		$\therefore 0=\dfrac{E}{L}\dfrac{R^2}{R^2+\omega^2L^2}\left[\dfrac{-\omega L^2}{R^2}\right]+c$	
		$\therefore 0=\dfrac{-E\omega L}{R^2+\omega^2L^2}+c$	
		$\therefore c=\dfrac{E\omega L}{R^2+\omega^2L^2}$	1
		$\therefore ie^{\frac{R}{L}t}=\dfrac{E}{L}\dfrac{R^2}{R^2+\omega^2L^2}\left[\dfrac{L}{R}\sin\omega te^{\frac{R}{L}t}-\dfrac{\omega L^2}{R^2}\cos\omega te^{\frac{R}{L}t}\right]+\dfrac{E\omega L}{R^2+\omega^2L^2}$	
		$\therefore i=\dfrac{ER}{R^2+\omega^2L^2}\left[\sin\omega t-\dfrac{\omega L}{R}\cos\omega t\right]+\dfrac{E\omega L}{R^2+\omega^2L^2}e^{\frac{-R}{L}t}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	OR	
		$L\frac{di}{dt}+Ri=E\sin\omega t$	
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}\sin\omega t$	
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{R}{L}$ and $Q=\frac{E}{L}\sin\omega t$	
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is	
		$i\cdot IF=\int Q\cdot IFdt+c$	
		$i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}\sin\omega t e^{\frac{R}{L}t}dt+c$	1
		$=\frac{E}{L}\int\sin\omega t e^{\frac{R}{L}t}dt+c-----(1)$	
		$\left[\because \int e^{ax}\sin bxdx=\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)\right]$	
		$\therefore \int e^{\frac{R}{L}t}\sin\omega tdt=\frac{e^{\frac{R}{L}t}}{\frac{R^2}{L^2}+\omega^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]$	1
		$\therefore$ equation (1) becomes	
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\frac{e^{\frac{R}{L}t}}{\frac{R^2}{L^2}+\omega^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]+c$	1
		$=\frac{ELe^{\frac{R}{L}t}}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]+c$	
		initially $i=0, \therefore$ when $t=0, i=0$	
		$\therefore 0=\frac{EL}{R^2+\omega^2L^2}[-\omega]+c$	
		$\therefore c=\frac{\omega EL}{R^2+\omega^2L^2}$	1
		$\therefore i\cdot e^{\frac{R}{L}t}=\frac{ELe^{\frac{R}{L}t}}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]+\frac{\omega EL}{R^2+\omega^2L^2}$	
		$\therefore i=\frac{EL}{R^2+\omega^2L^2}\left[\frac{R}{L}\sin\omega t-\omega\cos\omega t\right]+\frac{\omega EL}{R^2+\omega^2L^2}e^{-\frac{R}{L}t}$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	OR	
		$L\frac{di}{dt}+Ri=E\sin\omega t$	
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}\sin\omega t$	
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{R}{L}$ and $Q=\frac{E}{L}\sin\omega t$	
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is	
		$i\cdot IF=\int Q\cdot IFdt+c$	
		$i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}\sin\omega te^{\frac{R}{L}t}dt+c$	1
		$=\frac{E}{L}\int\sin\omega te^{\frac{R}{L}t}dt+c-----(1)$	
		$\left[\because\int e^{ax}\sin bxdx=\frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx-\tan^{-1}\left(\frac{b}{a}\right)\right)\right]$	
		$\therefore$ equation (1) becomes,	
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\frac{e^{\frac{R}{L}t}}{\sqrt{\frac{R^2}{L^2}+\omega^2}}\sin\left(\omega t-\tan^{-1}\left(\frac{\omega}{\frac{R}{L}}\right)\right)+c$	1
		$i=\frac{E}{\sqrt{R^2+L^2\omega^2}}\sin\left(\omega t-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)+ce^{\frac{-R}{L}t}$	1
		initially $i=0$	
		$\therefore$ when $t=0,i=0$	
		$\therefore 0=\frac{E}{\sqrt{R^2+L^2\omega^2}}\sin\left(-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)+c$	
		$\therefore c=\frac{-E}{\sqrt{R^2+L^2\omega^2}}\sin\left(-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)$	1
		$\therefore i=\frac{E}{\sqrt{R^2+L^2\omega^2}}\left(\sin\left(\omega t-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)-\sin\left(-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)e^{\frac{-R}{L}t}\right)$	1
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.		**Attempt any TWO of the following:**	**12**
	a)(i)	Using trapezoidal rule, evaluate the approximate value of $\int_0^4 \sqrt{x}\, dx,$ given by	
		<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$y=\sqrt{x}$</td><td>0</td><td>1</td><td>1.4142</td><td>1.7321</td><td>2</td></tr></table>	**03**
	Ans	$\int_a^b f(x)dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	
		$a=0, b=4$ and $h=1$	
		$\therefore \int_0^4 \sqrt{x}dx = \frac{1}{2}\left[(0+2)+2(1+1.4142+1.7321)\right]$	2
		$= 5.1463$	1
	a)(ii)	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule by using following data:	
		<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$y=\frac{1}{1+x^2}$</td><td>1</td><td>0.5</td><td>0.2</td><td>0.1</td><td>0.588</td><td>0.0385</td><td>0.027</td></tr></table>	**03**
	Ans	$\int_a^b f(x)dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	
		$a=0, b=6$ and $h=1$	
		$\therefore \int_0^6 \frac{1}{1+x^2}dx = \frac{1}{2}\left[(1+0.027)+2(0.5+0.2+0.1+0.588+0.0385)\right]$	2
		$=1.94$	1
	b)	Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ by Simpson's $1/3^{rd}$ rule by taking 6 sub intervals.	**06**
	Ans	Let $y=\frac{1}{1+x^2}$ $a=0, b=1$ and $n=6$	
		$\therefore h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$	1
		<table><tr><td>x</td><td>0</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{2}$</td><td>$\frac{2}{3}$</td><td>$\frac{5}{6}$</td><td>1</td></tr><tr><td>$y=\frac{1}{1+x^2}$</td><td>1</td><td>$\frac{36}{37}$</td><td>$\frac{9}{10}$</td><td>$\frac{4}{5}$</td><td>$\frac{9}{13}$</td><td>$\frac{36}{61}$</td><td>$\frac{1}{2}$</td></tr></table>	2
		Using Simpson's $1/3^{rd}$ rule	
		$\int_a^b f(x)dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	$\therefore \int_0^1 f(x)dx = \frac{1/6}{3}\left[\left(1+\frac{1}{2}\right)+4\left(\frac{36}{37}+\frac{4}{5}+\frac{36}{61}\right)+2\left(\frac{9}{10}+\frac{9}{13}\right)\right]$	2
		$\therefore \int_0^1 \frac{1}{1+x^2}dx = 0.7854$	1
		OR	
		Let $y=\frac{1}{1+x^2}$ $a=0, b=1$ and $n=6$	
		$\therefore h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}=0.1667$	1
		(table below)	2
		Using Simpson's 1/3rd rule	
		$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$	
		$\therefore \int_0^1 f(x)dx=\frac{0.1667}{3}\left[(1+0.5)+4(0.9730+0.8+0.5901)+2(0.9+0.6922)\right]$	2
		$\int_0^1 \frac{1}{1+x^2}dx=0.7855$	1
	c)	Using Simpson's 3/8th rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates.	**06**
	Ans	Here $n=6$	
		$y=e^{-x^2}$ $a=0,$ $b=0.6$	
		$\therefore h=\frac{b-a}{n}=\frac{0.6-0}{6}=0.1$	1
		(table below)	2
		Using Simpson's 3/8th rule.	
		$\int_a^b f(x)dx=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4...+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$	
		$\therefore \int_0^{0.6} e^{-x^2}dx=\frac{3(0.1)}{8}\left[(1+0.6977)+3(0.99+0.9608+0.8521+0.7788)+2(0.9139)\right]$	2
		$\therefore \int_0^{0.6} e^{-x^2}dx=0.5351$	1

