COURSE MATERIAL

II Year B. Tech I- Semester

MECHANICAL ENGINEERING

AY: 2022-23

THEORY OF MACHINES

R20A0308

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

DEPARTMENT OF MECHANICAL ENGINEERING

CONTENTS

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

VISION

- To establish a pedestal for the integral innovation, team spirit, originality and competence in the students, expose them to face the global challenges and become technology leaders of Indian vision of modern society.

MISSION

- To become a model institution in the fields of Engineering, Technology and Management.
- To impart holistic education to the students to render them as industry ready engineers.
- To ensure synchronization of MRCET ideologies with challenging demands of International Pioneering Organizations.

QUALITY POLICY

- To implement best practices in Teaching and Learning process for both UG and PG courses meticulously.
- To provide state of art infrastructure and expertise to impart quality education.
- To groom the students to become intellectually creative and professionally competitive.
- To channelize the activities and tune them in heights of commitment and sincerity, the requisites to claim the never - ending ladder of **SUCCESS** year after year.

For more information: www.mrcet.ac.in

VISION

To become an innovative knowledge center in mechanical engineering through state-of-the-art teaching-learning and research practices, promoting creative thinking professionals.

MISSION

The Department of Mechanical Engineering is dedicated for transforming the students into highly competent Mechanical engineers to meet the needs of the industry, in a changing and challenging technical environment, by strongly focusing in the fundamentals of engineering sciences for achieving excellent results in their professional pursuits.

Quality Policy

- ✓ To pursuit global Standards of excellence in all our endeavors namely teaching, research and continuing education and to remain accountable in our core and support functions, through processes of self-evaluation and continuous improvement.
- ✓ To create a midst of excellence for imparting state of art education, industry-oriented training research in the field of technical education.

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Department of Mechanical Engineering

PROGRAM OUTCOMES

Engineering Graduates will be able to:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **Life-long learning**: Recognize the need for and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSOs)

PSO1 Ability to analyze, design and develop Mechanical systems to solve the Engineering problems by integrating thermal, design and manufacturing Domains.

PSO2 Ability to succeed in competitive examinations or to pursue higher studies or research.

PSO3 Ability to apply the learned Mechanical Engineering knowledge for the Development of society and self.

Program Educational Objectives (PEOs)

The Program Educational Objectives of the program offered by the department are broadly listed below:

PEO1: PREPARATION

To provide sound foundation in mathematical, scientific and engineering fundamentals necessary to analyze, formulate and solve engineering problems.

PEO2: CORE COMPETANCE

To provide thorough knowledge in Mechanical Engineering subjects including theoretical knowledge and practical training for preparing physical models pertaining to Thermodynamics, Hydraulics, Heat and Mass Transfer, Dynamics of Machinery, Jet Propulsion, Automobile Engineering, Element Analysis, Production Technology, Mechatronics etc.

PEO3: INVENTION, INNOVATION AND CREATIVITY

To make the students to design, experiment, analyze, interpret in the core field with the help of other inter disciplinary concepts wherever applicable.

PEO4: CAREER DEVELOPMENT

To inculcate the habit of lifelong learning for career development through successful completion of advanced degrees, professional development courses, industrial training etc.

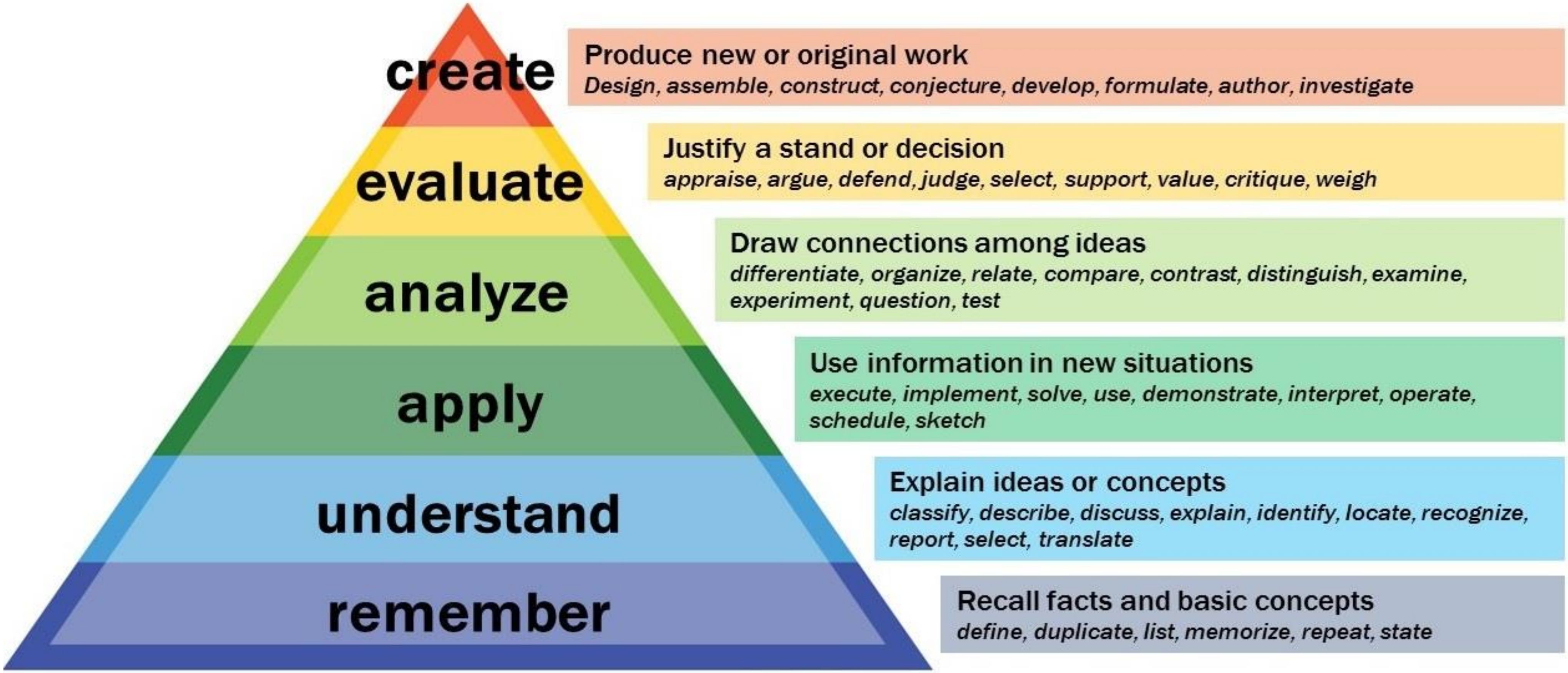
PEO5: PROFESSIONALISM

To impart technical knowledge, ethical values for professional development of the student to solve complex problems and to work in multi-disciplinary ambience, whose solutions lead to significant societal benefits.

Blooms Taxonomy

Bloom’s Taxonomy is a classification of the different objectives and skills that educators set for their students (learning objectives). The terminology has been updated to include the following six levels of learning. These 6 levels can be used to structure the learning objectives, lessons, and assessments of a course.

1. **Remembering**: Retrieving, recognizing, and recalling relevant knowledge from long- term memory.
2. **Understanding**: Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.
3. **Applying**: Carrying out or using a procedure for executing or implementing.
4. **Analyzing**: Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.
5. **Evaluating**: Making judgments based on criteria and standard through checking and critiquing.
6. **Creating**: Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.



create
Produce new or original work
Design, assemble, construct, conjecture, develop, formulate, author, investigate
evaluate
Justify a stand or decision
appraise, argue, defend, judge, select, support, value, critique, weigh
analyze
Draw connections among ideas
differentiate, organize, relate, compare, contrast, distinguish, examine, experiment, question, test
apply
Use information in new situations
execute, implement, solve, use, demonstrate, interpret, operate, schedule, sketch
understand
Explain ideas or concepts
classify, describe, discuss, explain, identify, locate, recognize, report, select, translate
remember
Recall facts and basic concepts
define, duplicate, list, memorize, repeat, state

MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY

II Year B.Tech. ME- I Sem **L/T/P/C**
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(R20A0308) THEORY OF MACHINES

Course Objectives:

1. To impart knowledge on various types of links and synthesis.
2. To impart skills to analyse the position, velocity and acceleration of mechanisms and Steering Gear Mechanisms
3. To study about gyroscope and its effects during precession motion of moving vehicles and turning moment diagrams.
4. To understand the working principles of different type brakes and clutches.
5. To familiarize higher pairs like cams and principles of cams design and governors.

UNIT-I

Introduction of Mechanisms and Machines:

Mechanisms : Elements or Links , Classification, Rigid Link, flexible and fluid link, Types of kinematic pairs , sliding, turning, rolling, screw and spherical pairs lower and higher pairs, closed and open pairs, constrained motion, completely, partially or successfully constrained and incompletely Constrained .

Machines: Mechanism and machines, classification of machines, kinematic chain inversion of mechanism, inversions of quadric cycle, chain, single and double slider crank chains.

UNIT-II

Kinematics: Velocity and acceleration - Motion of link in machine - Determination of Velocity and acceleration diagrams - Graphical method - Application of relative velocity method four bar chain.

Steering Gear Mechanisms: Conditions for correct steering Davis Steering gear Mechanism, Ackerman's steering gear mechanism.

UNIT–III

Precession: Gyroscopes, effect of precession motion on the stability of moving vehicles such as, aero planes and motor car.

Turning moment Diagrams: Single cylinder double acting steam engine, Four Stroke Cycle Internal Combustion Engine, Multi-cylinder Engine, and Flywheel.

UNIT–IV

Friction and Friction Drives: Introduction to friction, Laws of friction, Coefficient of friction, Inclined plane, Pivot and Collars, Friction clutches-centrifugal clutch.

Brakes: Types of brakes, Block and Shoe brakes, Internal expanding shoe brake, Braking effect in vehicle.

UNIT–V

Cams: Types of cams, Types of followers, Follower displacement programming, Derivatives of follower Motion, Layout of cam profiles-knife edge and roller follower.

Governors: introduction, Watt Governor, Porter Governor.

TEXT BOOKS:

1. Rattan S.S, “Theory of Machines” Tata McGraw-Hill Publishing Company Ltd., New Delhi, and 2nd edition -2005.
2. Sadhu Singh, “Theory of Machines,” Pearson Education (Singapore) Pvt. Ltd., Indian Branch, New

Delhi, 2ND Edi. 2006.

3. Jagadish Lal, ‘Theory of Machine’, Dhanpat Rai Publications, New Delhi..

REFERENCE BOOKS:

1. Shigley. J. V. and Uickers, J.J., “Theory of Machines & Mechanisms” OXFORD University press.2004
3. “Theory of Machines -I”, by A.S.Ravindra, Sudha Publications, Revised 5th Edi. 2004

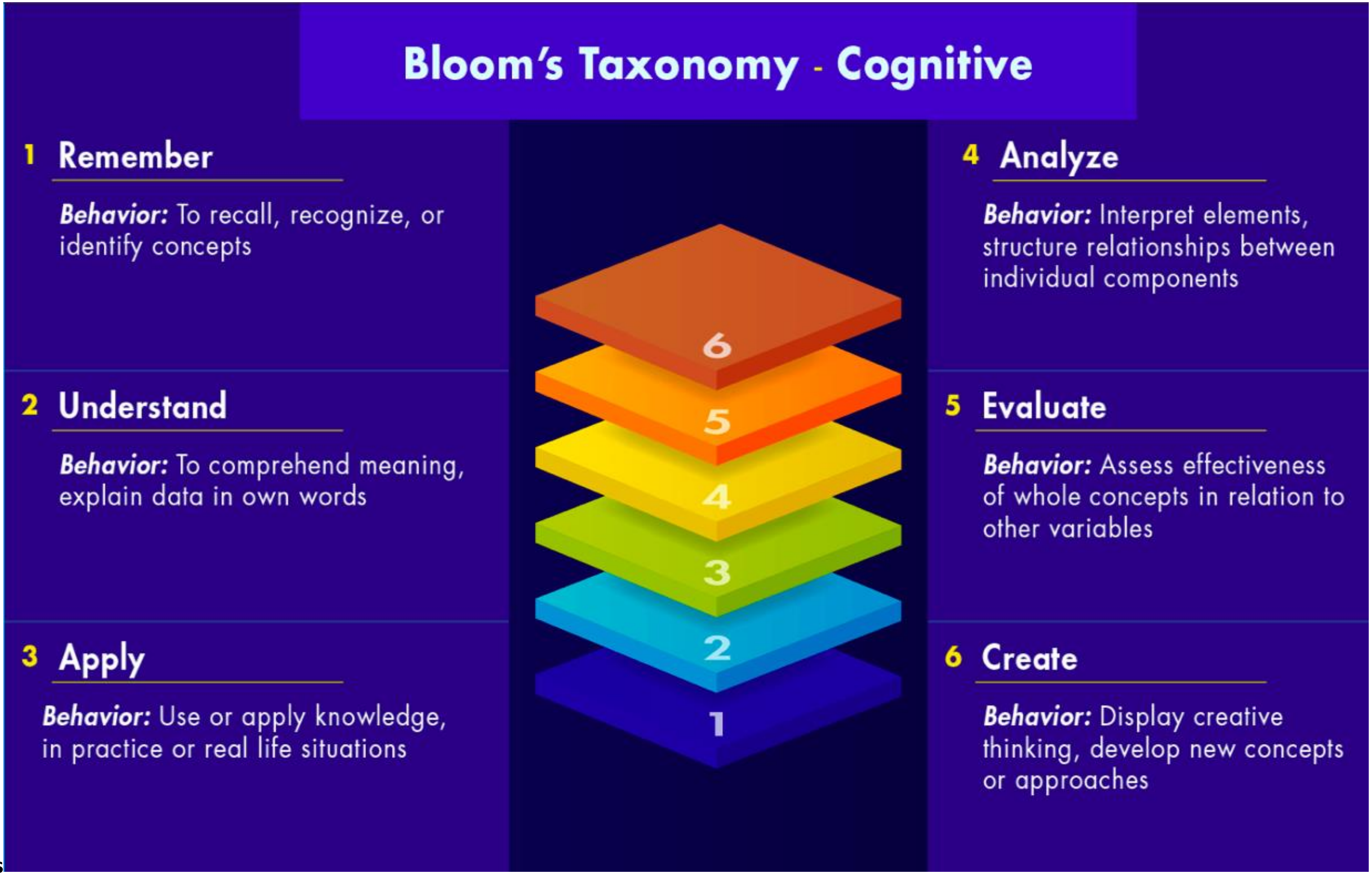
Course Outcomes:

1. Understand the principles of kinematic pairs, chains and their classification, DOF and inversions.
2. Analyze the planar mechanisms for position, velocity and acceleration and steering gear mechanism.
3. Knowledge acquired about Gyroscope and its precession motion and turning moment diagrams.
4. Acquire the knowledge on different type brakes and clutches.
5. Understand the concept of Design cams and followers for specified motion profiles and governors.

THEROY OF MACHINES (R20A0308)

COURSE OBJECTIVES

UNIT - 1	CO1: To impart knowledge on various types of Mechanisms and synthesis
UNIT - 2	CO2: To impart skills to analyse the position, velocity and acceleration of mechanisms and to familiarize higher pairs like cams and principles of cams design.
UNIT - 3	CO3: To understand the working principles of different type brakes and clutches.
UNIT - 4	CO4: Able to learn about the working of Belt, Rope and Chains.
UNIT - 5	CO5: To study the relative motion analysis and design of gears, gear trains.


Bloom's Taxonomy - Cognitive
1 Remember
Behavior: To recall, recognize, or identify concepts
2 Understand
Behavior: To comprehend meaning, explain data in own words
3 Apply
Behavior: Use or apply knowledge, in practice or real life situations
4 Analyze
Behavior: Interpret elements, structure relationships between individual components
5 Evaluate
Behavior: Assess effectiveness of whole concepts in relation to other variables
6 Create
Behavior: Display creative thinking, develop new concepts or approaches
6
5
4
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<u>COURSE OUTLINE</u>

UNIT – 1

NO OF LECTURE HOURS: 12

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES (2 to 3 objectives)
1.	Mechanisms	Definition of Mechanism Definition of Machine	Understanding the mechanics of rigid, fixed, deformable bodies (B2)
2.	Kinematic Link and Classification of Links	Definition of Link Definition of Pair Classification of Links Classification of Pairs	• State the basic concept of link and pair (B1) • Understanding the classification of links and pairs (B2)
3.	Constrained Motion and Classification	Definition of Constrained Motion Classification of Constrained Motion	• Describe the constrained motion (B1) • Understanding the direction of motion (B2)
4.	Mechanism and Machines	Definition of Machine. Determine the nature of chain. Definition of Grashof's law	Analyse machine and structure (B4) • Evaluate the nature of mechanism (B5)
5.	Inversion of Mechanism	Definition of inversion. Classification of inversion of mechanism	• Understanding the inversion of mechanisms and its classifications (B2)
6.	Inversions of Quadric Cycle	Working of 4-bar chain mechanisms	Understanding the important inversions of 4-bar mechanism (B2) • Analyse the inversion of 4-bar mechanism (B4)
7.	Inversion of Single Slider Crank Chains	Working of Single slider crank chain mechanisms	• Understanding the important inversions of single mechanism (B2) • Analyse the inversion of single mechanism (B4)

8.	Inversion of Double Slider Crank Chains	Working of Double slider crank chain mechanisms	Understanding the important inversions of single mechanism (B2)
9.	Problems	Practice on problems	Apply the formulas for couple (B3)
10.	Practice	Practice on problems	Apply the formulas for couple (B3)
11.	Straight Line Motion Mechanisms	Definition of Straight line motion mechanism Classification of exact straight line motion mechanism	Understanding the inversion of mechanisms and its classifications (B2)
12.	Approximate Straight Line Mechanism	Working of approximate straight-line mechanisms	Understanding the inversions of approximate straight line mechanism (B2) Analyse the approximate straight line mechanisms (B4)

UNIT – 2

NO OF LECTURE HOURS: 14

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES (2 to 3 objectives)
1.	Static and Dynamic force analysis	Definitions , Inertia force, Resultant Effect of a System of Forces Acting on a Rigid Body	Understand the concept static and Dynamic forces (B2)
2.	Free Body Diagrams & Inertia forces	Two, Three and Four Members, D'Alembert's Principle	Understand the concept of inertia forces(B3)
3.	Graphical method	Velocity and acceleration on a Link by Relative Velocity Method Rubbing Velocity at a Pin Joint	Understanding different types of graphical method for velocity and acceleration calculation (B2) Apply graphical method for various methods (B3)
4.	Relative velocity method four bar chain	Numerical examples to estimate the velocity and acceleration using relative velocity method	Apply relative velocity method to estimate the velocity and acceleration for four bar mechanisms (B3)
5.	Instantaneous centre of rotation	Definition of instantaneous centre of rotation Types of instantaneous centre of rotation	Understanding the Instantaneous axis (B2) Compare the two components of acceleration (B1)
6.	Three centers in line theorem	Aronhold Kennedy Theorem	Understanding the Three centers in line theorem (B2) Locate the instantaneous centres by Aronhold Kennedy's theorem (B5)
7.	Graphical determination of instantaneous center	Number of Instantaneous Centres in a Mechanism Numerical Examples using instantaneous centre of rotation	Evaluate instantaneous centers of the slider crank mechanism (B5) Apply graphical method for Instantaneous Centres (B3)

8.	Introduction to cams	Classification of Followers, cams	Understanding the difference between the cam and followers (B2)
9.	introduction	Terms used in radial cams, Motion of follower	Understanding the basic terms (B2)
10.	Cam profile introduction	Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity	Understanding the basic terms (B2) Apply the concept (B3)
11.	Cam profile introduction	Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity	Understanding the basic terms (B2) Apply the concept (B3)
12.	Cam profile introduction	Construction of cam profile f or a Radial cam	Apply the concept (B3)
13.	Problem	Problems on cam profile	Apply the concept (B3)
14.	Problems	Problems on cam profile	Apply the concept (B3)