Table for 6 b):

x	0	0.1667	0.3334	0.5001	0.6668	0.8335	1
$y=\frac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6922	0.5901	0.5

Table for 6 c):

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$y=e^{-x^2}$	1	0.99	0.9608	0.9139	0.8521	0.7788	0.6977

WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Q. No.	Sub Q. N.	Answer	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*	

22201

21718

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory.*

(2) Answer each next main Question on a new page.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any FIVE of following : 10

(a) If $f(x) = x^4 - 2x + 7$, find f(0) + f(2).

(b) State whether the function $f(x) = \frac{e^x + e^{-x}}{2}$ is odd or even.

(c) If $y = \log(x^2 + 2x + 5)$ then find $\frac{dy}{dx}$.

(d) Evaluate : $\int \frac{1 - \cos 2x}{1 + \cos 2x}\, dx$.

(e) Evaluate : $\int \frac{1}{2x + 5}\, dx$.

(f) Find the area under the parabola $y^2 = 4x$ bounded by the lines $x = 0$, $y = 0$, $x = 4$.

(g) State the trapezoidal rule of numerical integration.

2. Attempt any THREE of the following : 12

(a) If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

(b) If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

(c) Find maximum and minimum value of $y = x^3 - 18x^2 + 96x$.

(d) Find radius of curvature of the curve $y = x^3$ at (2, 8).

3. Attempt any THREE of the following : 12

(a) Find $\frac{dy}{dx}$ if $y = x^x + (\sin x)^x$.

(b) Find $\frac{dy}{dx}$ if $x^2 + 3xy + y^2 = 5$.

(c) Evaluate : $\int \frac{\log(\tan x/2)}{\sin x}\, dx$.

(d) Find the equation of the tangent to the circle $x^2 + y^2 + 6x - 6y - 7 = 0$ at a point it cuts the x-axis.

4. Attempt any THREE of the following : 12

(a) Evaluate : $\int \frac{1}{5 + 4\cos x}\, dx$.

(b) Evaluate : $\int \frac{x + 1}{x(x^2 - 4)}\, dx$.

(c) Evaluate : $\int \cos(\log x)\, dx$.

(d) Evaluate : $\int \frac{1}{x^2 + 4x + 9}\, dx$.

(e) Evaluate : $\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}}\, dx$.

5. Attempt any TWO of the following : 12

(a) Find the area of the loop of a curve $y^2 = x^2(1 - x)$.

(b) Attempt the following :

(i) Form the differential equation of $y = a \sin x + b \cos x$.

(ii) Solve : $\frac{dy}{dx} + \frac{y}{x} = x^2$.

(c) A resistance of 100 Ω and inductance of 0.1 henries are connected in series with a battery of 20 volts. Find the current in the circuit at any instant, if the relation between L, R and E is

$$L \frac{di}{dt} + Ri = E.$$

6. Attempt any TWO of the following : 12

(a) (i) Using trapezoidal rule, evaluate $\int_0^6 f(x)\, dx$ given by.

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.3333	0.25	0.2	0.6666	0.1428

(ii) Using Simpson's 1/3rd rule, evaluate $\int_1^2 \frac{1}{x} dx$ given by

x	1	1.25	1.5	1.75	2
y = f(x)	1	0.8	0.6666	0.5714	0.5

(b) Evaluate $\int_0^1 \frac{1}{1 + x^2} dx$. Using Simpson's 1/3rd rule divide the interval [0, 1] into six equal parts. Find approximate value of π.

(c) Evaluate $\int_0^6 \frac{1}{1 + x^2} dx$ by using Simpson's 3/8th rule.

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** Subject Code: 22201

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Attempt any FIVE of following:**	**10**
	a)	If $f(x)=x^4-2x+7$, find $f(0)+f(2)$	**02**
	Ans	$f(x)=x^4-2x+7$	
		$\therefore f(0)=(0)^4-2(0)+7=7$	½
		$\therefore f(2)=(2)^4-2(2)+7=19$	½
		$\therefore f(0)+f(2)=7+19$	½
		$\therefore f(0)+f(2)=26$	½
		--	
	b)	State whether the function $f(x)=\dfrac{e^x+e^{-x}}{2}$ is odd or even.	**02**
	Ans	$f(x)=\dfrac{e^x+e^{-x}}{2}$	
		$\therefore f(-x)=\dfrac{e^{-x}+e^{-(-x)}}{2}$	½
		$=\dfrac{e^{-x}+e^{x}}{2}$	½
		$=f(x)$	½
		$\therefore$ function is even.	½
		--	
	c)	If $y=\log\left(x^2+2x+5\right)$ then find $\dfrac{dy}{dx}$	**02**