UNIT – 3

NO OF LECTURE HOURS: 16

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES (2 to 3 objectives)
1.	friction	Introduction to friction, Law of friction, Coefficient of friction	State the basic concept of friction (B1)
2.	Friction``	Inclined plane, Pivot and Collars,	Remember the standard design formulas(B1) Understand the concept (B2)
3.	Pivot and Collars	Derivations	Understand the concept (B2)
4.	Problems`	Problems`	Remember the standard design formulas(B1) Apply the formulas(B3)
5.	Introduction of Clutches	Definition, Single Disc or plate clutch	Remember the standard design formulas(B1) Understand the concept (B2) Apply the formulas(B3) Remember the standard design formulas(B1) Apply the formulas(B3) Remember the standard design formulas(B1) Understand the concept (B2) Apply the formulas(B3)
6.	Problems	Problems on Single Disc or plate clutch.	
7.	CONE CLUTCH	Definition, Derivation	
8.	Problems	Problems on CONE CLUTCH	Remember the standard design formulas(B1) Apply the formulas(B3)

9.	centrifugal Clutch	Definition, Derivation.	Remember the standard design formulas(B1) Understand the concept (B2) Apply the formulas(B3)
10.	Problems	Problems on centrifugal Clutch	Remember the standard design formulas(B1) Apply the formulas(B3)
11.	Brakes and Dynamometers	Introduction and Types	Understand the concept (B2) Remember the standard design formulas(B1)
12.	Problems	brakes	Remember the standard design formulas(B1) Understand the concept (B2) Apply the formulas(B3)
13.	Brakes	Double Block or Shoe Brake	Understand the concept (B2) Remember the standard design formulas(B1)
14.	Brakes	Internal Expanding Brake	Understand the concept (B2) Remember the standard design formulas(B1)
15.	Problems	Double Block or Shoe Brake	Remember the standard design formulas(B1) Understand the concept (B2) Apply the formulas(B3)
16.	Problems	Internal Expanding Brake	Remember the standard design formulas(B1) Understand the concept (B2) Apply the formulas(B3)

UNIT – 4

NO OF LECTURE HOURS:12

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES (2 to 3 objectives)
1.	Belt	Introduction, types of belt drives, belts	Remember the standard design formulas(B1) Understand the concept (B2)
2.	Types of belts	Flat Belt, V Belt derivations, Velocity Ratio in Belt Drives	Remember the standard design formulas(B1) Understand the concept (B2)
3.	Belts	Law of Belting, Ratio of Friction Tensions in Belts, Power Transmitted by Belts	Remember the standard design formulas(B1) Understand the concept (B2)
4.	**Belts**	Problems	Remember the standard design formulas(B1) Apply the formulas(B3)
5.	**Belts**	problems	Remember the standard design formulas(B1) Apply the formulas(B3)
6.	Ropes	Introduction	Remember the standard design formulas(B1) Apply the formulas(B3)

7.	Problems	Problems	Remember the standard design formulas(B1) Apply the formulas(B3)
8.	Problems	Problems	Remember the standard design formulas(B1) Apply the formulas(B3)
9.	Chains	Introduction and terms	Remember the standard design formulas(B1) Understand the concept (B2)
10.	Chains	classifications	Understand the concept (B2) Remember the standard formulas(B1)
11.	Chains problems	Problems	Understand the concept (B2) Remember the standard formulas(B1
12.	Problems	Problems	Remember the standard design formulas(B1) Apply the formulas(B3)

UNIT – 5

NO OF LECTURE HOURS: 14

LECTUR E	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES (2 to 3 objectives)
1.	Introduction	POWER TRANSMISSION SYSTEMS, ADVANTAGES AND DISADVANTAGES OF GEAR DRIVE and classifications	Understand the concept (B2)
2.	Introduction	Terminology of gearing	Remember the concept (B1)
3.	LAW OF GEARING	Derivation, INVOLUTE TOOTH PROFILE	Understand the concept (B2)
4.	Velocity of sliding phenomena	LENGTH OF PATH OF CONTACT Deriavtion, contact ratio	Remember the standard design formulas(B1) Understand the concept (B2)
5.	**Problems**	Problems	Analyze the data(B4) Remember the standard design formulas(B1) Apply the formulas(B3))
6.	**Problems**	Problems	Remember the standard design formulas(B1) Understand the concept (B2)
7.	Velocity of sliding phenomena	INTERFERENCE AND UNDERCUTTING	Analyze the data(B4) Remember the standard design formulas(B1)

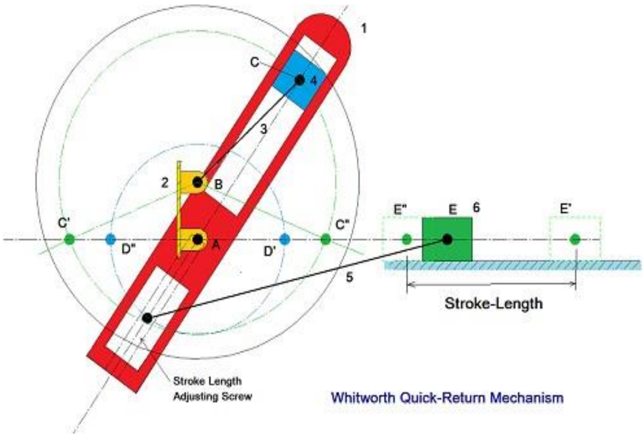
			Apply the formulas(B3)
8.	problems	Problems on law of gearing	Analyze the data(B4) Remember the standard design formulas(B1) Apply the formulas(B3)
9.	Problems	Problems on gears	Analyze the data(B4) Remember the standard design formulas(B1) Apply the formulas(B3)
10.	Gear trains	Introduction, Types of Gear Trains	Analyze the data(B4) Remember the standard design formulas(B1) Apply the formulas(B3)
11.	Problems	Problems on gear trains	Analyze the data(B4) Remember the standard design formulas(B1) Apply the formulas(B3)
12.	Gear trains	REVERTED GEAR TRAIN & problems	Analyze the data(B4) Remember the standard design formulas(B1) Apply the formulas(B3)
13.	Gear trains	EPICYCLIC GEAR TRAIN, VELOCITY RATIOS	Remember the standard design formulas(B1)
14.	VELOCITY RATIOS	Problems	Analyze the data(B4) Remember the standard design formulas(B1) Apply the formulas(B3)

UNIT 1

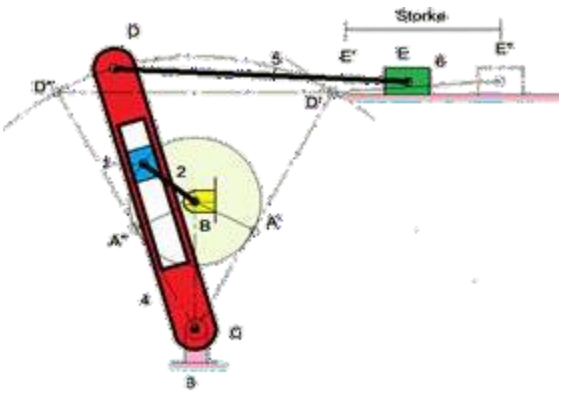
INTRODUCTION OF MECHANISMS AND MACHINES

1

Machines and Mechanisms



Whitworth Quick-Return Mechanism



Crank-Rocker Quick-Return Mechanism for Shaping Machine

Course Contents

1.1 Machine and Mechanism:

- **Mechanism:**

– If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*.

- **Machine:**

– A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.

- **Analysis:**

– *Analysis* is the study of motions and forces concerning different parts of an existing mechanism.

- **Synthesis:**

– *Synthesis* involves the design of its different parts.

1.2 Types of constrained motion:

1.2.1 Completely constrained motion:

– When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.

– For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (*i.e.* it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank.

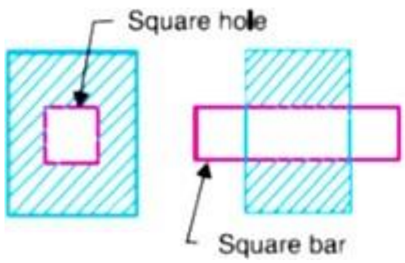


Fig. 1.1

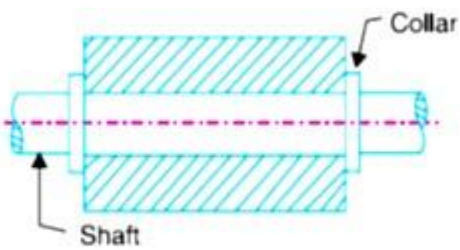


fig. 1.2

– The motion of a square bar in a square hole, as shown in Fig. 1.1, and the motion of a shaft with collars at each end in a circular hole, as shown in Fig. 1.2, are also examples of completely constrained motion.

1.2.2 Incompletely constrained motion:

– When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. 1.3, is an example of an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.

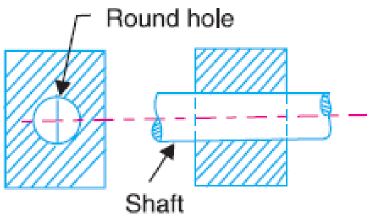


Fig. 1.3

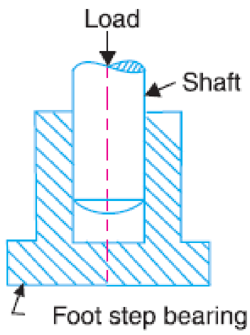


FIG. 1.4

1.2.3 Successfully constrained motion:

- When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig. 1.4.
- The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine

1.3 Types of Links:

- A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movements is known as a link.
- A link may also define as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.
- Links may be classified into binary, ternary and quaternary.

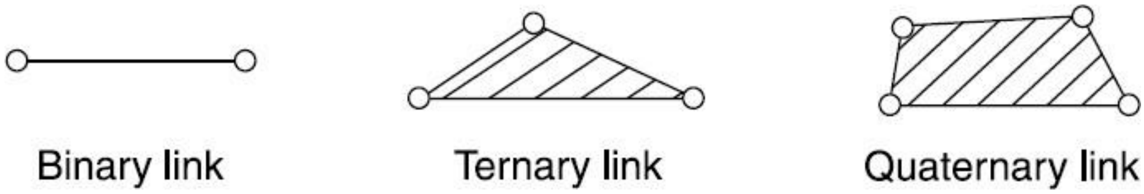


FIG. 1.4 Types of link

1.4 Kinematic Pair:

- When two kinematic links are connected in such a way that their motion is either completely or successfully constrained, these two links are said to form a kinematic pair.
- Kinematic pairs can be classified according to:

1.4.1 Kinematic pairs according to nature of contact:

a. Lower Pair:

- A pair of links having surfaced or area contact between the members is known as a lower pair. The contact surfaces of two links are similar.
- Examples: Nut turning on a screw, shaft rotating in a bearing.

b. Higher Pair:

- When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of two links are similar.
- Example: Wheel rolling on a surface, Cam and Follower pair etc.

1.4.2 Kinematic pairs according to nature of contact:

a. Closed Pair:

- When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

b. Unclosed Pair:

- When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g. cam and follower pair.

1.4.3 Kinematic pairs according to Nature of Relative Motion:

a. Sliding pair:

- When two links have a sliding motion relative to another; the kinematic pair is known as sliding pair.

b. Turning pair:

- When one link is revolve or turn with respect to the axis of first link, the kinematic pair formed by two links is known as turning pair.

c. Rolling pair:

- When the links of a pair have a rolling motion relative to each other, they form a rolling pair.

d. Screw pair:

- If two mating links have a turning as well as sliding motion between them, they form a screw pair.

e. Spherical pair:

- When one link in the form of sphere turns inside a fixed link, it is a spherical pair.

1.5 Types of Joint:

– The usual types of joints in a chain are:
  - Binary Joint
  - Ternary Joint
  - Quaternary Joint

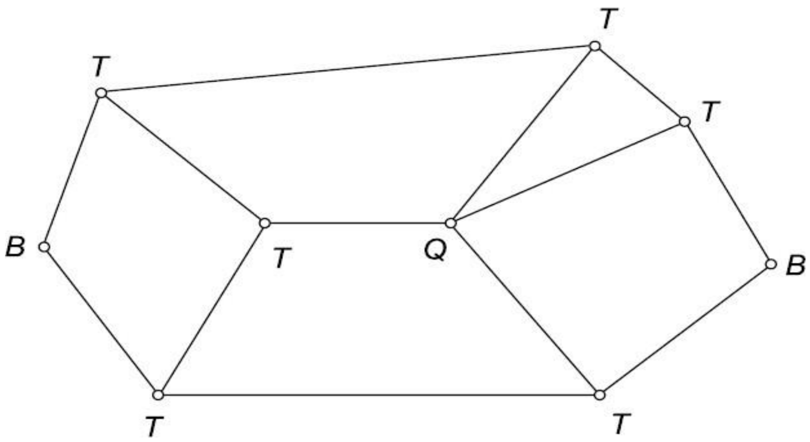


Fig1.5. Types of joint

a. Binary Joint:

- If two links are joined at the same connection, it is called a binary joint. For example, in fig. at joint B

b. Ternary Joint:

- If three links joined at a connection, it is known as a ternary link. For example point T in fig.

c. Quaternary Joint:

- If four links joined at a connection, it is known as a quaternary link. For example point Q in fig.

1.6 Degrees of Freedom:

– An unconstrained rigid body moving in space can describe the following independent motion:

a. Translational motion along any three mutually perpendicular axes x, y and z.

b. Rotational motion about these axes

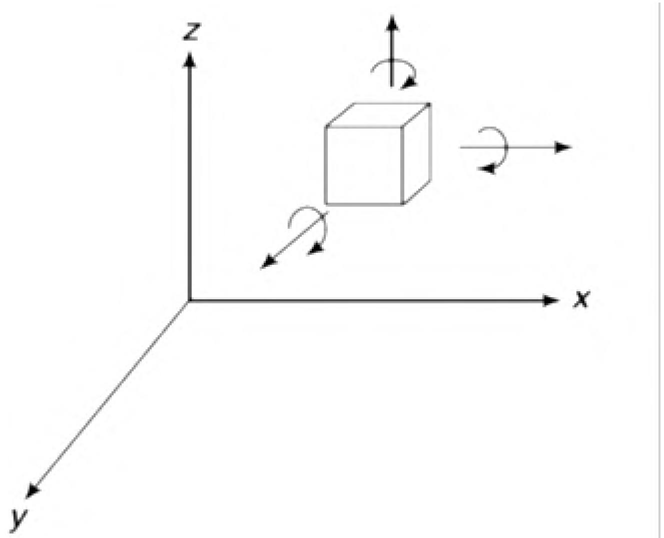


Fig.1.6 Degrees of freedom

- A rigid body possesses six degrees of freedom.
- Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.
- DOF = 6 – Number of Restraints

1.7 Kinematic chain

- Kinematic chain is defined as the combination of kinematic pairs in which each links forms a part of two kinematic pairs and the relative motion between the links is either completely constrained or successfully constrained.
- Examples: slider-crank mechanism
- For a kinematic chain

$$N = 2P - 4 = 2(j + 2) / 3$$

- Where N = no. of links, P = no. of Pairs and j = no. of joints
- When,

LHS > RHS, then the chain is locked
LHS = RHS, then the chain is constrained
LHS < RHS, then the chain is unconstrained

1.8 Kutzbach Criterion

- DOF of a mechanism in space can be determined as follows:
- In mechanism one link should be fixed. Therefore total no. of movable links are in mechanism is (N-1)
- Any pair having 1 DOF will impose 5 restraints on the mechanism, which reduces its total degree of freedom by 5P1.
- Any pair having 2 DOF will impose 4 restraints on the mechanism, which reduces its total degree of freedom by 4P2
- Similarly, the other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of mechanism. Thus,
- Thus,

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5 - 0P_6$$

- Hence,

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$$

- The above equation is the general form of **Kutzbach criterion.** This is applicable to any type of mechanism including a spatial mechanism.

1.9 Grubler's criterion

- If we apply the Kutzbach criterion to planer mechanism, then equation of Kutzbach criterion will be modified and that modified equation is known as Grubler's Criterion for planer mechanism.
- Therefore in planer mechanism if we consider the links having 1 to 3 DOF, the total number of degree of freedom of the mechanism considering all restraints will becomes,

$$F = 3\,(N\text{-}1) - 2\,P_1 - 1\,P_2$$

- The above equation is known as **Grubler's criterion** for planer mechanism.
- Sometimes all the above empirical relations can give incorrect results, e.g. fig (a) has 5 links, 6 turning pairs and 2 loops. Thus, it is a structure with zero degree of freedom.

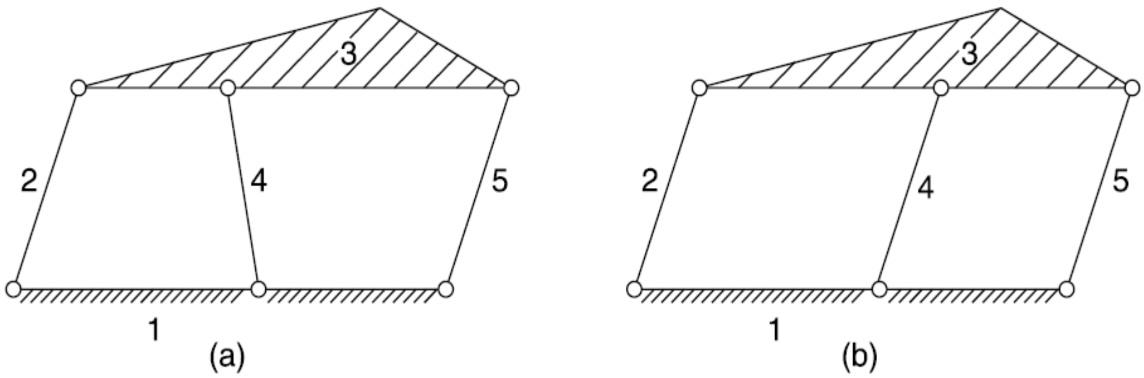


Fig. 1.7

- However, if the links are arranged in such a way as shown in fog. (b), a double parallelogram linkage with one degree of freedom is obtained. This is due to the reason that the lengths of links or other dimensional properties are not considered in these empirical relations.
- Sometimes a system may have one or more link which does not introduce any extra constraint. Such links are known as redundant links and should not be counted to find the degree of freedom. For example fig. (B) has 5 links, but the function of the mechanism is not affected even if any one of the links 2, 4 and 5 are removed. Thus, the effective number of links in this case is 4 with 4 turning pairs, and thus 1 degree of freedom.
- In case of a mechanism possessing some redundant degree of freedom, the effective degree of freedom is given by,

$$F = 3\,(N - 1) - 2\,P_1 - 1P_2 - F_r$$

- Where F = no. of redundant degrees of freedom

1.10 The Four-Bar chain

- A four bar chain is the most fundamental of the plane kinematic chains. It is a much proffered mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints.
- When one of the link fixed, it is known as mechanism or linkage. A link that makes complete revolution is called the crank. The link opposite to the fixed link is called coupler, and the forth link is called a lever or rocker if it oscillates or another crank if it rotates.
- It is impossible to have a four-bar linkage if the length of one of the link is greater than the sum of other three. This has been shown in fig.

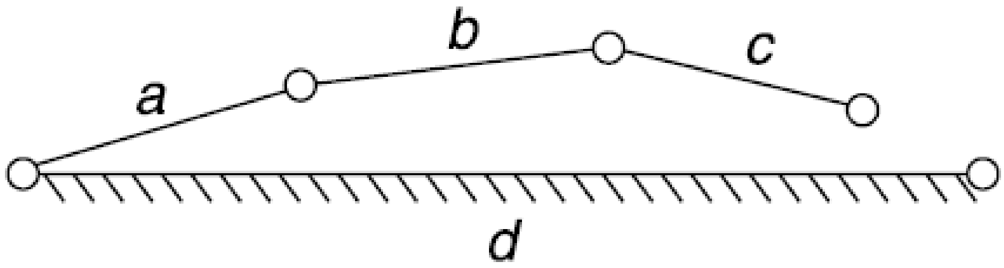


Fig. 1.7 Four bar chain

1.11 Grashof's law:

- We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. 5.18. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths.
- According to Grashof's 's law for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

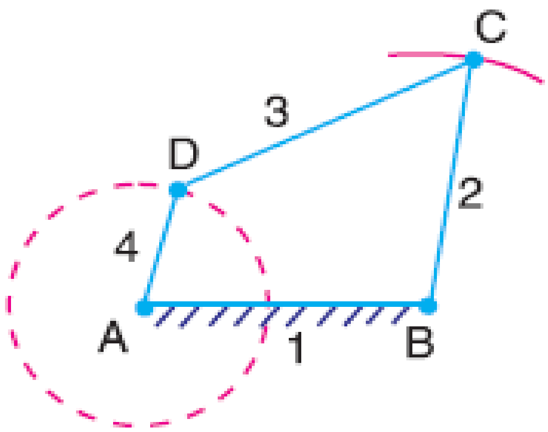


Fig. 1.8 Grashof's law

- A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof's law. Such a link is known as crank or driver. In Fig.5.18, AD (link 4) is a crank.
- The link BC (link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link CD (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link AB (link 1) is known as frame of the mechanism.