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	c)	$y=\log\left(x^2+2x+5\right)$	
	Ans	$\therefore \frac{dy}{dx}=\frac{1}{x^2+2x+5}\cdot\frac{d}{dx}\left(x^2+2x+5\right)$	1
		$=\frac{1}{x^2+2x+5}\cdot\left(2x+2\right)=\frac{2\left(x+1\right)}{x^2+2x+5}$	1
		---	
	d)	Evaluate $\int\frac{1-\cos 2x}{1+\cos 2x}\,dx$	**02**
	Ans	$\int\frac{1-\cos 2x}{1+\cos 2x}\,dx$	
		$=\int\frac{2\sin^2 x}{2\cos^2 x}\,dx$	½
		$=\int\tan^2 x\,dx$	½
		$=\int\left(\sec^2 x-1\right)dx$	½
		$=\tan x-x+c$	½
		---	
	e)	Evaluate $\int\frac{1}{2x+5}dx$	**02**
	Ans	$\int\frac{1}{2x+5}dx=\frac{1}{2}\left[\log\left(2x+5\right)\right]+c$	2
		OR	
		$\int\frac{1}{2x+5}dx=\frac{1}{2}\int\frac{1}{x+\frac{5}{2}}dx$	
		$=\frac{1}{2}\left[\log\left(x+\frac{5}{2}\right)\right]+c$	2
		---	
	f)	Find the area under the parabola $y^2=4x$, bounded by the lines $x=0, y=0$, $x=4$	**02**
	Ans	Area $A=\int_a^b y\,dx$	
		$=\int_0^4 2\sqrt{x}\,dx$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	Area $A=\int_0^4 2x^{1/2}dx$	
		$=\left[2\frac{x^{3/2}}{\frac{3}{2}}\right]_0^4$	
		$=\left[\frac{4}{3}x^{3/2}\right]_0^4$	½
		$=\frac{4}{3}\left[4^{3/2}-0\right]$	½
		$=10.667$	½
		--	
	g)	State the trapezoidal rule of numerical integration.	**02**
	Ans	Trapezoidal rule	
		$\int_a^b f(x)dx=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$	2
		where $h=\frac{b-a}{n}$	
		--	
2.		**Attempt any THREE of the following:**	**12**
	a)	If $x^y=e^{x-y}$ then prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$	**04**
	Ans	$x^y=e^{x-y}$	
		$\log x^y=\log e^{x-y}$	½
		$\therefore y\log x=(x-y)\log e$	½
		$\therefore y\log x=x-y$	
		$y\log x+y=x$	
		$y(\log x+1)=x$	
		$\therefore y=\frac{x}{1+\log x}$	1
		$\therefore \frac{dy}{dx}=\frac{(1+\log x)\frac{d(x)}{dx}-x\frac{d(1+\log x)}{dx}}{(1+\log x)^2}$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	a)	$\therefore \frac{dy}{dx}=\frac{(1+\log x)-x\left(\frac{1}{x}\right)}{(1+\log x)^2}$	1
		$\therefore \frac{dy}{dx}=\frac{1+\log x-1}{(1+\log x)^2}$	½
		$\therefore \frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$	½
		--	
	b)	If $x=a(\theta-\sin\theta), y=a(1-\cos\theta)$, then find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$	**04**
	Ans	$x=a(\theta-\sin\theta)$ $\quad$ $y=a(1-\cos\theta)$	
		$\therefore \frac{dx}{d\theta}=a(1-\cos\theta)$ $\quad$ $\frac{dy}{d\theta}=a\sin\theta$	1+1
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\therefore \frac{dy}{dx}=\frac{a\sin\theta}{a(1-\cos\theta)}=\frac{\sin\theta}{(1-\cos\theta)}$ OR $\frac{dy}{dx}=\frac{\sin\theta}{(1-\cos\theta)}=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=\cot\frac{\theta}{2}$	1
		at $\theta=\frac{\pi}{4}$	
		$\therefore \frac{dy}{dx}=\frac{\sin\frac{\pi}{4}}{\left(1-\cos\frac{\pi}{4}\right)}=\frac{\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}$	
		$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{2}-1}$ or 2.414 OR $\frac{dy}{dx}=\cot\frac{\frac{\pi}{4}}{2}=\cot\frac{\pi}{8}=2.414$	1
		--	
	c)	Find maximum and minimum value of $y=x^3-18x^2+96x$	**04**
	Ans	Let $y=x^3-18x^2+96x$	
		$\therefore \frac{dy}{dx}=3x^2-36x+96$	½
		$\therefore \frac{d^2y}{dx^2}=6x-36$	½
		Consider $\frac{dy}{dx}=0$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
2.	c)	$3x^2-36x+96=0$	½
		$x^2-12x+32=0$	
		$\therefore x=8$ or $x=4$	½
		at $x=8$	
		$\frac{d^2y}{dx^2}=6(8)-36=12<0$	½