1.12 Inversion of Mechanism:

- When the number of links in kinematic chain is more than three, the chain is known as mechanism. When one link of the kinematic chain at a time is fixed, give the different mechanism of the kinematic chain. The method of generating different mechanism by fixing a link is called the inversion of mechanism.
- The number of inversion is equal to the numbers of links in the kinematic chain.
- The inversion of mechanism may be classified as:
  - **a.** Inversion of four-bar chain
  - **b.** Inversion of single slider crank chain
  - **c.** Inversion of double slider crank chain

1.13 Inversion of Four-Bar chain

1.13.1 <u>First inversion: coupled wheel of locomotive</u>

- The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links is shown in Fig.

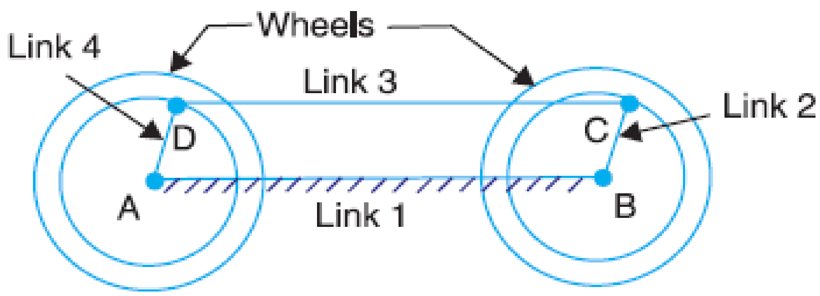


Fig. 1.9 coupled wheel of locomotive

- In this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to Centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

1.13.2 Second inversion: Beam Engine

- A part of the mechanism of a beam engine (also known as cranks and lever mechanism) which consists of four links is shown in Fig. 1.10.
- In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

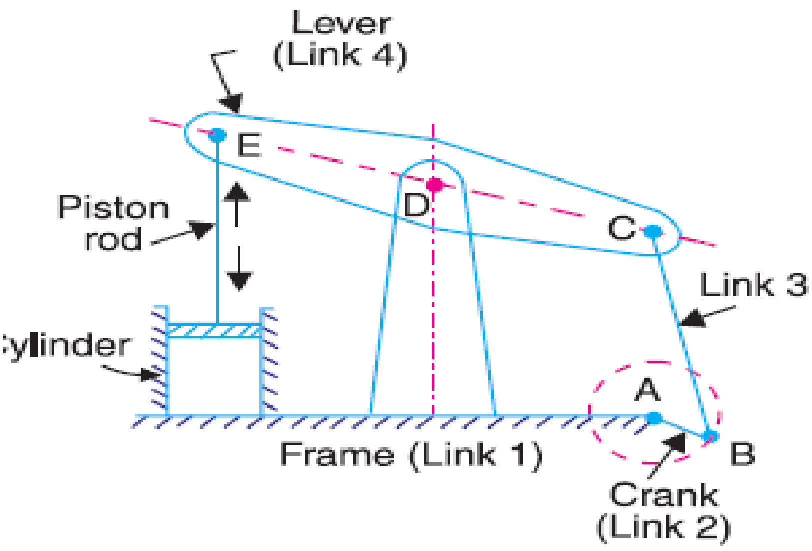


Fig. 1.10 beam engine

– In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

1.13.3 Third inversion: watts indicator mechanism

– A Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links is shown in Fig.

– The four links are: fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers.

– The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

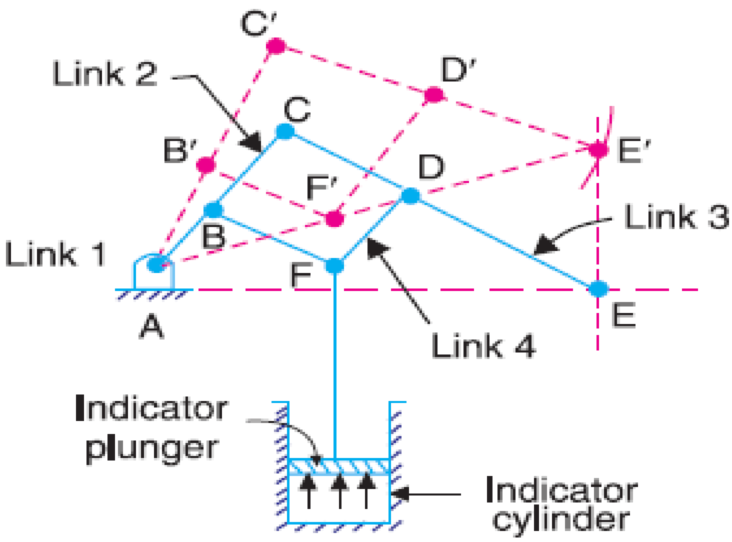


Fig. 1.11 watts indicator mechanism

1.14 The slider-crank chain

- When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a single slider-crank chain or simply a slider-crank chain.
- It is also possible to replace two sliding pairs of a four-bar chain to get a double slider-crank chain. In a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced.
- The distance e between the fixed pivot O and the straight line path of the slider is called the offset and the chain so formed an offset slider-crank chain.
- Different mechanisms obtained by fixing different links of a kinematic chain are known as its inversions.

1.14.1 First inversion

- This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and slider respectively. (fig.a)
- **Applications:**
  - a Reciprocating engine
  - b Reciprocating compressor

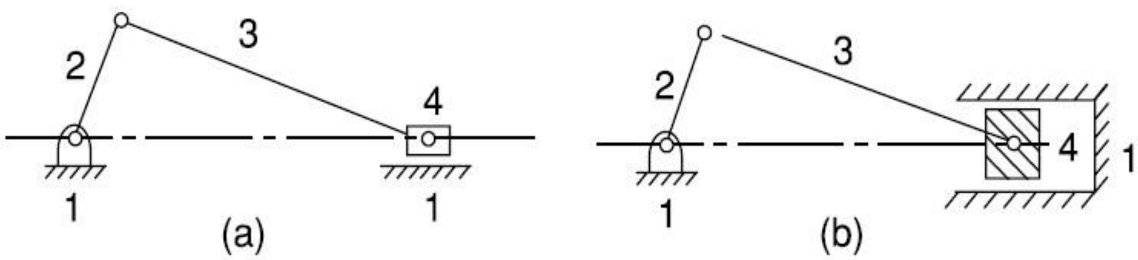


Fig. 1.12 first inversion

1.14.2 Second inversion

- Fixing of the link 2 of a slider-crank chain results in the second inversion.
- **Applications:**
  - a Whitworth quick-return mechanism
  - b Rotary engine

1.14.3 Third Inversion

- By Fixing of the link 3 of the slider-crank mechanism, the third inversion is obtained. Now the link 2 again acts as a crank and the link 4 oscillates.
- **Applications:**
  - a **Oscillating cylinder engine**
  - b Crank and slotted-lever mechanism

1.14.4 Fourth Inversion

- If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained. Link 3 can oscillates about the fixed pivot B on the link 4. This makes the end A of the link 2 to oscillate about B and the end O to reciprocate along the axis of the fixed link 4.

– **Application: Hand Pump**

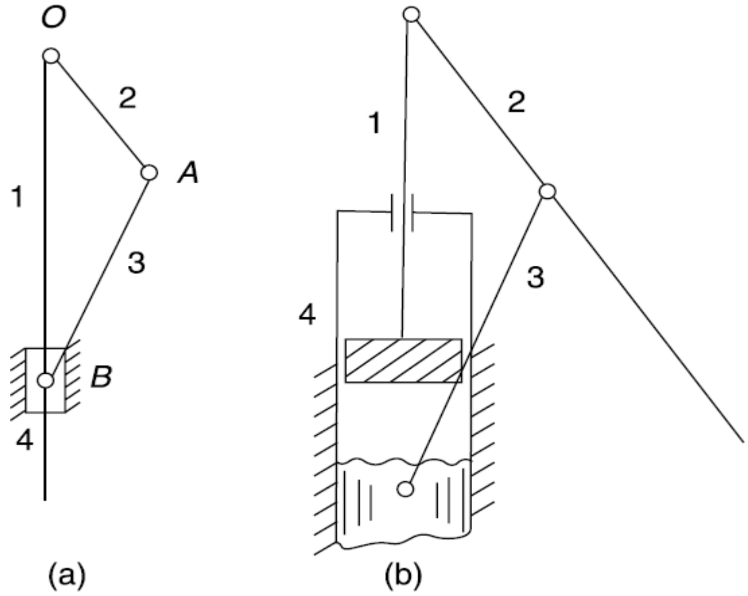


Fig. 1.13 hand pump

– Fig.1.13 shows a hand-pump. Link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

1.15 Whitworth Quick-Return Mechanism:

- This mechanism used in shaping and slotting machines.
- In this mechanism the link CD (link 2) forming the turning pair is fixed; the driving crank CA (link 3) rotates at a uniform angular speed and the slider (link 4) attached to the crank pin at a slides along the slotted bar PA (link 1) which oscillates at D.
- The connecting rod PR carries the ram at R to which a cutting tool is fixed and the motion of the tool is constrained along the line RD produced.

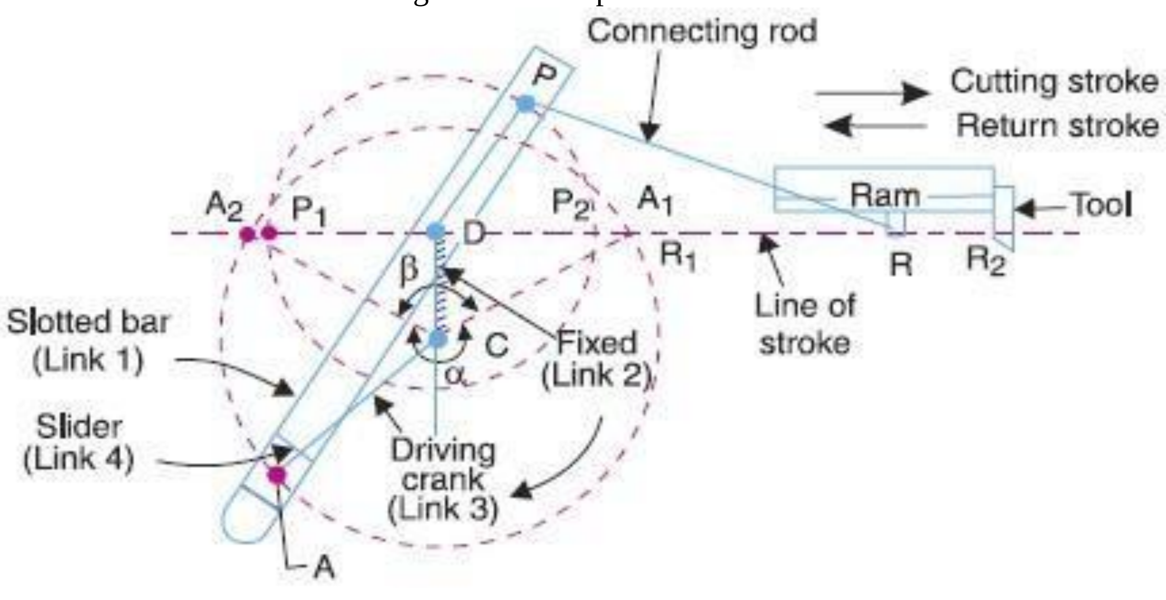


Fig. 1.14 Whitworth quick returns mechanism

– The length of effective stroke = 2 PD. And mark P1R1 = P2 R2 = PR.

$$\frac{\textit{time of cutting stroke}}{\textit{time of return}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} = \frac{360° - \beta}{\beta}$$

1.16 Rotary engine

– Sometimes back, rotary internal combustion engines were used in aviation. But now- a-days gas turbines are used in its place.

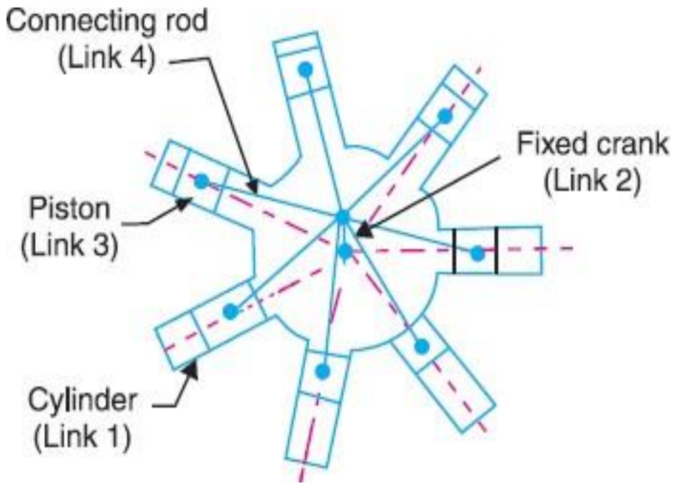


Fig. 1.15 rotary engine

– It consists of seven cylinders in one plane and all revolves about fixed center D, as shown in Fig. 5.25, while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

1.17 Oscillating cylinder engine

– The arrangement of oscillating cylinder engine mechanism, as shown in Fig. Is used to convert reciprocating motion into rotary motion.

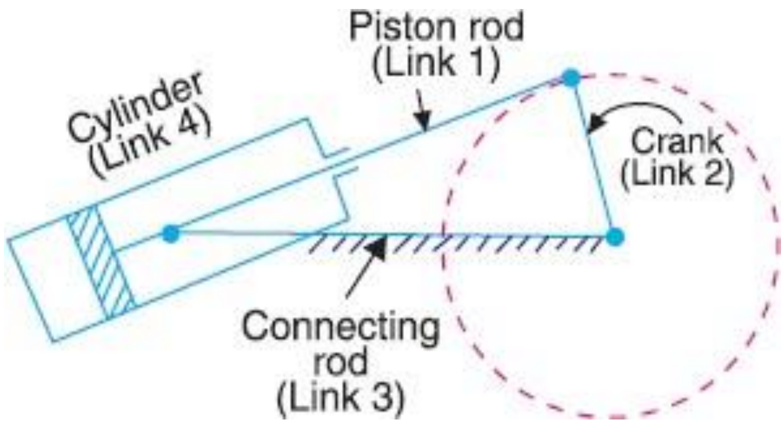


Fig. 1.16 oscillating cylinder engine

– In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at *A*.

1.18 Crank and slotted-lever Mechanism

– This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.
– In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed center C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A.
– A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced.

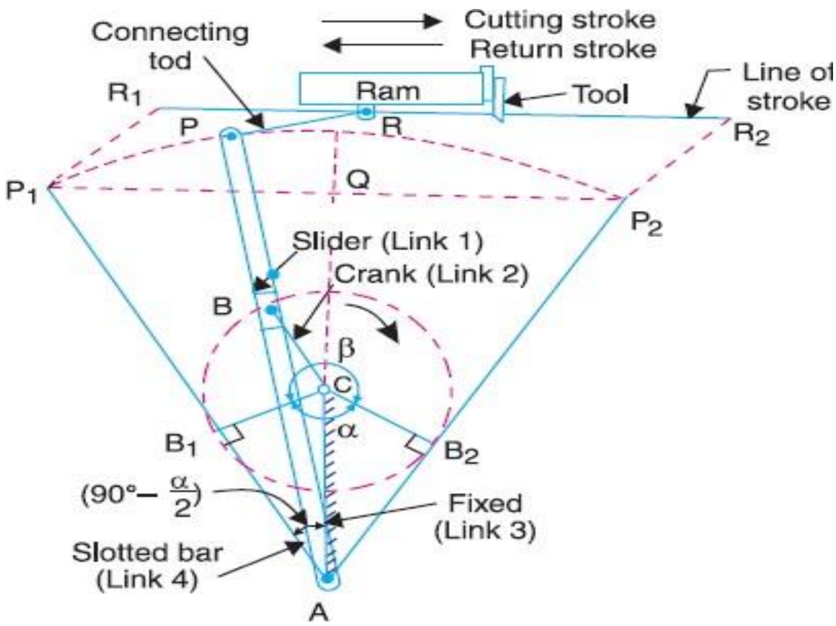


Fig.1.17 Crank and slotted lever mechanism

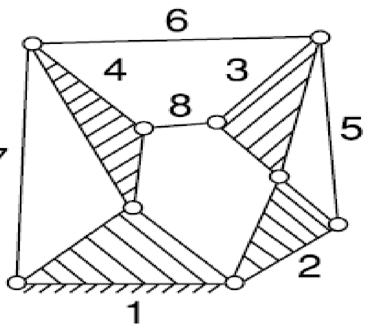
– In the extreme positions, *AP*1 and *AP*2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position *CB*1 to *CB*2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position *CB*2 to *CB*1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\textit{time of cutting stroke}}{\textit{time o}\quad \textit{return}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} = \frac{360° - \alpha}{\alpha}$$

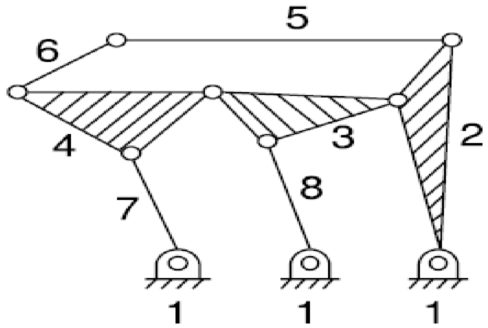
1.19 Example based on Degrees of Freedom:

1 **For the kinematic linkages shown in following fig. calculate the following:**

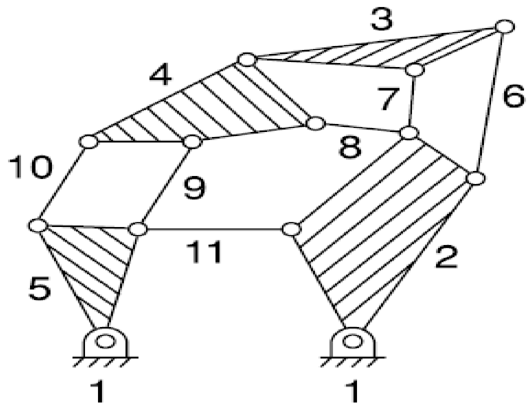
The numbers of binary links (N_b)
The numbers of ternary links (Nt)
The numbers of other (quaternary) links (No)
The numbers of total links (n)
The numbers of loops (L)
The numbers of joints or pairs (P1)
The numbers of degrees of freedom (F)



(a)



(b)



(c)

a $N_b = 4$; $N_t = 4$; $N_0 = 0$; N = 8; L = 4; $P_1 = 11$ (by counting) $P_1 = (N + L – 1) = 11$

$F = 3 (N – 1) – 2P_1$

$F = 3 (8 – 1) – 2×11 = -1$ or,

v $F = N – (2 L + 1)$

$F = 8 – (2 ×4 + 1) = -1$

b $N_b = 4$; $N_t = 4$; $N_0 = 0$; N = 8; L = 3; $P_1 = 10$ (by counting) $P_1 = (N + L – 1) = 10$

2.1 Straight Line Mechanisms

- It permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called ***straight line mechanisms***.
  1. In which only turning pairs are used
  2. In which one sliding pair is used.
- These two types of mechanisms may produce exact straight line motion or approximate straight line motion.
- <u>**Need of Straight Line:**</u>
  1. Sewing Machine converts rotary motion to up/down motion.
  2. Want to constrain pistons to move only in a straight line.
  3. How do you create the first straight edge in the world? (Compass is easy)
  4. Windshield wipers, some flexible lamps made of solid pieces connected by flexible joints.

2.2 Exact Straight Line Motion Mechanisms Made Up Of Turning Pairs

- The principle adopted for a mathematically correct or exact straight line motion is described in Fig.2.1
- Let O be a point on the circumference of a circle of diameter OP. Let OA be any chord and B is a point on OA produced, such that

$$OA \times OB = \text{constant}$$

- The triangles OAP and OBQ are similar.