		$\therefore y$ is minimum at $x=8$	
		$y_{\min}=(8)^3-18(8)^2+96(8)$	
		$=128$	½
		at $x=4$	
		$\frac{d^2y}{dx^2}=6(4)-36=-12<0$	½
		$\therefore y$ is maximum at $x=4$	
		$y_{\max}=(4)^3-18(4)^2+96(4)$	
		$=160$	½
		--	
	d)	Find radius of curvature of the curve $y=x^3$ at $(2,8)$	**04**
	Ans	$y=x^3$	
		$\therefore \frac{dy}{dx}=3x^2$	½
		$\therefore \frac{d^2y}{dx^2}=6x$	½
		at $(2,8)$	
		$\frac{dy}{dx}=3(2)^2=12$	½
		$\frac{d^2y}{dx^2}=6(2)=12$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(12)^2\right]^{\frac{3}{2}}}{12}$	1
		$\therefore \rho=145.50$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
3.		**Attempt any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $y = x^x + (\sin x)^x$	**04**
	Ans	Let $u = x^x$	
		$\therefore \log u = \log x^x$	
		$\log u = x \log x$	½
		$\frac{1}{u}\frac{du}{dx} = x\frac{1}{x} + \log x(1)$	1
		$\therefore \frac{du}{dx} = u(1+\log x)$	
		$\therefore \frac{du}{dx} = x^x(1+\log x)$	
		Let $v = (\sin x)^x$	
		$\therefore \log v = \log(\sin x)^x$	
		$\log v = x\log(\sin x)$	½
		$\frac{1}{v}\frac{dv}{dx} = x\frac{1}{\sin x}\cos x + \log(\sin x)(1)$	1
		$\frac{1}{v}\frac{dv}{dx} = x\cot x + \log(\sin x)$	
		$\frac{dv}{dx} = v(x\cot x + \log(\sin x))$	
		$\frac{dv}{dx} = (\sin x)^x(x\cot x + \log(\sin x))$	
		$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$	
		$\therefore \frac{dy}{dx} = x^x(1+\log x) + (\sin x)^x(x\cot x + \log(\sin x))$	1
		--	
	b)	Find $\frac{dy}{dx}$ if $x^2 + 3xy + y^2 = 5$	**04**
	Ans	$x^2 + 3xy + y^2 = 5$	
		$2x + 3\left[x\frac{dy}{dx} + y(1)\right] + 2y\frac{dy}{dx} = 0$	2
		$2x + 3x\frac{dy}{dx} + 3y + 2y\frac{dy}{dx} = 0$	½
		$(3x+2y)\frac{dy}{dx} = -2x - 3y$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	b)	$\frac{dy}{dx}=\frac{-2x-3y}{3x+2y}=\frac{-(2x+3y)}{3x+2y}$	1
		--	
	c)	Evaluate: $\int\frac{\log\left(\tan x/2\right)}{\sin x}dx$	**04**
	Ans	$\int\frac{\log\left(\tan x/2\right)}{\sin x}dx$	
		Put $\log\left(\tan x/2\right)=t$	½
		$\frac{1}{\tan x/2}\sec^2 x/2\left(\frac{1}{2}\right)dx=dt$	½
		$\left(\frac{1}{2}\right)\frac{\cos x/2}{\sin x/2}.\frac{1}{\cos^2 x/2}dx=dt$	
		$\therefore\frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx=dt$	
		$\therefore\frac{1}{\sin x}dx=dt$	½
		$\therefore\int t\ dt$	1
		$=\frac{t^2}{2}+c$	1
		$=\frac{\left(\log\left(\tan x/2\right)\right)^2}{2}+c$	½
		--	
	d)	Find the equation of tangent to the circle $x^2+y^2+6x-6y-7=0$ at a point it cuts the x-axis	**04**
	Ans	$x^2+y^2+6x-6y-7=0$	
		$\because$ tangent cuts x-axis $\therefore\ y=0$	
		$\therefore x^2+(0)^2+6x-6(0)-7=0$	
		$\therefore x^2+6x-7=0$	
		$\therefore x=1$ and $x=-7$ $\therefore$ Points are $(1,0)$ and $(-7,0)$	1
		$\because x^2+y^2+6x-6y-7=0$	
		$\therefore 2x+2y\frac{dy}{dx}+6-6\frac{dy}{dx}=0$	½
		$\therefore(2y-6)\frac{dy}{dx}=-2x-6$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	$\therefore \frac{dy}{dx} = \frac{-2x-6}{2y-6}$	½
		at $(1,0)$	
		Slope $= \frac{dy}{dx} = \frac{-2(1)-6}{2(0)-6} = \frac{-8}{-6} = \frac{4}{3}$	½
		$\therefore$ equation is	
		$y - y_1 = m(x - x_1)$	
		$y - 0 = \frac{4}{3}(x-1)$	½
		$3y = 4x - 4$	
		$4x - 3y - 4 = 0$	
		at $(-7,0)$	
		Slope $= \frac{dy}{dx} = \frac{-2(-7)-6}{2(0)-6} = \frac{8}{-6} = \frac{-4}{3}$	½
		$\therefore$ equation is	
		$y - 0 = \frac{-4}{3}(x+7)$	½
		$3y = -4x - 28$	
		$4x + 3y + 28 = 0$	