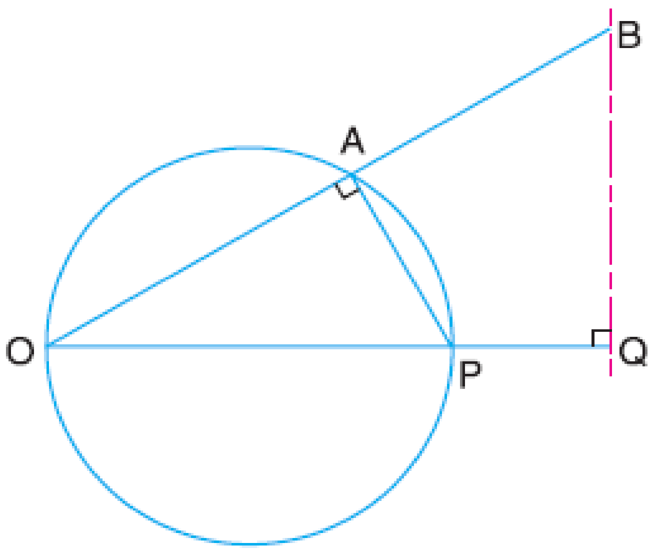


Fig. 2.1 Exact straight line motion mechanism

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$OP \times OQ = OA \times OB$$

$$OQ = \frac{OA \times OB}{OP}$$

- But OP is constant as it is the diameter of a circle; therefore, if $OA \times OB$ is constant, then OQ will be constant.
- Hence

$$OA \times OB = constant$$

- So point B moves along the straight line.

2.3 Peaucellier Mechanism (Exact Straight Line)

- It consists of a fixed link OO1 and the other straight links O1A, OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig. 2.2
- The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O1A. In Fig. 2.2
- AC = CB = BD = DA
- OC = OD
- $OO_1 = O_1A$

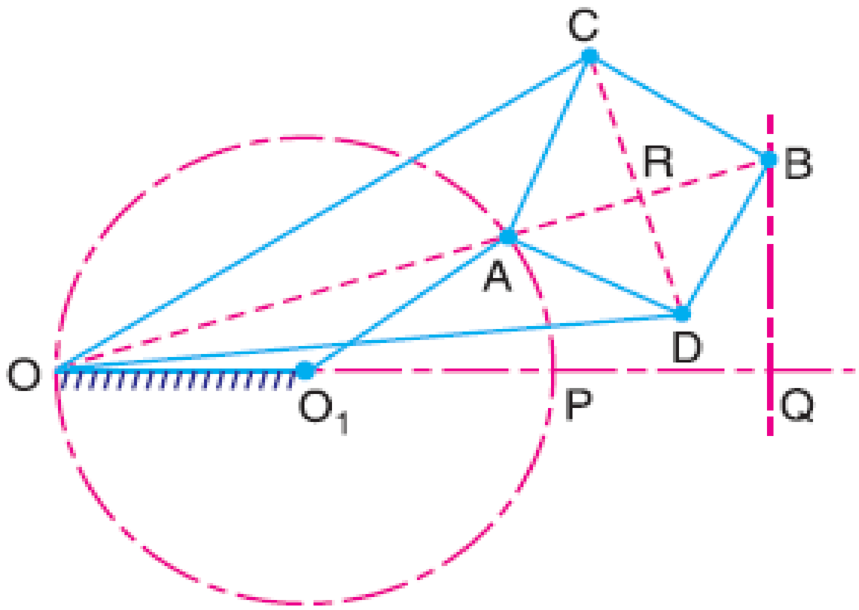


Fig. 2.2 Peaucellier Mechanism

- From right angled triangles ORC and BRC, we have

$$OC^2 = OR^2 + RC^2 \qquad \text{(I)}$$

$$BC^2 = RB^2 + RC^2 \qquad (ii)$$

- From (i) and (ii)

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR - RB)(OR + RB)$$

$$= OB \times OA$$

- Since *OC* and *BC* are of constant length, therefore the product *OB* × *OA* remains constant.

Hart's Mechanism

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.
- It consists of a fixed link OO1 and other straight links O1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig. 2.3.
- The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio. A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to FD and CE.

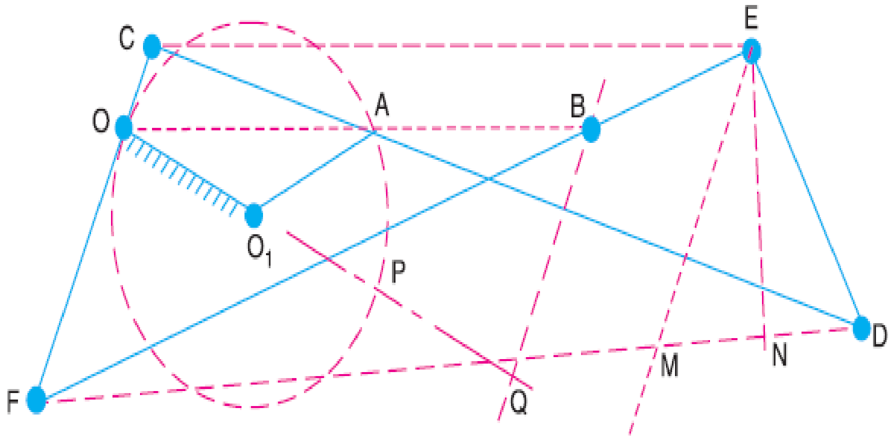


Fig. 2.3 Hart's Mechanism

- Here, FC = DE & CD = EF
- The point O, A and B divide the links FC, CD and EF in the same ratio.
- From similar triangles CFE and OFB,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or CB} = \frac{CE \times OF}{FC} \ldots \ldots \text{(i)}$$

- From similar triangle FCD and OCA

$$\frac{FD}{FC} = \frac{OA}{OC} \quad or\ OA = \frac{FD \times OC}{FC} \quad \ldots .\ (ii)$$

- From above equations,

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC}$$

$$= FD \times CE \times \frac{OC \times OF}{FC^2}$$

- Since the lengths of OC, OF and FC are fixed, therefore

$$OA \times OB = FD \times CE \times cons. \ldots (iii)$$

- From point E, draw EM parallel to CF and EN perpendicular to FD.

$$FD \times CE = FD \times FM \quad (CE = FM)$$

$$= (FN + ND)(FN - MN)$$

$$= FN^2 - ND^2 \ (\mathrm{MN} = \mathrm{ND})$$

$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$ (From right angle triangles FEN and EDN)

$$= E^2 - ED^2 = constant \qquad (iv)$$

– From equation (iii) and (iv),

$$OA \times OB = constant$$

Exact Straight Line Motion consisting of one sliding pair-Scott Russell's Mechanism

– A is the middle point of PQ and OA = AP = AQ. The instantaneous center for the link PAQ lies at I in OA produced and is such that IP is perpendicular to OP.

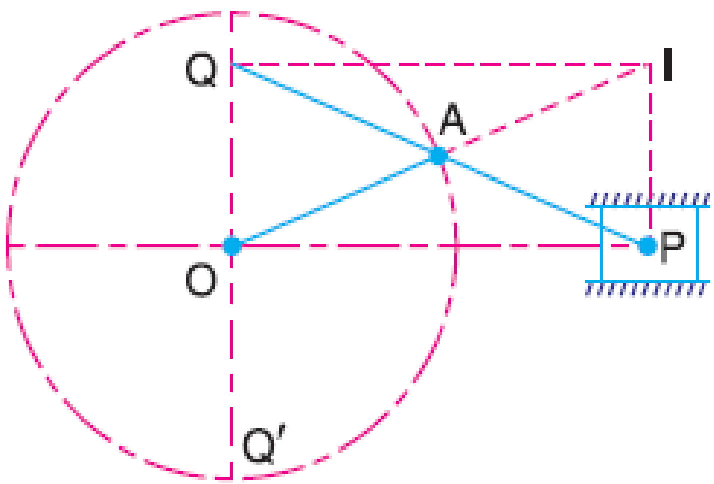


Fig. 2.4 Scott Russell's Mechanism

– Join IQ. Then Q moves along the perpendicular to IQ. Since OPIQ is a rectangle and IQ is perpendicular to OQ, therefore Q moves along the vertical line OQ for all positions of QP. Hence Q traces the straight line OQ′.

– If OA makes one complete revolution, then P will oscillate along the line OP through a distance 2 OA on each side of O and Q will oscillate along OQ′ through the same distance 2 OA above and below O. Thus, the locus of Q is a copy of the locus of P.

Approximate straight line motion mechanisms

Watt's Mechanism

- It has four links as shown in fig. OB, O1A, AB and OO1.

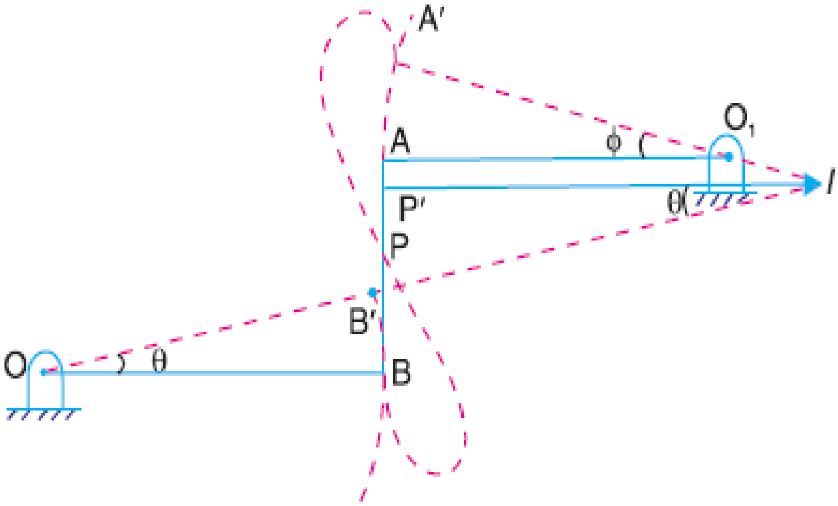


Fig. 2.5 watt's mechanism

- OB and O1A oscillates about centers O and O1 respectively. P is a point on AB such that,

$$\frac{O_1}{OB} = \frac{PB}{PA}$$

- As OB oscillates the point P will describe an approximate straight line.

Modified Scott-Russel Mechanism

- This is similar to Scott-Russel mechanism but in this case AP is not equal to AQ and the points P and Q are constrained to move in the horizontal and vertical directions.

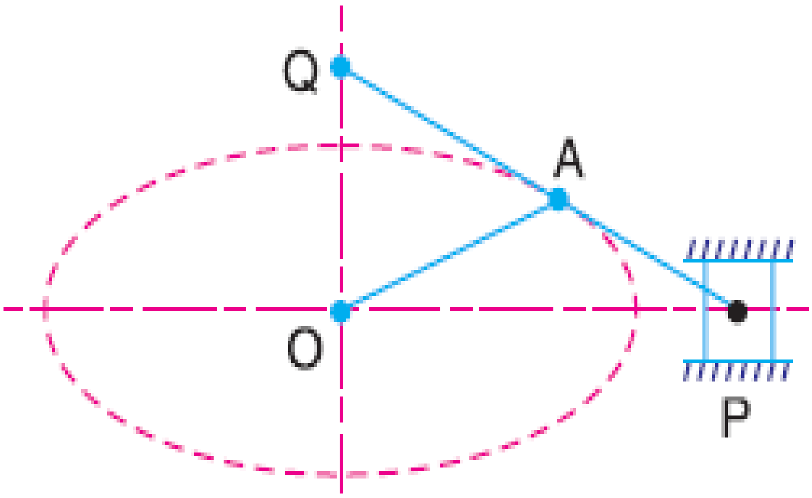


Fig. 2.6 Modified Scott-Russel Mechanisms

- A little consideration will show that it forms an elliptical trammel, so that any point *A* on *PQ* traces an ellipse with semi-major axis *AQ* and semi minor axis *AP*.
- If the point *A* moves in a circle, then for point *Q* to move along an approximate straight line, the length *OA* must be equal (*AP*)2 / *AQ*. This is limited to only small displacement of *P*.

Grasshopper Mechanism

- In this mechanism, the centers O and O1 are fixed. The link OA oscillates about O through an angle AOA1 which causes the pin P to move along a circular arc with O1 as center and O1P as radius.

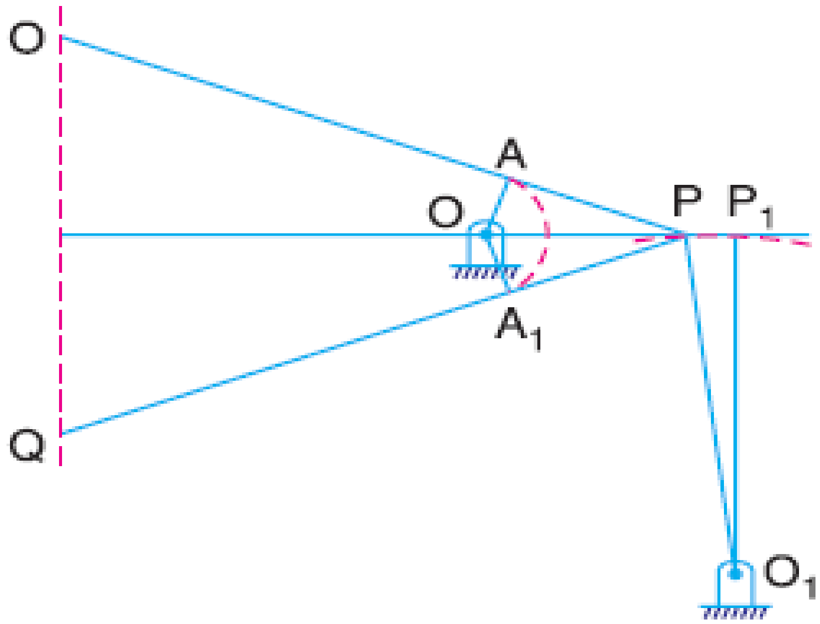


Fig. 2.7 Grasshopper Mechanism

- For small angular displacements of OP on each side of the horizontal, the point Q on the extension of the link PA traces out an approximately a straight path QQ′. if the lengths are such that

$$OA = \frac{AP^2}{AQ}$$

Tchebicheff's Mechanism

- It is a four bar mechanism in which the crossed links OA and O1B are of equal length, as shown in Fig. 2.8.
- The point P, which is the mid-point of AB, traces out an approximately straight line parallel to OO1.

- The proportions of the links are, usually, such that point P is exactly above O or O1 in the extreme positions of the mechanism i.e. when BA lies along OA or when BA lies along BO_1.

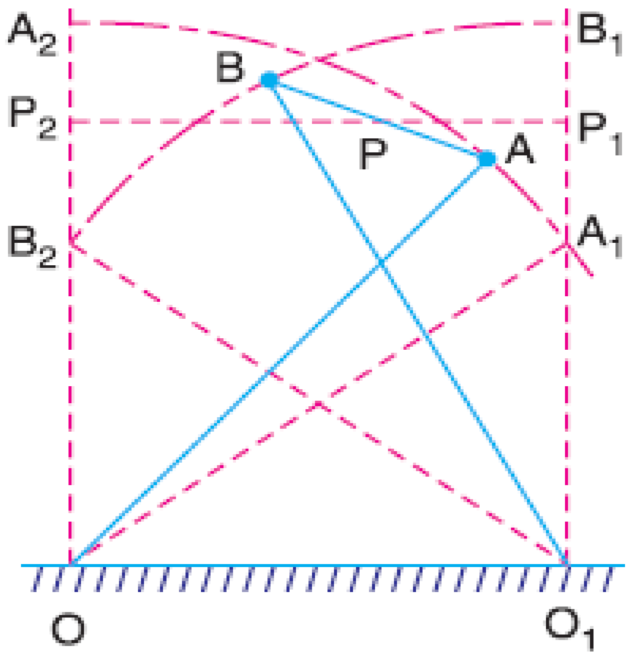


Fig. 2.8 Tchebicheff's mechanism

- It may be noted that the point *P* will lie on a straight line parallel to OO_1, in the two extreme positions and in the mid position, if the lengths of the links are in proportions

$$AB : OO_1 : OA = 1 : 2 : 4.5.$$

Roberts Mechanism

- It is also a four bar chain mechanism, which, in its mean position, has the form of a trapezium. The links OA and O1 B are of equal length and OO1 is fixed. A bar PQ is rigidly attached to the link AB at its middle point P.

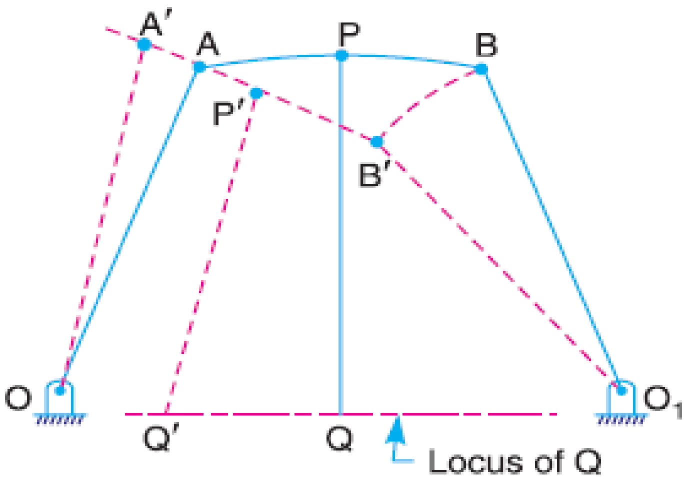


Fig. 2.9 Robert's Mechanism

Industry applications

1. Compliant mechanisms used in new age industries.
2. Mechanical Components form Specialized Motion-Control Systems
3. Mechanism for Planar Manipulation with Simplified Kinematics
4. Five linkages for straight-line motion
5. Seven linkages for transport mechanisms

Question Bank for Assignments

1. Explain inversions of a four bar chain in detail?
2. Explain the working of any two inversions of a single slider crank chain with neat sketches.
3. What is inversion of mechanism? Describe various inversions of double slider crank mechanism with sketches.
4. Explain with neat sketch the working of crank and slotted lever quick return motion mechanism. Deduce the expression for length of stroke in terms of link lengths.
5. State and explain Whitworth quick return mechanism. Also derive an equation for ratio of time taken for return strokes and forward strokes.
6. Define Kinematic pair and discuss various types of kinematic pairs with example.

Tutorial Questions

1. What is a machine? Giving example, differentiate between a machine and a structure.

2. Write notes on complete and incomplete constraints in lower and higher pairs, illustrating your answer with neat sketches.

3. Explain different kinds of kinematic pairs giving example for each one of them.

4. Explain the terms: 1. Lower pair, 2. Higher pair, 3. Kinematic chain, and 4. Inversion.

5. In what way a mechanism differ from a machine?

6. What is the significance of degrees of freedom of a kinematic chain when it functions as a mechanism? Give examples.

7. Explain Grubler's criterion for determining degree of freedom for mechanisms. Using Grubler's criterion for plane mechanism, prove that the minimum number of binary links in a constrained mechanism with simple hinges is four.

8. Sketch and explain the various inversions of a slider crank chain.

9. Sketch and describe the four bar chain mechanism. Why it is considered to be the basic chain?

10. Show that slider crank mechanism is a modification of the basic four bar mechanism.

11. Sketch slider crank chain and its various inversions, stating actual machines in which these are used in practice.

12. Sketch and describe the working of two different types of quick return mechanisms. Give examples of their applications. Derive an expression for the ratio of times taken in forward and return stroke for one of these mechanisms.

13. Sketch and explain any two inversions of a double slider crank chain.

UNIT 2

VELOCITY AND ACCELERATION ANALYSIS

& CAMS

Course Objectives:

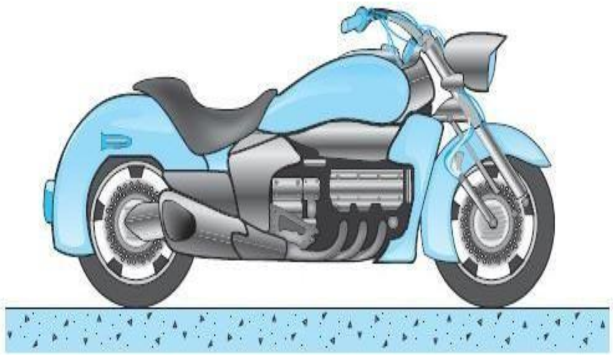
To impart skills to analyze the position, velocity and acceleration of mechanisms and to familiarize higher pairs like cams and principles of cams design.