4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate : $\int \frac{1}{5+4\cos x} dx$	**04**
	Ans	$\int \frac{1}{5+4\cos x} dx$	
		Put $\tan\frac{x}{2} = t \quad \therefore \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2dt}{1+t^2}$	1
		$\therefore \int \frac{dx}{5+4\cos x} = \int \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$	½
		$= 2\int \frac{1}{t^2+9} dt$	
		$= 2\int \frac{1}{t^2+3^2} dt$	1
		$= 2 \times \frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right) + c$	1
		$= \frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right) + c$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	b)	Evaluate: $\int \frac{x+1}{x\left(x^2-4\right)}\, dx$	**04**
	Ans	$\int \frac{x+1}{x\left(x^2-4\right)}\, dx = \int \frac{x+1}{x(x-2)(x+2)} dx$	
		Let $\frac{x+1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$	½
		$x+1 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$	
		put $x=0 \quad \therefore A = \frac{-1}{4}$	½
		put $x=2 \quad \therefore B = \frac{3}{8}$	½
		put $x=-2 \quad \therefore C = \frac{-1}{8}$	½
		$\frac{x+1}{x(x-2)(x+2)} = \frac{\frac{-1}{4}}{x} + \frac{\frac{3}{8}}{x-2} + \frac{\frac{-1}{8}}{x+2}$	
		$\int \frac{x+1}{x(x-2)(x+2)} dx = \int \left(\frac{\frac{-1}{4}}{x} + \frac{\frac{3}{8}}{x-2} + \frac{\frac{-1}{8}}{x+2} \right) dx$	½
		$= \frac{-1}{4}\log x + \frac{3}{8}\log(x-2) - \frac{1}{8}\log(x+2) + c$	½+½+½
		--	
	c)	Evaluate: $\int \cos(\log x)\, dx$	**04**
	Ans	$\int \cos(\log x)\, dx$	
		Put $\log x = t \Rightarrow x = e^t$	½
		$\therefore \frac{1}{x} dx = dt$	
		$\therefore dx = x dt$	
		$\therefore dx = e^t dt$	1
		$\therefore \int e^t \cos t dt$	1
		$= \frac{e^t}{1+1}(1\cos t + 1\sin t) + c$	1
		$= \frac{x}{2}\left(\cos(\log x) + \sin(\log x)\right) + c$	½