Course Outcomes:

Analyze the planar mechanisms for position, velocity and acceleration and design cams and followers for specified motion profiles

2

Velocity and AccelerationAnalysis



Course Contents

2.1 Introduction

- There are many methods for determining the velocity of any point on a linkin a mechanism whose direction of motion (i.e. path) and velocity of some other point on the same link is known in magnitude and direction, yet the following two methods:
    1. Instantaneous centremethod
    2. Relative velocitymethod
- The instantaneous centre method is convenient and easy to apply in simple mechanisms, whereas the relative velocity method may be used to any configurationdiagram.

2.2 Velocity Of Two Bodies Moving In StraightLines

2.2.1 Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig. 2.1 (a) and2.2

(a) respectively.

2.2.2 Consider two bodies A and B moving along parallel lines in the same direction withabsolute velocities v_A and v_B such that $v_A > v_B$, as shown in Fig. 2.1 (a). The relative velocity of A with respect toB,

$$v_{AB} = vector\ difference\ of\ v_A\ and\ v_B = \vec{v_A} - \vec{v_B}$$

2.2.3 From Fig. 2.1 (b), the relative velocity of A with respect to B (i.e. v_{AB}) may be writtenin the vector form as follows:

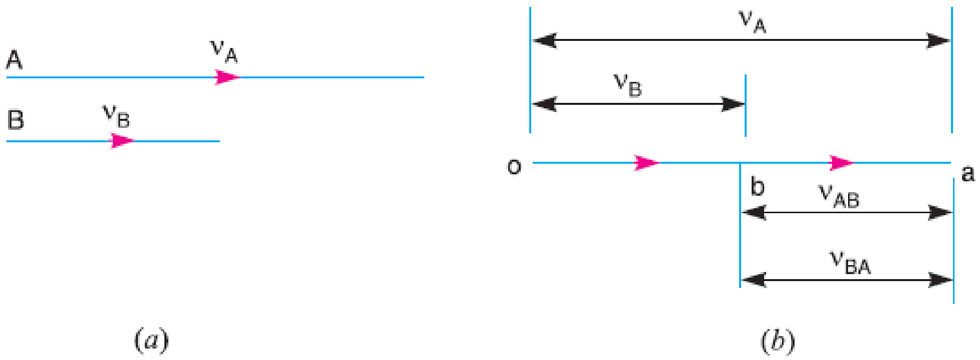


Fig. 3.1 relative velocity of two bodies moving along parallel line

2.2.4 Similarly, the relative velocity of B with respect toA,

$$v_{AB} = vector\ difference\ of\ v_A\ and\ v_B$$

2.2.5 Now consider the body B moving in an inclined direction as shown in Fig. 2.2 (a). The relative velocity of A with respect to B may be obtained by thelaw of parallelogramof velocities or triangle law of velocities. Take any fixed point o and draw vector oa to represent v_A in magnitude and direction to some suitable scale. Similarly, drawvector obto represent v_B in magnitude and direction to the same scale. Then vector barepresents the relative velocity of A with respect to B as shown in Fig. 7.2 (b). Inthe similar way as discussed above, the relative velocity of A with respect toB,

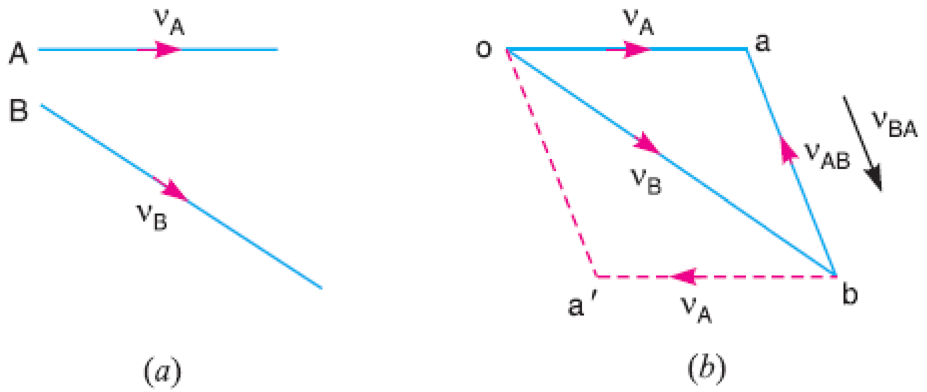


Fig. 3.2 relative velocity of two bodies moving along inclined line

$$v_{AB}=vectordiffereceofv_Aandv_B$$

2.2.6 Similarly, the relative velocity of B with respect toA

$$v_{BA}=vectordiffereceofv_Bandv_A$$

2.2.7 From above, we conclude that the relative velocity of a point A with respect to B(v_{AB}) and the relative velocity of point B with respect to A (v_{BA}) are equal in magnitude but opposite in direction

$$v_{AB}=-v_{BA}$$

2.3 Motion Of ALink

2.3.1 Consider two points A and B on a rigid link A B, as shown in Fig. 2.3 (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular toAB.

2.3.2 Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.

2.3.3 The relative velocity of B with respect to A (i.e. v_{BA}) is represented by the vector ab and is perpendicular to the line A B as shown in Fig. 2.3(b).

2.3.4 We know that the velocity of the point B with respect toA

$$v_{BA}=\omega\times AB \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

2.3.5 SimilarlythevelocityofthepointC onAB withrespecttoA

$$v_{CA}=\omega\times AC \;\ldots\ldots\ldots\ldots\ldots..\ldots\ldots\ldots(ii)$$

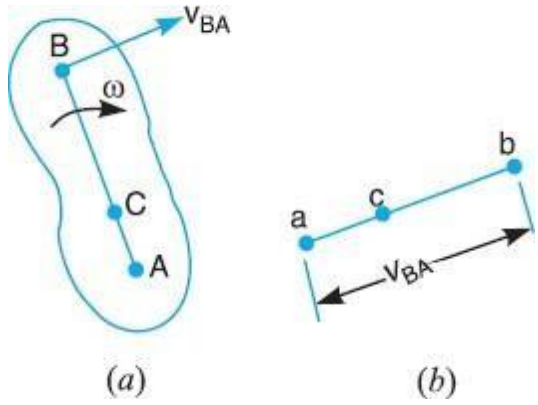


Fig. 3.3 Motion of a Link

2.3.6 F
orm
equation
(i)
and(ii),

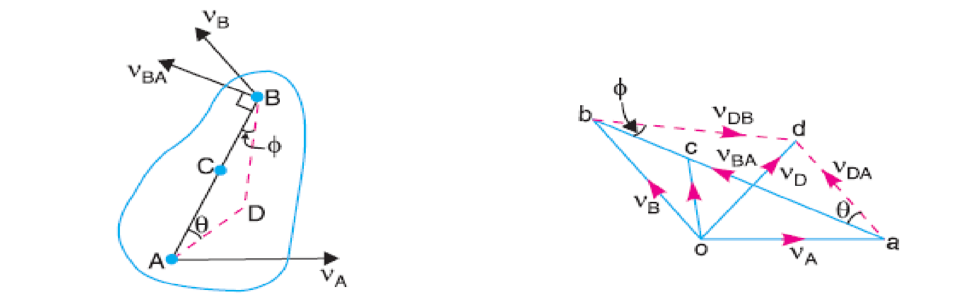
$$\frac{v_{CA}}{v_{BA}} = \frac{\omega \times AC}{\omega \times AB} = \frac{AC}{AB} \ldots \ldots \ldots \ldots \ldots \ldots .... (iii)$$

Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the linkAB.

2.4 Velocity Of A Point On A Link By Relative VelocityMethod

2.4.1 Consider two points A and B on a link as shown in Fig. 2.4 (a). Let the absolute velocity of the point A i.e. v_A is known in magnitude and direction and the absolute velocity of the point B i.e. v_B is knownin direction only. Then the velocity of B maybe determined by drawing the velocity diagram as shown in Fig. 2.4 (b). The velocity diagram is drawn as follows:

1 Take some convenient point o, known as thepole.
2 Through o, draw oa parallel and equal to v_A, to some suitablescale.
3 Through a, draw a line perpendicular to AB ofFig. 2.4 (a). This line will represent the velocity of B with respect to A, i.e.v_{BA}.
4 Through o, draw a line parallel to vBintersecting the line of vBA atb
5 Measure ob, which gives the required velocity of point B (v_B), to the scale



(a) Motion of points on a link.

(b) Velocity diagram.

Fig. 2.4

2.5 Velocities InSlider CrankMechanism

2.5.1 In the previous article, we have discussed the relative velocity method for the velocity of any point on a link, whose direction of motion and velocity of some other point on the same link is known. The same method may also be applied for the velocities in a slider crankmechanism.

2.5.2 A slider crank mechanism is shown in Fig. 2.5 (a). The slider A is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity ω rad/s. Therefore, the velocity of B i.e. v_B is known in magnitude and direction. The slider reciprocates alongthe line of strokeAO.

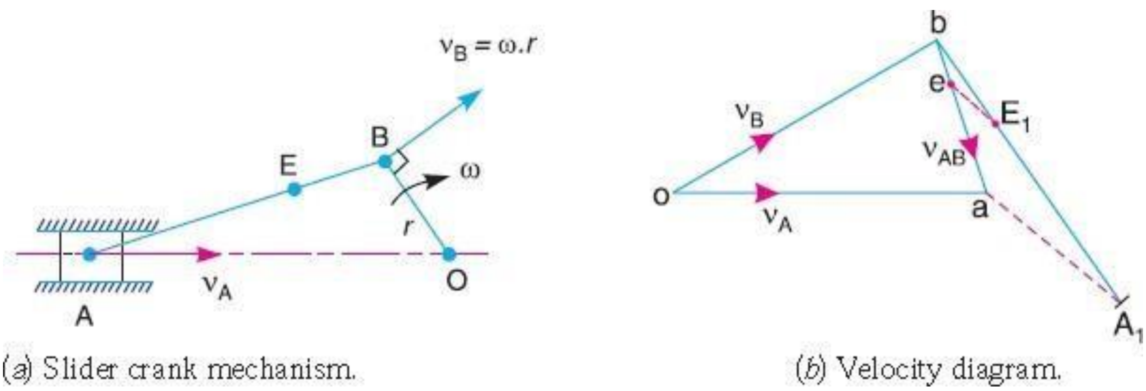


Fig. 2.5

2.5.3 The velocity of the slider A (i.e. v_A) may be determined by relative velocity method as discussed below:

1. From any point o, draw vector ob parallel to the direction of v_B (or perpendicular to OB) such that ob = $v_B = \omega.r$, to some suitable scale, as shown in Fig. 2.5(b).
2. Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB. Now draw vector baperpendicular to A B to represent the velocity of A with respect to B i.e.v_{AB}.
3. From point o, draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors baand oa intersect at a. Now oarepresents the velocity of the slider I.e. v_A, to thescale.

2.5.4 The angular velocity of the connecting rod A B (ω_{AB}) may be determined asfollows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$

2.6 Rubbing Velocity At A PinJoint

2.6.1 The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of thepin.

Consider two links OA and OB connected by a pin joint at O as shown infig.

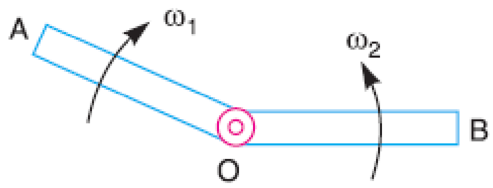


Fig. 3.6 Links connected by pin joints

2.6.2 L
e
t
,

ω1 = angular velocity of link OA

ω2 = angular velocity of link OB

According to thedefinition,

Rubbing velocity at the pin jointO

$= (\omega_1 - \omega_1) \times r$ **if the links move in the samedirection**

$= (\omega_1 + \omega_1) \times r$ **if the links move in the samedirection**

2.7 Examples Based OnVelocity

2.7.1 In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, whilethe link CD = 80 mm oscillatesabout D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

– **Given :** N_{BA} = 120 r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 12.568$ rad/s

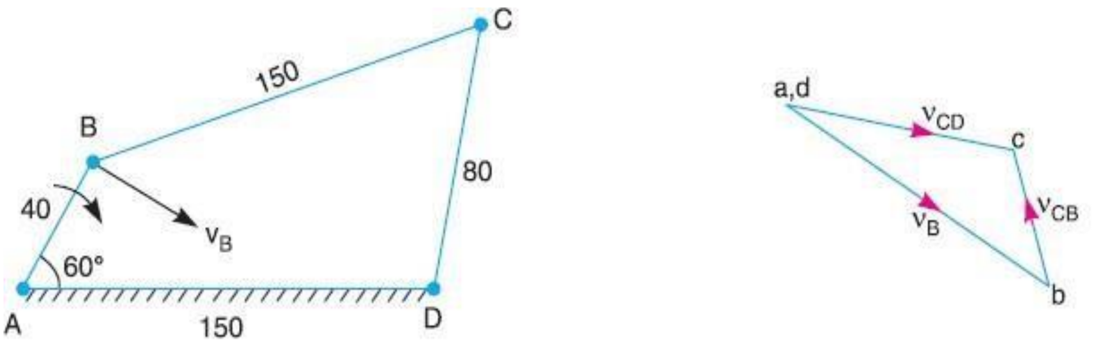
– Since the length of crank A B = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixedpoint),

– Since the length of crank A B = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixedpoint),

$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503$ m/s

– Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector abperpendicular to B A, to some suitable scale, torepresent the velocity of B with respect to A or simply velocity of B (i.e. v_{BA} or v_B) suchthat

Vector ab = $v_{BA} = v_B = 0.503$ m/s



(a) Space diagram (All dimensions in mm).   (b) Velocity diagram.

Fig. 3.7

- Now from point b, draw vector bcperpendicular to CB to represent the velocity ofC with respect to B (i.e. v_{CB}) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bcand dc intersect atc.

  By measurement, we find that

$$V_{CD} = v_C = \text{vector dc} = 0.385 \text{ m/s}$$

- Angular velocity of link CD,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 rad/s$$

2.7.2 The crankand connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centreposition,determine:

1. Velocity of piston, 2. Angular velocity of connecting rod, 3.Velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4.velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. Position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.

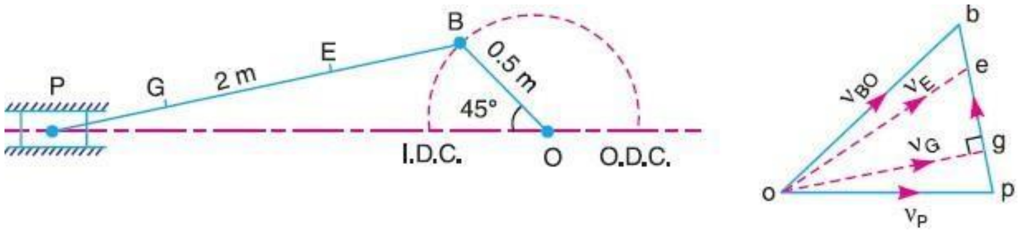
- **Given:**

–N_{BO} = 180 r.p.m. or $\omega_{BO} = 2\pi \times 180/60 = 18.852$ rad/s

- Since the crank length *OB* = 0.5 m, therefore linear velocity of *B* with respect to*O* or velocity of *B* (because *O* is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

- First of all draw the space diagram and then draw the velocity diagram as shown in fig.



(a) Space diagram. (b) Velocity diagram.

Fig. 3.8

- By measurement, we find that velocity of pistonP,

$$v_P = vector\ op = 8.15\ m/s$$

- Fromthevelocitydiagram,wefindthatthevelocityofPwithrespecttoB

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}$$

- Since the length of connecting rod PB is 2 m, therefore angular velocity ofthe connectingrod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4 rad/s$$

$$v_E = vector\ oe = 8.5\ m/s$$

- We know that velocity of rubbing at the pin ofcrank-shaft

$$= \frac{d_0}{2} \times \omega_{BO} = 0.47\ m/s$$

- Velocity of rubbing at the pin ofcrank

$$= \frac{d_B}{2}(\omega_{BO} + \omega_{PB}) = 0.6675 m/s$$

- Velocity of rubbing at the pin ofcrank

$$= \frac{d_c}{2} \times \omega_{PB} = 0.051 m/s$$

- By measurement we findthat

$$vector\ bg = 5\ m/s$$

- By measurement we find linear velocity of pointG

$$v_G = vector\ og = 8\ m/s$$

2.7.2 In Fig. , the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are: OA = 28 mm; AB = 44 mm; BC 49 mm; and BD = 46 mm. The centredistance between the canters of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC isvertical.

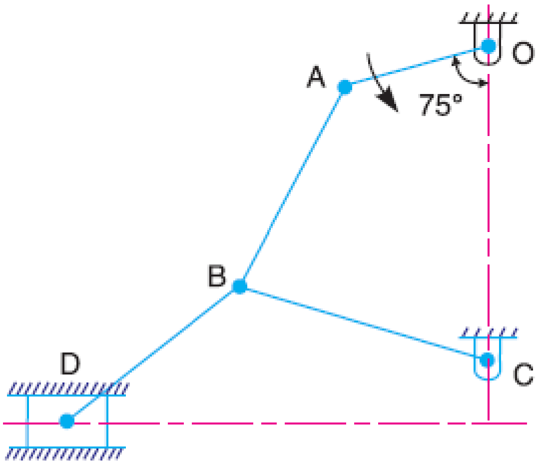


Fig. 3.9

– **Given :**

– N_{AO} = 180 r.p.m. or ω_{BO} = 2 π × 180/60 = 18.852 rad/s

– OA = 28mm

$$v_{OA} = v_A = \omega_{AO} \times AO = 1.76\ m/s$$

– Since the points O and C are fixed, therefore these points are marked as one point, inthe velocity diagram. Now from point o, draw vector oaperpendicular to O A, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A suchthat

$$vector\ oa = v_{OA} = v_A = 1.76\ m/s$$

– From point a, draw vector abperpendicular to A B to represent the velocity of Bwith respect A (i.e. v_{BA}) and from point c, draw vector cbperpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (i.e. v_{BC} or v_B). The vectors ab and cb intersect atb.

– From point b, draw vector bdperpendicular to BD to represent the velocity ofD with respecttoB(i.e.v_{DB})andfrompointo,drawvectorodparalleltothepathofmotion of the slider D which is horizontal, to represent the velocity of D (i.e. v_D). The vectors bdand od intersect atd.

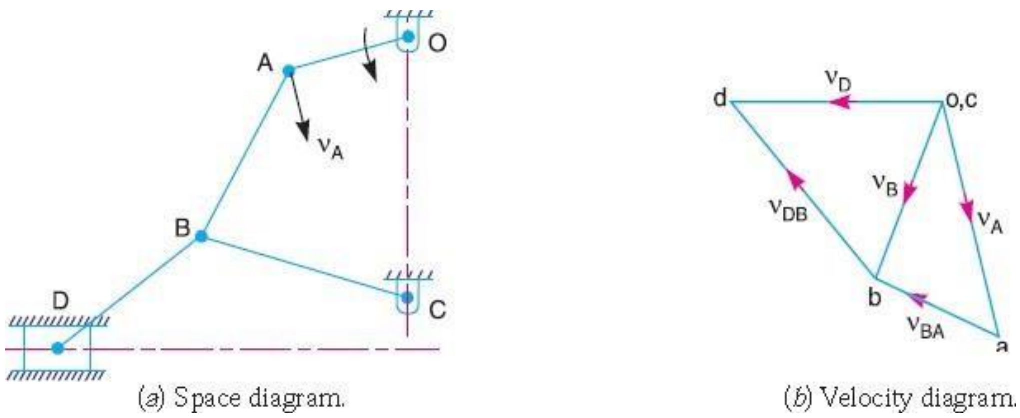


Fig.3.10

– By measurement, we find that velocity of sliderD,

$$v_D = \ vector\ od = 1.6\ m/s$$

– By measurement from velocity diagram, we find that velocity of D with respect to B,

$$v_{DB} = \ \ vector\ bd = 1.7\ m/s$$

– Therefore angular velocity of linkBD

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 rad/s$$

The mechanism, as shown in Fig. 7.11, has the dimensions of various links as follows:

AB = DE = 150 mm; BC = CD = 450 mm; EF = 375 mm. The crank AB makes an angle of
45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC.
The block F moves in the horizontal guides, being driven by the linkEF. Determine: 1. velocity of the block F, 2. angular velocity of DC, and 3. rubbing speed at the pin C which is 50 mm in diameter.

– Given:

–N_{BA} = 120 r.p.m. or ω_{BA} = 2 π × 120/60 = 4 π rad/s

– Since the crank length A B = 150 mm = 0.15 m, therefore velocity of Bwith respect to A or simply velocity of B (because A is a fixedpoint),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\pi \times 0.15 = 1.885\ m/s$$

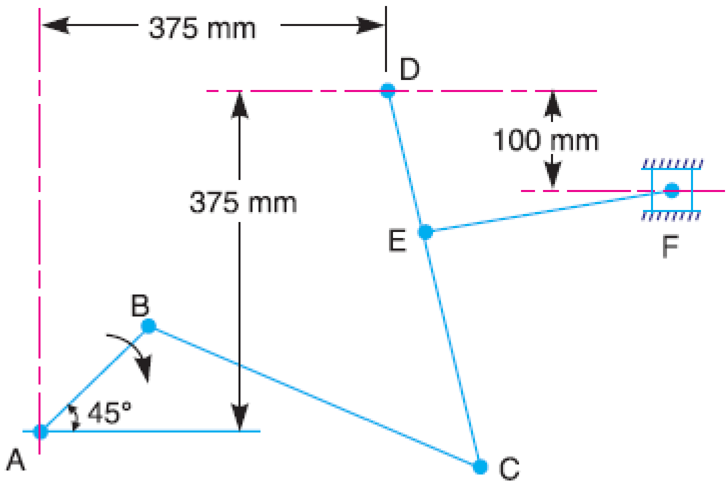
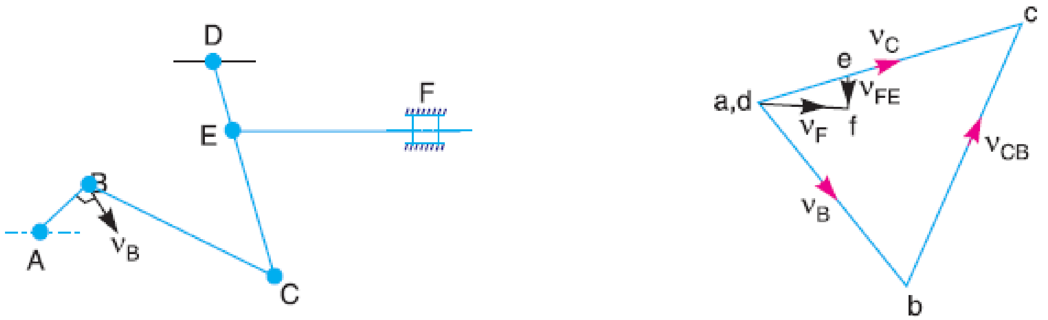


Fig.3.11

– Since the points A and D are fixed, therefore these points are marked as one point as shown in Fig. (b). Now from point a, draw vector abperpendicular to AB, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B, such that

Vector ab = v_{BA} = v_B = 1.885 m/s

– The point C moves relative to B and D, therefore draw vector bcperpendicular to BC to represent the velocity of C with respect to B (i.e. vCB), and from point d, draw vector dc perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (i.e. vCDor vC). The vectors bcand dc intersectat c.



(a) Space diagram. (b) Velocity diagram.

Fig. 3.12

– Since the point E lies on DC, therefore divide vector dc in e in the same ratio as E divides CD in Fig. (a). In otherwords

ce/cd = CE/CD

– From point e, draw vector efperpendicular to EF to represent the velocity of F with respect to E (i.e. v_{FE}) and from point d draw vector df parallel to the path of motion of F, which is horizontal, to represent the velocity of F i.e. v_F. The vectors ef and df intersect atf.

v_F= vector df = 0.7 m/s

– By measurement from velocity diagram, we find that velocity of C with respect to D,

v_{CD}= vector *dc* = 2.25 m/s

$$\omega_{DC} = \frac{v_{CD}}{DC} = 5\,\frac{rad}{s}$$

– From velocity diagram, we find that velocity of C with respect toB,

v_{CB}= vector *bc*= 2.25 m/s

–Angular velocity ofBC,

$$\omega_{CD} = \frac{v_{CD}}{BC} = \frac{2.25}{0.45} = 5 rad/s$$

2.8 Velocity Of A Point On A Link By Instantaneous CentreMethod

2.8.1 The instantaneous centremethod of analyzing the motion in a mechanism is based upon the concept that any displacement of a body (or a rigid link) having motion in oneplane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centreor virtual centreofrotation.

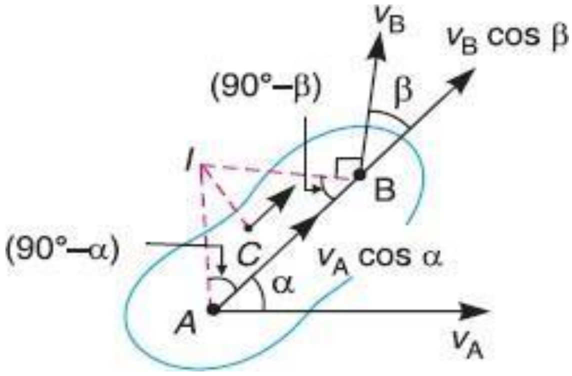


Fig. 3.13 velocity of a point on a link

2.8.2 The velocities of points A and B, whose directions are given a link.by angles α and β as shown in Fig. If vAis known in magnitudeand direction and vB in direction only, then the magnitude of vB may be determined by the instantaneous centremethodas discussed below:

2.8.3 Draw AI and BI perpendiculars to the directions vA and vB respectively. Let these lines intersect at I, which is known as instantaneous centre or virtual centreof the link. The complete rigid link is to rotate or turn about the centreI.

2.8.4 Since A and B are the points on a rigid link, therefore there cannot be any relative motion between them along lineAB.

2.8.5 Now resolving the velocities alongAB,

$$\boldsymbol{v_A \times \cos\alpha = v_B \times \cos\beta}$$

$$\frac{\boldsymbol{v_A}}{\boldsymbol{v_B}} = \frac{\boldsymbol{\cos\beta}}{\boldsymbol{\cos\alpha}} = \frac{\boldsymbol{\sin(90-\beta)}}{\boldsymbol{\sin(90-\alpha)}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (\boldsymbol{i})$$

2.8.6 Applying Lami'stheorem to triangleABI,

$$\frac{\boldsymbol{AI}}{\boldsymbol{\sin(90-\beta)}} = \frac{\boldsymbol{BI}}{\boldsymbol{\sin(90-\alpha)}}$$

$$\frac{\boldsymbol{AI}}{\boldsymbol{BI}} =$$

2.8.7 Hence

$\sin(90-\beta)$ … …

… $\sin(90-\alpha)$ … … … …… … … …. … (ii)

…

$$\frac{v_A}{v_B} = \frac{AI}{BI}$$

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

2.8.8 If C is any other point on link,then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

2.9 Properties OfInstantaneousMethod

2.9.1 The following properties of instantaneous centreare important:

1 A rigid link rotates instantaneously relative to another link at the instantaneous centrefor the configuration of the mechanismconsidered.

2 The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (i.e. instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link. In other words, the velocity of the instantaneous centre relative to any third rigid link will be same whether the instantaneous centreis regarded as a point on the first rigid link or on the second rigidlink

2.10 Number OfInstantaneous Centre In AMechanism:

2.10.1 The number of instantaneous centresin a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number 3 of instantaneous centresis the number of combinations of n links taken two at a time. Mathematically, number of instantaneouscentres

$$N = \frac{n\,(n-1)}{2}\text{, } where\ n = Number\ of\ Link$$

2.11 Location of Instantaneouscentres:

2.11.1 The following rules may be used in locating the instantaneous centresin amechanism

:

1 When the two links are connectedby a pin joint (or pivotjoint), theinstantaneous centrelies on the centreof the pin as shown in Fig. (a). such an instantaneous centreis of permanent nature, but if one of the links is fixed, the instantaneous centrewill be of fixedtype.

2 When the two links have a pure rolling contact (i.e. link 2 rolls without slipping upon the fixed link 1 which may be straight or curved),the instantaneous centrelies on their point of contact, as shown in Fig.(b). The velocity of any point A on the link2 relative to fixed link 1 will be perpendicular to I12 A and is proportional to I12A.

3 When the two links have a sliding contact, the instantaneous centrelies on the common normal at the point of contact. We shall consider the following three cases:

a. When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig.(c), the instantaneous centrelies at infinity and each point on the slider have the samevelocity.

b. When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig.(d),the instantaneous centrelies on the centreof curvature of the curvilinear path in the configuration at thatinstant.

c. When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. 6.6 (e), the instantaneous centrelies at the centreof curvature i.e. the centreof the circle, for all configuration of thelinks.

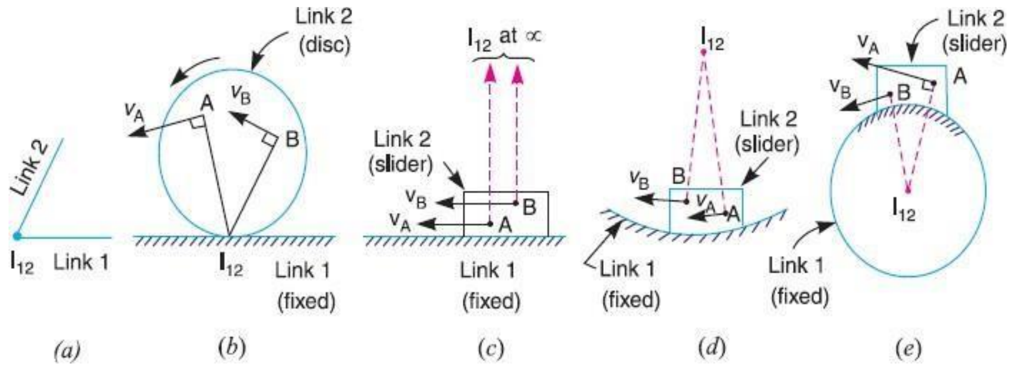


Fig. 3.14 Location of Instantaneous centres

2.12 Kennedy'sTheorem

2.12.1 The Aronhold Kennedy's theorem states that "if three bodies move relatively to each other, they have three instantaneous centresand lie on a straightline."

2.12.2 Consider three kinematic links A , B and C having relative plane motion. The numberof instantaneous centres(N) is givenby

$$N = \frac{n\,(n-1)}{2} = \frac{3\,(3-1)}{2} = 3$$

2.12.3 The two instantaneous centresat the pin joints of B with A , and C with A (i.e. $I_{ab}$and I_{ac}) are the permanent instantaneous centre According to AronholdKennedy's theorem, the third instantaneous centreI_{bc}must lie on the line joining $I_{ab}$and I_{ac}. In order to prove this let us consider that the instantaneous centreI_{bc}lies outside the line joining $I_{ab}$and I_{ac} as shown in Fig. The point I_{bc}belongs to both the links B and C. Let us consider the point I_{bc}on the link B. Its velocity v_{BC} must beperpendicular to the line joining $I_{ab}$and I_{bc}. Now consider the point I_{bc}on the link C. Its velocity v_{BC} must be perpendicular to the line joining I_{ac}andI_{bc}.

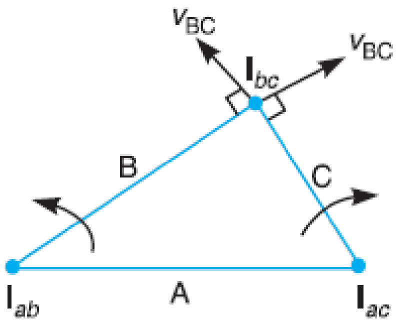


Fig. 3.15 Aronhold Kennedy's theorem

2.12.4 We have already discussed that the velocity of the instantaneous centreis samewhether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point Ibc cannot be perpendicular to both lines IabIbcand IacIbcunless the point Ibclies on the line joining the points Iab and Iac. Thus the three instantaneous centres(Iab, Iac and Ibc) must lie on the same straight line. The exact location of Ibcon line IabIacdepends upon the directionsand magnitudesof the angular velocities of B and C relative toA.

2.13 Acceleration Diagram for aLink

2.13.1 Consider two points A and B on a rigid link as shown in Fig. (a). Let the point B moves with respect to A, with an angular velocity of **ω** rad/s and let **α** rad/s2 be the angular acceleration of the linkAB.

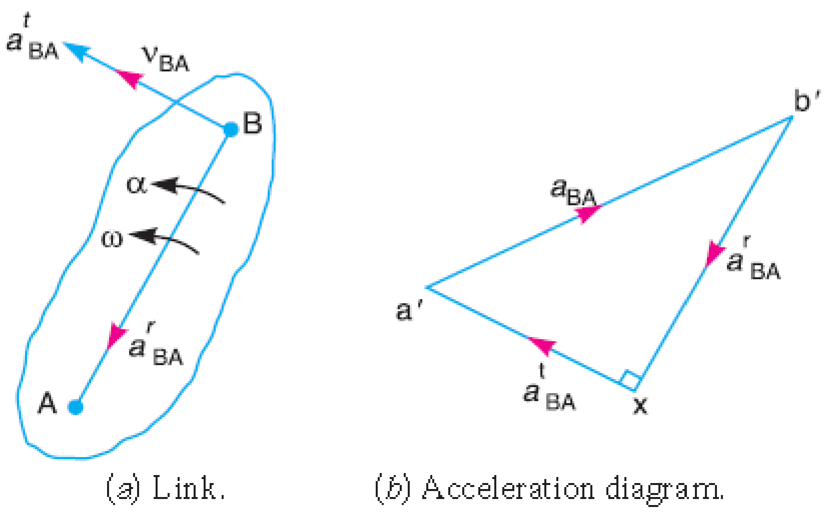


Fig. 3.16 Acceleration of a link

2.13.2 We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components.

1. The centripetal or radial component, which is perpendicular to the velocity of the particle at the giveninstant.
2. The tangential component, which is parallel to the velocity of the particle at the given

2.13.3 Thus for a link A B, the velocity of point B with respect to A (i.e. vBA) is perpendicular to the link A B as shown in Fig.(a). Since the point B moves with respect to A with an angular velocity of **ω** rad/s, therefore centripetal or radial component of the

acceleration of B with respect to A

$$a^r_{BA} = \omega^2 \times Length\ of\ link\ AB = \omega^2 \times AB = v^2_{BA} / AB$$

2.13.4 This radial component of acceleration acts perpendicular to the velocity vBA, In other words, it acts parallel to the linkAB.

We know that tangential component of the acceleration of B with respect to A ,

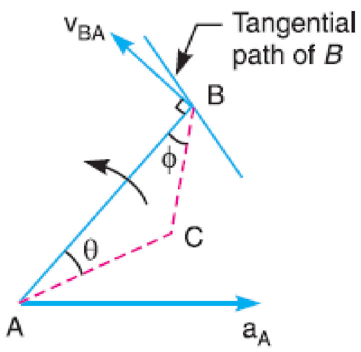
$$a^t_{BA} = \alpha \times Length\ of\ link\ AB = \alpha \times AB$$

2.13.5 This tangential component of acceleration acts parallel to the velocity vBA. In other words, it acts perpendicular to the linkAB.

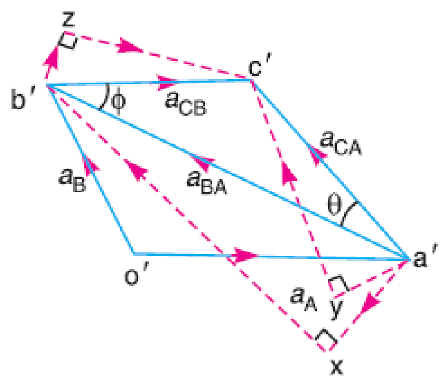
2.13.6 In order to draw the acceleration diagram for a link A B, as shown in Fig. 8.1 (b), from any point b', draw vector b'x parallel to BA to represent the radial component of acceleration of B with respect toA.

2.14 Acceleration of a Point on aLink

2.14.1 Consider two points A and B on the rigid link, as shown in Fig. (a). Let the acceleration of the point A i.e. aAis known in magnitude and direction and the direction of path of B is given. The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussedbelow.



(a) Points on a Link.



(b) Acceleration diagram.

Fig. 3.17 acceleration of a point on a link

2.14.2 From any point o', draw vector o'a' parallel to the direction of absolute acceleration at point A i.e. a_A,to some suitable scale, as shown in Fig. 8.2(b).

2.14.3 We know that the acceleration of B with respect to A i.e. aBA has the following two components:

1 Radial component of the acceleration of B with respect to A i.e. a^r_{BA}

2.14.4 Draw vector a’x parallel to the link AB suchthat,

$$\text{vector } a'x = a^r_{BA} = v^2_{BA}/AB$$

2.14.5 From point x, draw vector xb’ perpendicular to AB or vector a’x and through o’draw

a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B

2.14.6 By joining the points a' and b' we may determine the total acceleration of B with respect to A i.e. a_{BA}. The vector a' b' is known as acceleration image of the linkAB.

2.14.7 For any other point C on the link, draw triangle a' b' c' similar to triangle ABC. Now vector b' c' represents the acceleration of C with respect to B i.e. a_{CB}, and vector a'c'

represents the acceleration of C with respect to A i.e. a_{CA}. As discussedabove, a_{CB} and a_{CA} will each have two components as follows:

a. a_{CB} has twocomponents; a^r_{CB} and a^t_{CB} as shown by triangle b’zc’ infig.b
b. a_{CA} has two components; a^r_{CA} and a^t_{CA} as shown by trianglea’yc’

2.14.8 The angular acceleration of the link AB is obtained by dividing the tangential component of acceleration of B with respect to A to the length ofthe link.

$$\alpha_{AB} = a^t_{BA}/AB$$

2.15 Acceleration in Slider CrankMechanism

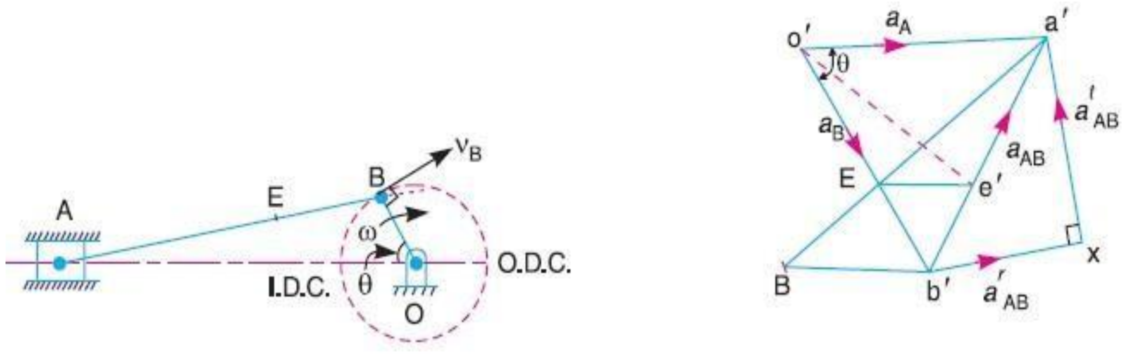
2.15.1 A slider crank mechanism is shown in Fig. 8.3 (a). Let the crank OB makes an angle **θ** with the inner dead centre(I.D.C) and rotates in a clockwise direction about the fixedpoint O with uniform angular velocity ω_{BO}rad/s

2.15.2 Velocity of B with respect to O or velocity of B (because O is a fixedpoint),

$$v_{BO} = v_B = \omega_{BO} \times OB \text{ acting tangentially at } B$$

2.15.3 We know that centripetal or radial acceleration of B with respect to O or acceleration of B (Because O is a fixedpoint)

$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{BO}$$



(a) Slider crank mechanism. (b) Acceleration diagram.