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	OR	
		$\int \cos(\log x)\ dx$	
		Put $\log x = t \Rightarrow x = e^t$	
		$\therefore \frac{1}{x}dx = dt$	
		$\therefore dx = xdt$	
		$\therefore dx = e^t dt$	1
		$\therefore I = \int e^t \cos t dt$	
		$= \cos t \int e^t dt - \int \left(\int e^t dt \frac{d}{dt} \cos t \right) dt$	
		$= \cos t\ e^t - \int e^t (-\sin t) dt$	1
		$= \cos t\ e^t + \int e^t \sin t dt + c$	
		$= \cos t\ e^t + e^t \sin t - \int e^t \cos t dt + c$	1
		$\therefore I = \cos t\ e^t + e^t \sin t - I + c$	
		$\therefore 2I = \cos t\ e^t + e^t \sin t + c$	
		$\therefore I = \frac{e^t}{2}(\cos t + \sin t) + c$	½
		$\therefore I = \frac{x}{2}(\cos(\log x) + \sin(\log x)) + c$	½
		OR	
		$I = \int \cos(\log x)\ dx$	
		$\therefore I = \int \cos(\log x).1\ dx$	½
		$\therefore I = \cos(\log x) \int 1dx - \int \left(\int 1dx \frac{d}{dx} \cos(\log x) \right) dx$	
		$\therefore I = \cos(\log x) x - \int x \left(\frac{-\sin(\log x)}{x} \right) dx$	1
		$\therefore I = x\cos(\log x) + \int \sin(\log x)\ dx$	
		$\therefore I = x\cos(\log x) + \int \sin(\log x).1dx$	½
		$\therefore I = x\cos(\log x) + \left[\sin(\log x) x - \int x \left(\frac{\cos(\log x)}{x} \right) dx \right]$	
		$\therefore I = x\cos(\log x) + x\sin(\log x) - \int \cos(\log x)\ dx$	1
		$\therefore I = x\cos(\log x) + x\sin(\log x) - I + c$	
		$\therefore 2I = x(\cos(\log x) + \sin(\log x)) + c$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	c)	$\therefore I = \frac{x}{2}\left[\cos(\log x)+\sin(\log x)\right]+c$	1
		--	
	d)	Evaluate : $\int \frac{1}{x^2+4x+9}\,dx$	**04**
	Ans	$\int \frac{1}{x^2+4x+9}\,dx$	
		Third term $=\left(\frac{1}{2}\times 4\right)^2 = 4$	1
		$=\int \frac{1}{x^2+4x+4-4+9}\,dx$	1
		$=\int \frac{1}{(x+2)^2+\left(\sqrt{5}\right)^2}\,dx$	1
		$=\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+c$	1
		OR	
		$\int \frac{1}{x^2+4x+9}dx$	
		Third term $=\frac{(M.T.)^2}{4(F.T.)} = 4$	1
		$=\int \frac{1}{x^2+4x+4-4+9}\,dx$	1
		$=\int \frac{1}{(x+2)^2+\left(\sqrt{5}\right)^2}\,dx$	1
		$=\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+c$	1
		--	
	e)	Evaluate $\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}\,dx$	**04**
	Ans	$\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}\,dx$ -------(1)	
		$I=\int_1^5 \frac{\sqrt{9-(1+5-x)}}{\sqrt{9-(1+5-x)}+\sqrt{(1+5-x)+3}}\,dx$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	$\therefore I=\int_1^5 \frac{\sqrt{x+3}}{\sqrt{x+3}+\sqrt{9-x}}\,dx --------(2)$	½
		add (1) and (2)	
		$I+I=\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+3}}\,dx+\int_1^5 \frac{\sqrt{x+3}}{\sqrt{x+3}+\sqrt{9-x}}\,dx$	½
		$\therefore 2I=\int_1^5 \frac{\sqrt{9-x}+\sqrt{x+3}}{\sqrt{9-x}+\sqrt{x+3}}\,dx$	
		$\therefore 2I=\int_1^5 1\,dx$	
		$\therefore 2I=[x]_1^5$	½
		$\therefore 2I=5-1$	
		$\therefore 2I=4$	1
		$I=2$	½
		--	
5.		**Attempt any TWO of the following:**	**12**
	a)	Find the area of the loop of a curve $y^2=x^2(1-x)$.	**06**
	Ans	$y^2=x^2(1-x)$	
		$y=x\sqrt{(1-x)}$	
		at $y=0,\ x^2(1-x)=0$	
		$\therefore x=0\ ,\ 1$	1
		$\therefore A_1=\int_0^1 y\,dx$	
		$=\int_0^1 x\sqrt{1-x}\,dx$	1
		$=\int_0^1 (1-x)\sqrt{x}\,dx$	1
		$=\int_0^1 \left(\sqrt{x}-x^{3/2}\right)dx$	
		$=\left[\frac{2}{3}x^{3/2}-\frac{2}{5}x^{5/2}\right]_0^1$	1
		$=\left[\frac{2}{3}-\frac{2}{5}\right]-0$	
		$=\frac{4}{15}$ or 0.267	1
		$\therefore$ Area of loop $=2\times A_1=2\times\frac{4}{15}=\frac{8}{15}$ or 0.533	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	a)	*OR*	
		$y^2 = x^2(1-x) \quad \therefore y = x\sqrt{(1-x)}$	
		at $y=0,\ x^2(1-x)=0$	
		$\therefore x = 0\ ,\ 1$	1
		$\therefore\ A = \int_0^1 y dx$	
		$= \int_0^1 x\sqrt{1-x}dx$	1
		put $t = 1-x$, $\therefore dt = -dx$, $\therefore -dt = dx$; when $x \to 0$ to 1, $t \to 1$ to 0	1
		$= -\int_1^0 (1-t)\sqrt{t}dt$	
		$= -\int_1^0 \left(\sqrt{t} - t^{3/2}\right)dt$	
		$= -\left[\frac{2}{3}t^{3/2} - \frac{2}{5}t^{5/2}\right]_1^0$	1
		$= -\left[0 - \left(\frac{2}{3} - \frac{2}{5}\right)\right]$	
		$= \frac{4}{15}$ or 0.267	1
		$\therefore$ Area of loop $= 2\times A_1 = 2\times\left(\frac{4}{15}\right) = \frac{8}{15}$ or 0.533	1
		--	
	b)	**Attempt the following:**	**06**
	(i)	Form the differential equation of $y = a\sin x + b\cos x$	**03**
	Ans	$y = a\sin x + b\cos x$	
		$\therefore \frac{dy}{dx} = a\cos x - b\sin x$	1
		$\therefore \frac{d^2y}{dx^2} = -a\sin x - b\cos x$	1
		$\therefore \frac{d^2y}{dx^2} = -(a\sin x + b\cos x)$	½
		$\frac{d^2y}{dx^2} = -y$	½
		$\frac{d^2y}{dx^2} + y = 0$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	b)(ii)	Solve : $\frac{dy}{dx}+\frac{y}{x}=x^2$	**03**
	Ans	$\frac{dy}{dx}+\frac{y}{x}=x^2$ Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{1}{x}$ and $Q=x^2$	½
		$IF=e^{\int\frac{1}{x}dx}=e^{\log x}=x$	1
		$\therefore$ Solution is $y\cdot IF=\int Q\cdot IFdx+c$	
		$y\cdot x=\int x\cdot^2 xdx+c$	½
		$xy=\int x^3dx+c$	
		$xy=\frac{x^4}{4}+c$	1
		---	
	c)	A resistance of 100Ω and inductance of 0.1 henries are connected in series with a battary of 20 volts. find the current in the circuit at any instant , if the relation between L,R and E is $L\frac{di}{dt}+Ri=E$	**06**
	Ans	$L\frac{di}{dt}+Ri=E$	
		$\therefore \frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$ Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{R}{L}$ and $Q=\frac{E}{L}$	½
		$IF=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$	1
		$\therefore$ Solution is $i\cdot IF=\int Q\cdot IFdt+c$	
		$i\cdot e^{\frac{R}{L}t}=\int\frac{E}{L}e^{\frac{R}{L}t}dt+c$	½
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{L}\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}+c$	1
		$i\cdot e^{\frac{R}{L}t}=\frac{E}{R}e^{\frac{R}{L}t}+c$	
		Initially at t = 0 , i = 0 $\therefore$ c = $\frac{-E}{R}$	1
		$\therefore i\cdot e^{\frac{R}{L}t}=\frac{E}{R}e^{\frac{R}{L}t}+\left(\frac{-E}{R}\right)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	$i=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$	1
		When R=100 , L = 0.1 , E= 20	
		$i=\frac{20}{100}\left(1-e^{-\frac{100}{0.1}t}\right)$	1
		$i=0.2\left(1-e^{-1000t}\right)$	
6.		**Attempt any TWO of the following:**	**12**
	a)(i)	Using trapezoidal rule, evaluate $\int_0^6 f(x)\,dx$	**03**