Fig. 3.18 acceleration in the slider crank mechanism

2.15.4 The acceleration diagram, as shown in Fig. 8.3 (b), may now be drawn as discussed below:

1. Draw vector o' b' parallel to BO and set off equal in magnitude of a=a, to some BO suitablescale.
2. From point b', draw vector b'x parallel to BA. The vector b'xrepresents theradial componentof the acceleration of A with respect to B whose magnitude is given by :

$$a^r_{AB} = v^2_{AB}/BA$$

3. From point x, draw vector xa' perpendicular to b'x. The vector xa' represents the tangential components of the acceleration of A with respect toB.
4. Since the point A reciprocates along AO, therefore the acceleration mustbe parallel to velocity. Therefore from o', draw o' a' parallel to A O, intersecting the vector xa' at a'.
5. The vector b' a', which is the sum of the vectors b' x and x a', represents the total acceleration of A with respect to B i.e. a_{AB}. The vector b'a' represents theacceleration of the connecting rodAB.
6. The acceleration of any other point on A B such as E may be obtained by dividing the vector b' a' at e' in the same ratio as E divides A B in Fig. 8.3 (a). In other words

$$a'e'/a'b' = AE/AB$$

7. The angular acceleration of the connecting rod A B may be obtained by dividingthe tangential component of the acceleration of A with respect to B to the length of AB. In other words, angular acceleration ofAB,

$$\alpha_{AB} = a^t_{AB}/AB$$

2.16 Examples Based onAcceleration

3.16.1 The crank of the slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine:

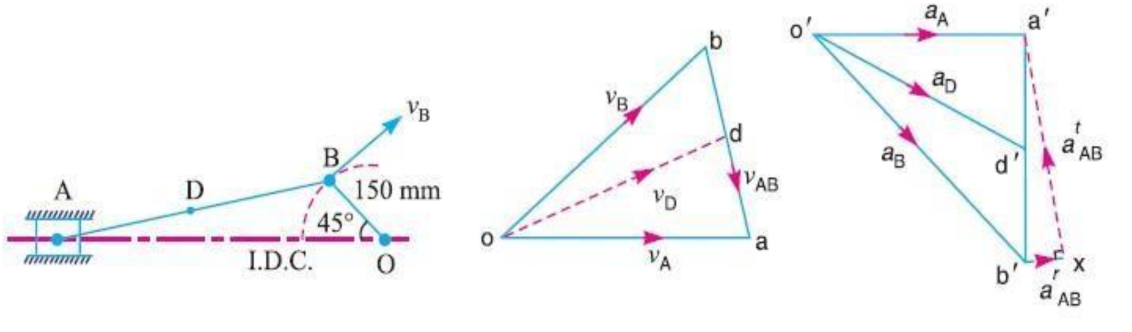
1. **Linear velocity and acceleration of the midpointof the connecting rod, and**
2. **Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centreposition**

- **Given:**
- N_{BO} = 300 r.p.m. or ω_{BO} = 2 π × 300/60 = 31.42 rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m
- WeknowthatlinearvelocityofBwithrespecttoO orvelocityofB,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713\ m/s$$

- Draw vector obperpendicular to BO, to some suitable scale, to represent thevelocity of B with respect to O or simply velocity of B i.e. v_{BO} or vB, suchthat

vectorob = v_{BO} = v_B = 4.713 m/s



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 3.19

- From point b, draw vector baperpendicular to BA to represent the velocity of A with respect to B i.e. v_{AB} , and from point odraw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A i.e. v_A. The vectors baand oa intersect ata.
- By measurement we find the velocity A with respect toB,

$$v_{AB} = vector\ ba = 3.4\ m/s$$

$$v_A = \text{vector oa} = 4\ \text{m/s}$$

- In order to find the velocity of the midpoint D of the connectingrod A B, dividethe vector baat d in the same ratio as D divides A B, in the space diagram. In other words,

$$bd/ba = BD/BA$$

- By measurement, we findthat

$$v_D = \text{vector od} = 4.1\ \text{m/s}$$

- Weknowthattheradial componentoftheacceleration ofBwithrespectto Oorthe acceleration ofB,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1\ m/s^2$$

- Andtheradialcomponentoftheaccelerationof A withrespecttoB,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3\ m/s^2$$

$$vector\ \ o'b' = a^r_{BO} = a_B = 148.1\ m/s^2$$

- By measurement, we findthat

$$a = \ ector\ o'd' = 117\ m/s^2$$

- We know that angular velocity of the connecting rodAB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 rad/s^2$$

- From the acceleration diagram, we findthat

$$a^t_{AB} = 103 m/s^2$$

- We know that angular acceleration of the connecting rodAB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67\ rad/s^2$$

Q. An engine mechanism is shown in Fig. 8.5. The crank CB = 100 mm and the connecting rod BA = 300 mm with centreof gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s².Find:

1. Velocity of G and angular velocity of AB,and

2. Acceleration of G and angular acceleration ofAB.

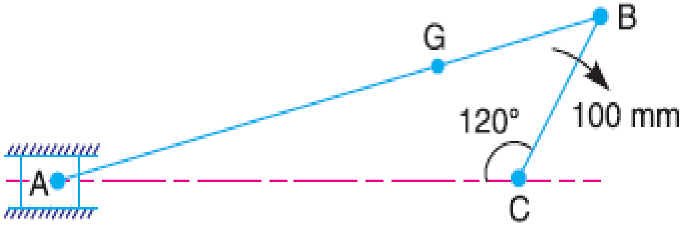


Fig. 3.20

- **Given :**
- ω_{BC}= 75 rad/s ; α_{BC} = 1200 rad/s^2, CB = 100 mm = 0.1 m; B A = 300 mm = 0.3m
- WeknowthatvelocityofB withrespecttoCorvelocityofB

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5\ m/s$$

- Since the angular acceleration of the crankshaft, α_{BC} = 1200 rad/s^2, thereforetangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120\ m/s^2$$

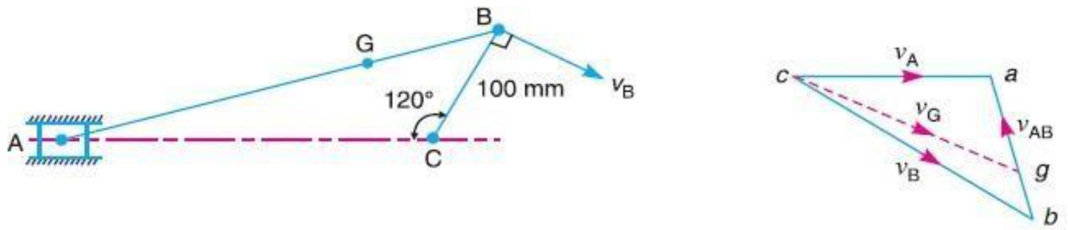
$$vector\ cb = v_{BC} = v_B = 7.5\ \frac{m}{s}$$

- By measurement, we find that velocity ofG,

$$v = ector\ cg = 6.8\ m/s$$

- Fromvelocitydiagram,wefindthatthevelocityofAwithrespecttoB,

$$v_{AB} = vector\ ba = 4\ m/s$$

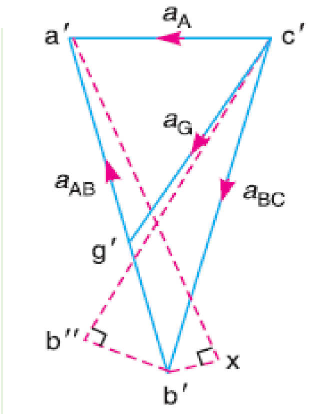


(a) Space diagram. (b) Velocity diagram.

Fig. 3.21

- We know that angular velocity ofAB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3\ rad/s$$



(c) Acceleration diagram.

Fig. 3.22

– We know that radial component of the acceleration of B with respect to C

$$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(7.5)^2}{0.1} = 562.5\ m/s^2$$

– And radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{A}}{CB} = \frac{(4)^2}{0.3} = 53.3\ m/s^2$$

$$vector\ c'b'' = a^r_{BC} = 562.5\ m/s^2$$

$$vector\ b''b' = a^t_{BC} = 120 m/s^2$$

$$vector\ b'x = a^r_{AB} = 53.3 m/s^2$$

– By measurement we find that acceleration of G,

$$a = vector\ xa' = 414\ m/s^2$$

– From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = vector\ xa' = 546 m/s^2$$

– Angular acceleration of AB

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820\ rad/s^2$$

Q. In the mechanism shown in Fig. 8.7, the slider C is moving to the right with a velocity of 1 m/s and an acceleration of 2.5 m/s2. The dimensions of various links are AB = 3 m inclined at 45° with the vertical and BC = 1.5 m inclined at 45° with the horizontal. Determine: 1. the magnitude of vertical and horizontal component of the acceleration of the point B, and 2. the angular acceleration of the links AB and BC.

– **Given:**

– v_C = 1 m/s ; a_C = 2.5 m/s2; AB = 3 m ; BC = 1.5m

– Here,

$$vector = v_{CD} = v_c = 1 m/s$$

– By measurement, we find that velocity of B with respect to A

$$v_B = vector\ ab = 0.72\ m/s$$

– Velocity of B with respect to C

$$v_B = vector\ cb = 0.72\ m/s$$

– Weknowthatradialcomponentofaccelerationof B withrespecttoC,

$$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(0.72)^2}{1.5} = 0.346\ m/s^2$$

– And radial component of acceleration of B withrespect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(0.72)^2}{3} = 0.173\ m/s^2$$

$$vectro\ d'c' = a_{cd} = a_c = 2.5\ m/s^2$$

$$vect'x = a^r_{BC} = 0.346\ \frac{m}{s^2}$$

$$vecto\ 'y = a^r_{BA} = 0.173\ m/s^2$$

– Bymeasurement,

$$vector\ b'b'' = 1.13\ m/s^2$$

– By measurement from acceleration diagram, we find that tangential component of acceleration of the point B with respect toA

$$a^t_{BA} = ctor\ yb' = 1.41 m/s^2$$

– And tangential component of acceleration of the point B with respect toC,

$$a^t_{BC} = vector\ xb' = 1.94 m/s^2$$

– we know that angular velocity ofAB,

$$\alpha_{AB} = \frac{v^t_{BA}}{AB} = 0.47\ rad/s^2$$

– And aglular accelerationofBC,

$$\alpha_{BC} = \frac{a^t_{BC}}{CB} = \frac{1.94}{1.5}\ rad/s^2$$

CAMS

Introduction

– A cam is a rotating machine element which gives reciprocating or oscillatingmotion to another element known asfollower.
– The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is pre-determined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modernmachinerytoday.
– The cams are widely used for operating the inlet and exhaust valves of internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathesetc.

Classification ofFollowers

The followers may be classified as discussed below :

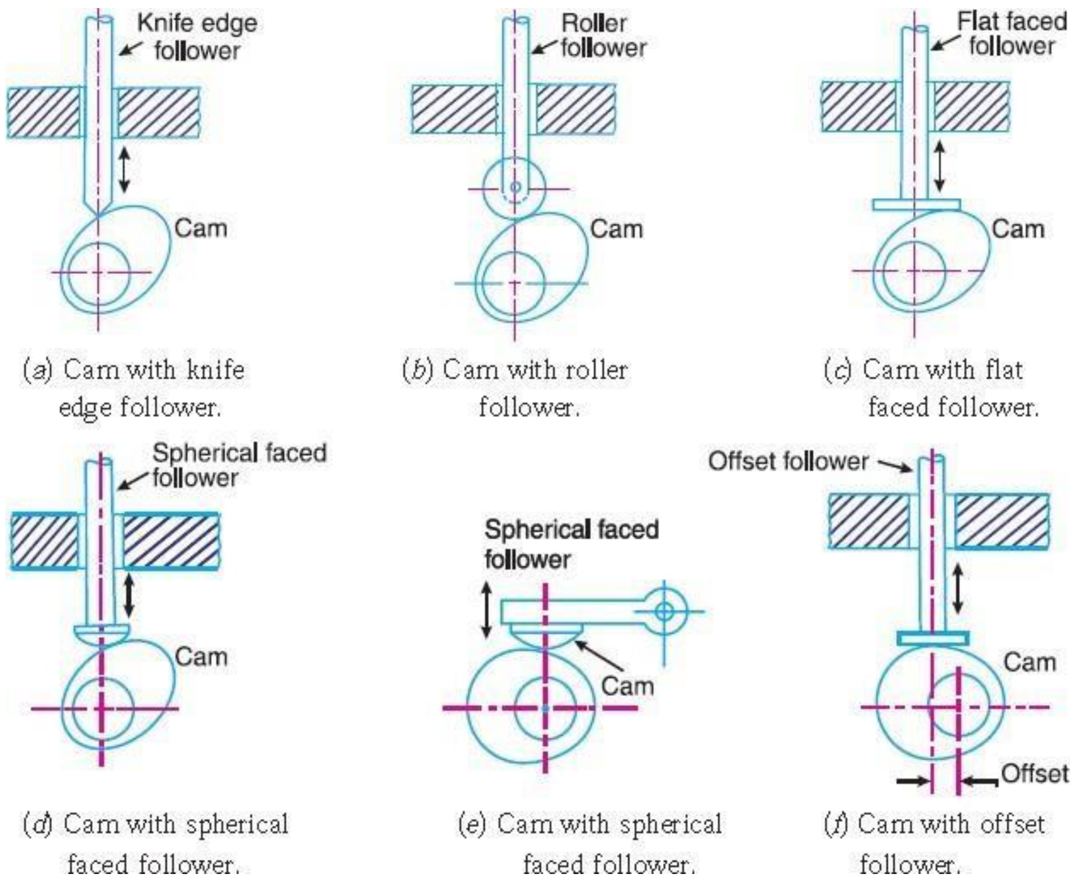


Fig. 4.1 classification of follower

According to surface in contact

aKnife edge follower

- When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig. 7.1(a).
- The sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface). It is seldom used in practice because thesmall area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.

b Rollerfollower

- When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. 7.1 (b). Since the rolling motion takes place between the contacting surfaces (i.e. the roller and thecam), therefore the rate of wear is greatlyreduced.
- In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraftengines.

c Flat faced or mushroomfollower

- When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower, as shown in Fig. 7.1 (c). It may be noted that the side thrust between the follower and the guide is much reduced in case of flat facedfollowers.
- The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off-setting the axisof the follower, as shown in Fig. 7.1 (f) so that when the cam rotates, the follower also rotates about its ownaxis.
- The flat faced followers are generally used where space is limited such as incams whichoperatethevalves ofautomobileengines.

d Spherical facedfollower

- When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. 7.1 (d). It may be noted that when a flat-faced follower is used in automobile engines, high surface stresses are produced. In order to minimize these stresses, the flat end of thefollower is machined to a sphericalshape.

According to the motion of f ollower

a Reciprocating or TranslatingFollower

- When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followersas shown inFig. 7.1 (a) to (d) are all reciprocating or translatingfollowers.

b Oscillating or RotatingFollower

- When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower,it is called oscillating or rotating follower. The follower, as shown in Fig 7.1 (e), is an oscillating or rotating follower.

According to the path of motion of the follower

a Radial Follower

- When the motion of the follower is along an axis passing through the centreof the cam, it is known as radial follower. The followers, as shown inFig.
  7.1 (a) to (e), are all radial followers.

b Off-set Follower

- When the motion of the follower is along an axis away from the axis ofthe cam centre, it is called off-set follower. The follower, as shown inFig.
  7.1 (f), is an off-set follower.

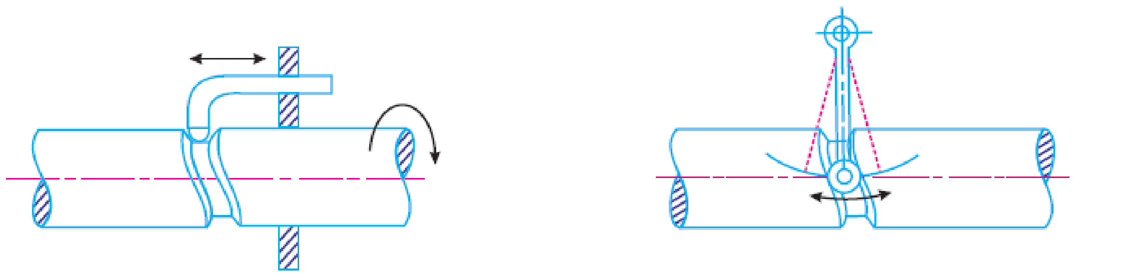
Classification of cams

a Radial or Disccam

- In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in Fig. 7.1 are all radial cams.

b Cylindricalcam

- In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in Fig. 7.2 (a) and (b)respectively.



(*a*) Cylindrical cam with reciprocating follower.

(*b*) Cylindrical cam with oscillating follower.

Fig. 4.2 cylindrical cam

Terms used in radialcams

a Basecircle

- It is the smallest circle that can be drawn to the camprofile.

b Tracepoint

- It is a reference point on the follower and is used to generate the pitch curve. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centreof the roller represents the tracepoint.

c Pressureangle

- It is the angle between the direction of the followermotion and a normal to the pitch curve. This angle is very importantin designing a cam profile. If the pressure angle is too large, a reciprocatingfollower will jam in its bearings.

d **Pitchpoint**

- It is a point on the pitch curve having the maximum pressureangle.

e **Pitchcircle**

- It is a circle drawn from the centreof the cam through the pitchpoints.

f **Pitchcurve**

- It is the curve generated by the trace point as thefollower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of theroller.

g **Primecircle**

- It is the smallest circle that can be drawn from the centreof the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a rollerfollower, the primecircleislargerthanthebasecirclebytheradiusoftheroller.

h **Lift orStroke**

- It is the maximum travel of the follower from its lowest position to the topmostposition.

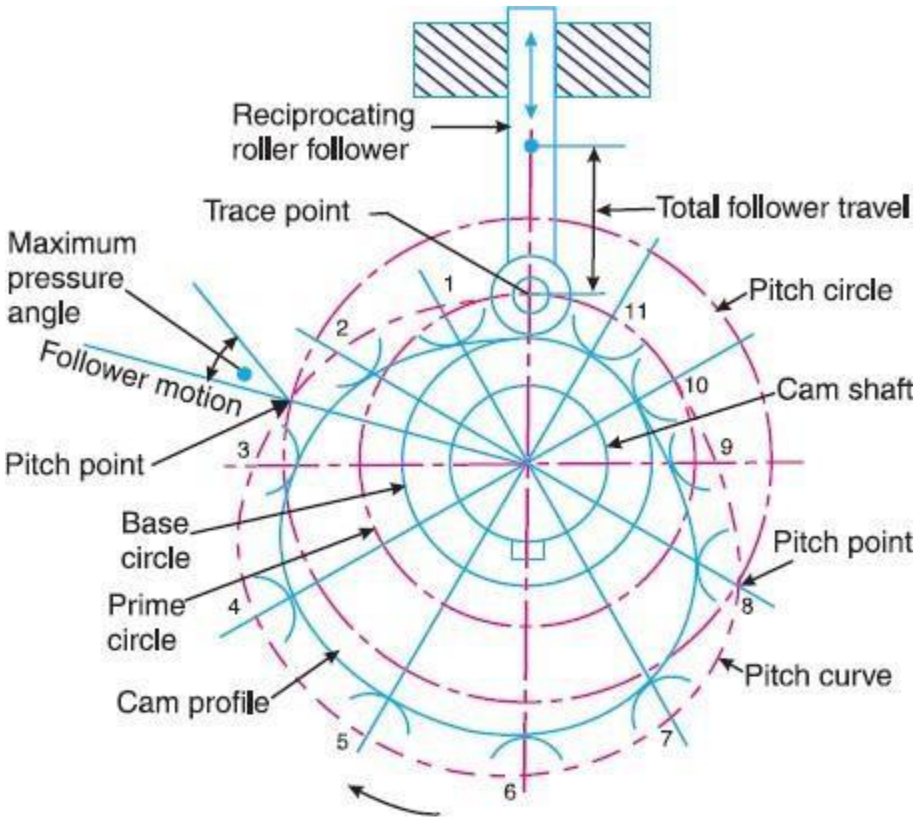


Fig. 4.3 terms used in radial cams

Motion of f ollower

The follower, during its travel, may have one of the following motions:

a Uniform velocity

b Simple harmonicmotion

c Uniform acceleration and retardation

d Cycloidalmotion

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with UniformVelocity

The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in Fig. 4.4 (a), (b) and (c) respectively.