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.3333	0.25	0.2	0.6666	0.1428

Ans

$$\int_a^b f(x)\,dx=\frac{h}{2}\left[\left(y_0+y_n\right)+2\left(y_1+y_2+...+y_{n-1}\right)\right]$$

$a=0, b=6$ and $h=1$

$$\therefore\int_0^6 f(x)\,dx=\frac{1}{2}\left[\left(1+0.1428\right)+2\left(0.5+0.3333+0.25+0.2+0.6666\right)\right]$$ 2

$$=2.5213$$ 1

---

a)(ii) Using Simpson's $\frac{1}{3}^{rd}$ rule, evaluate $\int_1^2 \frac{1}{x}\,dx$ given by **03**

x	1	1.25	1.5	1.75	2
$y=f(x)$	1	0.8	0.6666	0.5714	0.5

Ans

$$\int_a^b f(x)\,dx=\frac{h}{3}\left[\left(y_0+y_n\right)+4\left(y_1+y_3+...+y_{n-1}\right)+2\left(y_2+y_4+...+y_{n-2}\right)\right]$$

Let $y=f(x)=\frac{1}{x}$ $a=1, b=2$ and $h=0.25$

$$\therefore\int_1^2 f(x)\,dx=\frac{0.25}{3}\left[\left(1+0.5\right)+4\left(0.8+0.5714\right)+2\left(0.6666\right)\right]$$ 2

$$=0.6932$$ 1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ Using Simpson's $1/3^{rd}$ rule divide the interval $[0,1]$ into six equal parts. Find approximate value of π.	**06**
	Ans	Let $y=\frac{1}{1+x^2}$ $a=0, b=1$ and $n=6$	
		$\therefore h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}$	1

x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$y=\frac{1}{1+x^2}$	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$

(2)

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

$$\therefore \int_0^1 f(x)dx=\frac{1/6}{3}\left[\left(1+\frac{1}{2}\right)+4\left(\frac{36}{37}+\frac{4}{5}+\frac{36}{61}\right)+2\left(\frac{9}{10}+\frac{9}{13}\right)\right]$$ (1)

$$=0.7854$$ (1)

$$\because \int_0^1 \frac{1}{1+x^2}dx=0.7854$$

$$\therefore \left[\tan^{-1}x\right]_0^1=0.7854$$ (½)

$$\left[\tan^{-1}(1)\right]-\left[\tan^{-1}(0)\right]=0.7854$$

$$\frac{\pi}{4}=0.7854$$

$$\pi=3.142$$ (½)

OR

Let $y=\frac{1}{1+x^2}$ $a=0, b=1$ and $n=6$

$\therefore h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}=0.1667$ (1)

x	0	0.1667	0.3334	0.5001	0.6668	0.8335	1
$y=\frac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6922	0.5901	0.5

(2)

Using Simpson's $1/3^{rd}$ rule

$$\int_a^b f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	$\therefore \int_0^1 f(x)\,dx = \frac{0.1667}{3}\left[(1+0.5)+4(0.9730+0.8+0.5901)+2(0.9+0.6922)\right]$	1
		$= 0.7855$	1
		$\int_0^1 \frac{1}{1+x^2}\,dx = 0.7855$	
		$\left[\tan^{-1} x\right]_0^1 = 0.7855$	½
		$\left[\tan^{-1}(1)\right]-\left[\tan^{-1}(0)\right] = 0.7855$	
		$\frac{\pi}{4} = 0.7855$	
		$\pi = 3.142$	½
		---	
	c)	Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ Using Simpson's $3/8^{th}$ rule.	**06**
	Ans	Consider $n=6$	
		$y=\frac{1}{1+x^2}$ $a=0,\quad b=6$	
		$\therefore h=\frac{b-a}{n}=\frac{6-0}{6}=1$	1
		(table below)	2
		Using Simpson's $3/8^{th}$ rule.	
		$\int_a^b f(x)\,dx=\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4 ...+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$	
		$\therefore \int_0^6 \frac{1}{1+x^2}\,dx=\frac{3(1)}{8}\left[(1+0.0270)+3(0.5+0.2+0.0588+0.0385)+2(0.1)\right]$	2
		$\therefore \int_0^6 \frac{1}{1+x^2}\,dx = 1.3571$	1
		Note: If the student has considered any value of n and attempted to solve give appropriate marks.	

x	0	1	2	3	4	5	6
$y=\frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	0.0385	0.0270

Q. No.	Sub Q. N.	Answer	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.*	