The abscissa (base) represents the time (i.e. the number of seconds required for the cam to complete one revolution) or it mayrepresentthe angular displacement of the cam in degrees. The ordinate represents the displacement, or velocity or acceleration of thefollower.

Since the follower moves with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words,AB1 and C1D must be straight lines.

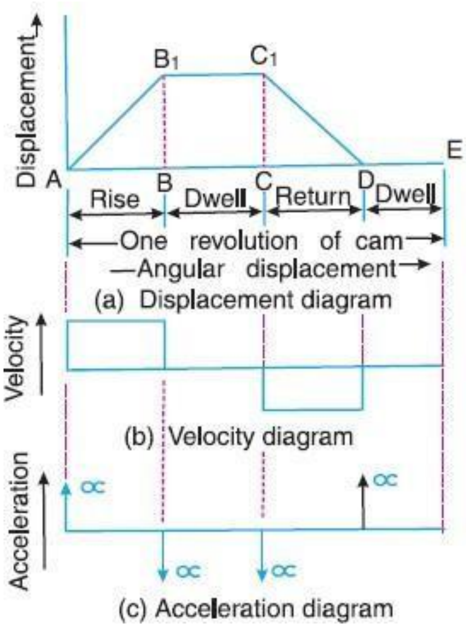


Fig. 4.4 displacement, velocity and acceleration diagrams

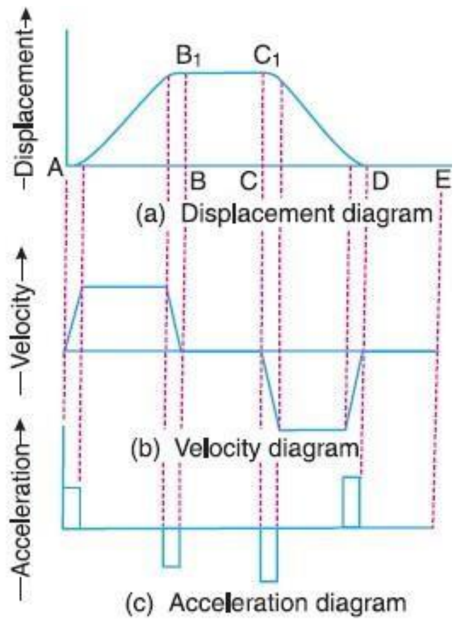


Fig. 4.5 modified displacement,velocity accelerationdiagrams

A little consideration will show that the follower remains at rest during part of the cam rotation. The periods during which the follower remains at rest are known as dwell periods, as shown by lines B1C1 and DE in Fig. 4.4 (a). From Fig. **4.2** (c), we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are however,impracticable.

In order to have the acceleration and retardation within the finite limits, it is necessary to modify the conditions which govern the motion of the follower. This may be done by rounding off thesharp corners of the displacement diagram atthe beginning and at the end of each stroke, as shown in Fig. 4.5 (a). By doing so, the velocity of the follower increases gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke as shown in Fig. 4.5(b).

The modified displacement, velocity and acceleration diagrams are shown in Fig.4.5. The round corners of the displacement diagram are usually parabolic curves because the parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion are shown in Fig. 4.6 (a), (b) and (c) respectively. The displacement diagram is drawn as follows:

a Draw a semi-circle on the follower stroke asdiameter.

b Divide the semi-circle into any number of even equal parts (sayeight).

c Divide the angular displacements of the cam during out stroke and return stroke intothe
same number of equal parts.

d The displacement diagram is obtained by projecting the points as shown inFig. 7.6 (a).

The velocity and acceleration diagrams are shown in Fig. 4.6 (b) and (c) respectively. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve.

We see from Fig. 4.6 (b) that the velocity of the follower is zeroat the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximumat the beginning and at the ends of the stroke and diminishes to zero atmid-stroke.

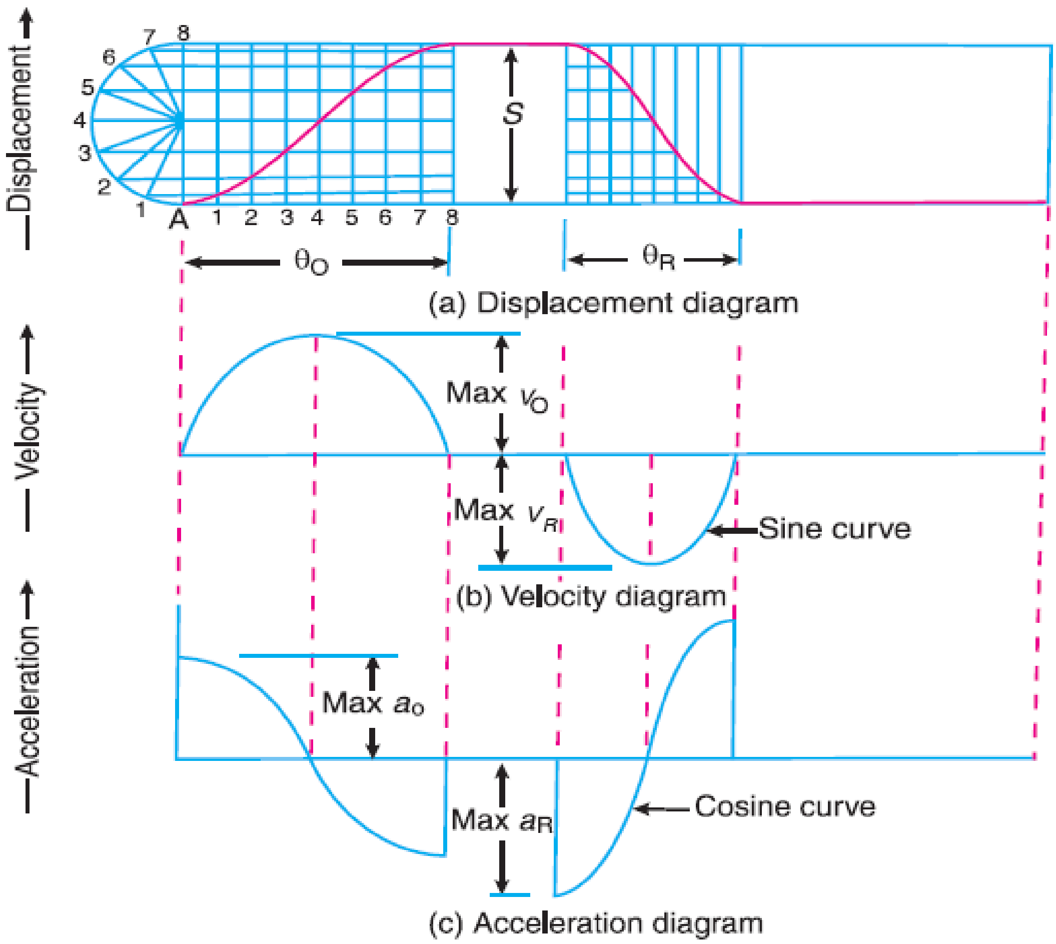


Fig. 7.6 acceleration diagram

4.3.1Se= Stroke of the follower t

,

Θ_0 and Θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively

ω = angular velocity of cam

Time required for the outstroke of the follower in second

$$t_0 = \frac{0}{\omega}$$

Consider a point P moving at uniform speed ω_p radians per sec round the circumference of a circle with the stroke S as diameter, as shown in Fig. 7.7 the point (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P'.

Peripheral speed of the point P'

$$v_p = \frac{\pi \times s}{2} \times \frac{1}{t_0} = \frac{\pi \times s}{2} \times \frac{\omega}{\theta_0}$$

and maximum velocity of the follower on the outstroke,

$$v_0 = v_p = \frac{\pi \times s}{2} \times \frac{\omega}{\theta_0} = \frac{\pi \times \omega \times s}{2\,\theta_0}$$

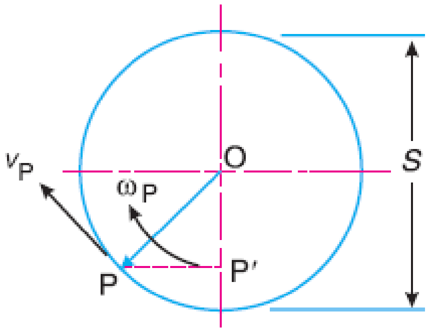


Fig. 7.7 motion of a point

We know that the centripetal acceleration of the point P

$$a_p = \frac{v_p^2}{op} = \left(\frac{\times \times s}{2\,\theta_0}\right)^2 \times \frac{2}{s} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta_0)^2}$$

Maximum acceleration of the follower on theoutstroke,

$$a_0 = a_p = \frac{\pi^2 \times {}^2 \times s}{2 \times (\theta_0)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \times \omega \times S}{2\theta_R}$$

and maximum acceleration of the follower on the return stroke

$$a_R = \frac{\pi^2 \omega^2 S}{2(\theta_R)^2}$$

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with UniformAcceleration and Retardation

The displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown in Fig. 4.8 (a), (b) and (c) respectively.We see that the displacement diagram consists of a parabolic curve and may be drawn as discussed below:

Divide the angular displacement of the cam during outstroke (Θ) into any even number of equal parts and draw vertical lines through these points as shown in fig. 4.8 (a)

Divide the stroke of the follower (S) into the same number of equal even parts.

Join Aato intersect the vertical line through point 1 at B. Similarly, obtain the other points C, D etc. as shown in Fig. 20.8 (a). Now join these pointsto obtain theparabolic curve for the out stroke of the follower.

In the similar way as discussed above, the displacement diagram for the follower during return stroke may be drawn.

We know that time required for the follower during outstroke,

$$t_0 = \frac{0}{\omega}$$

and time required for the follower during return stroke,

$$t_R = \frac{\theta_R}{\omega}$$

Mean velocity of the follower during outstroke

$$v_0 = \frac{S}{t_0}$$

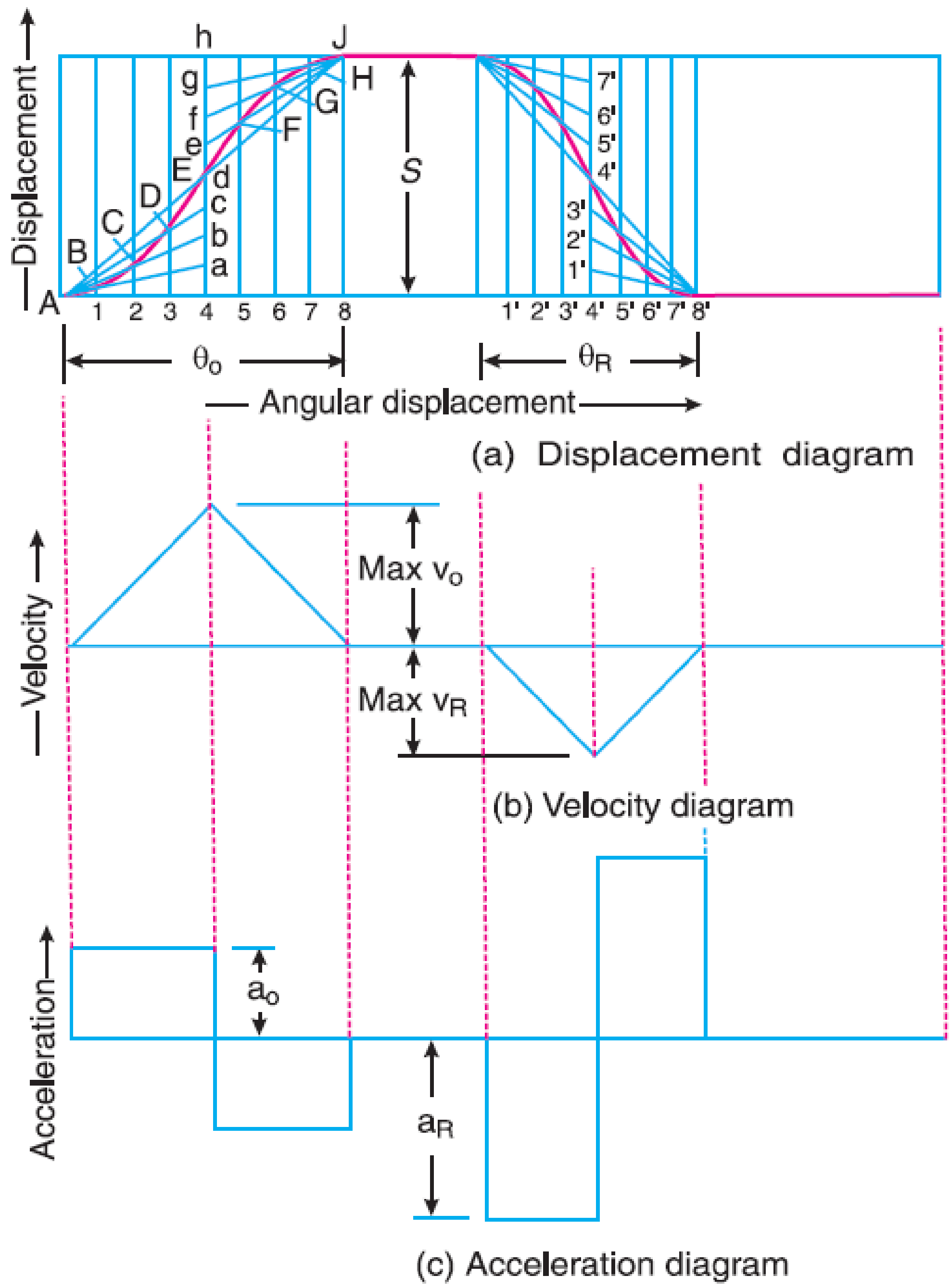


Fig. 4.8 Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

Since the maximum velocity of follower is equal to twice the mean velocity, therefore maximum velocity of the follower during outstroke,

$$v_0 = \frac{2\,S}{t_0} = \frac{2\,\omega\,S}{\theta_0}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\,\omega\,S}{\theta_R}$$

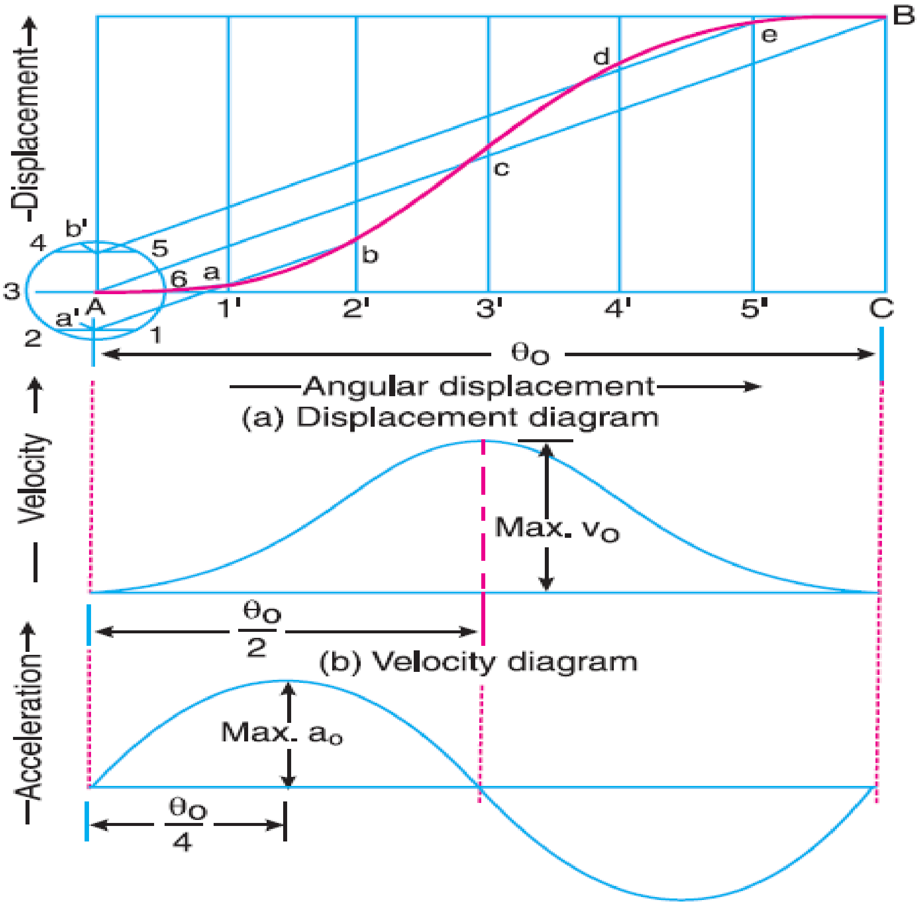
Maximum acceleration of the follower during outstroke,

$$a_0 = \frac{v_0}{t_0/2} = \frac{2 \times 2\,\omega\,s}{t_0\theta_0} = \frac{4\,\omega^2.\,S}{(\,)^2_0}$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\,\omega^2\,S}{(\,)^2}$$

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with cycloidal Motion



(c) Acceleration diagram

- The displacement, velocity and acceleration diagrams when the follower moves with cycloidalmotion are shown in Fig. (a), (b) and (c) respectively. We know that cycloid is a curve traced by a point on a circle when the circle rolls withoutslipping on a straightline.
- We know that displacement of the follower after time tseconds,

$$= S\left[\frac{\theta}{\theta_0} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

- Velocity of the follower after time tseconds,

$$\frac{dx}{dt} = S\left[\frac{1}{\theta_0} \times \frac{d\theta}{dt} - \frac{2\pi\theta}{\theta_0}\cos\left(\frac{2\pi\theta}{\theta_0}\right)\frac{d\theta}{dt}\right]$$

$$= \frac{S}{\theta_0} \times \frac{d\theta}{dt}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$= \frac{\omega S}{\theta_0}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

- The velocity is maximum,when

$$\cos\left(\frac{2\pi\theta}{\theta_0}\right) = -1$$

$$\frac{2\pi\theta}{\theta_0} = \pi$$

$$= \frac{\theta_0}{2}$$

- Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega S}{\theta_R}$$

- Now, acceleration of the follower after time tsec,

$$\frac{d^2x}{dt^2} = \frac{\omega S}{\theta_0}\left[\frac{2}{\theta_0}\sin\left(\frac{2\pi\theta}{\theta_0}\right)\frac{d\theta}{dt}\right]$$

$$= \frac{2\pi\omega^2 S}{(\theta_0)^2}\sin\left(\frac{2\pi\theta}{\theta_0}\right)$$

- The acceleration is maximum,when

$$\sin\left(\frac{2\pi\theta}{\theta_0}\right) = 1$$

$$= \frac{\theta_0}{4}$$

$$a_0 = \frac{2\pi\omega^2 S}{(\theta_0)^2}$$

$$a_R = \frac{2\,\pi\,\omega^2 S}{(\theta)^2}$$

Construction of cam profile f or a Radial cam

In order to draw the cam profile for a radial cam, first of all the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam isdrawn.

In constructing the cam profile, the principle of kinematic inversion is used, i.e. the cam is imagined to be stationary and the follower is allowed to rotate in the opposite direction to the camrotation.

Examples based on cam profile

Draw the profile of a cam operating a knife-edge follower having a lift of 30 mm. the cam raises the follower with SHM for 150⁰ of the rotation followed by a period of dwell for 60⁰. The follower descends for the next 100⁰ rotation of the cam with uniform velocity, again followed by a dwell period. The cam rotatesat a uniform velocity of 120 rpm and has a least radius of 20 mm. what will be the maximum velocity and acceleration of the follower during the lift and the return?

– S = 30 mm :Øa = 150⁰ ; N = 120 rpm ;

–$\delta_1 = 60°$; r_c = 20 mm :$\delta_2 = 50°$

– **Duringascent:**

$$\omega = \frac{2\,\pi\,N}{60} = \frac{2\,\pi \times 120}{60} = 12.57\,rad/s$$

$$v_{max} = \frac{\pi \times \omega \times s}{2\,\theta_0} = \frac{\pi \times 12.57 \times 30}{2 \times 150 \times \frac{\pi}{180}} = 226.3$$

$$a_{max} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta^0)^2} = \frac{\pi^2 \times 12.57^2 \times 30}{2 \times (150 \times \frac{\pi}{180})^2} = 7.413\ m/s^2$$

– **Duringdescent:**

$$v_{max} = \frac{\omega\,S}{\emptyset_d}$$

$$v_{max} = \frac{12.57 \times 30}{100 \times \frac{\pi}{180}} = 216\ mm/s$$

$$\boldsymbol{f_{max} = 0}$$

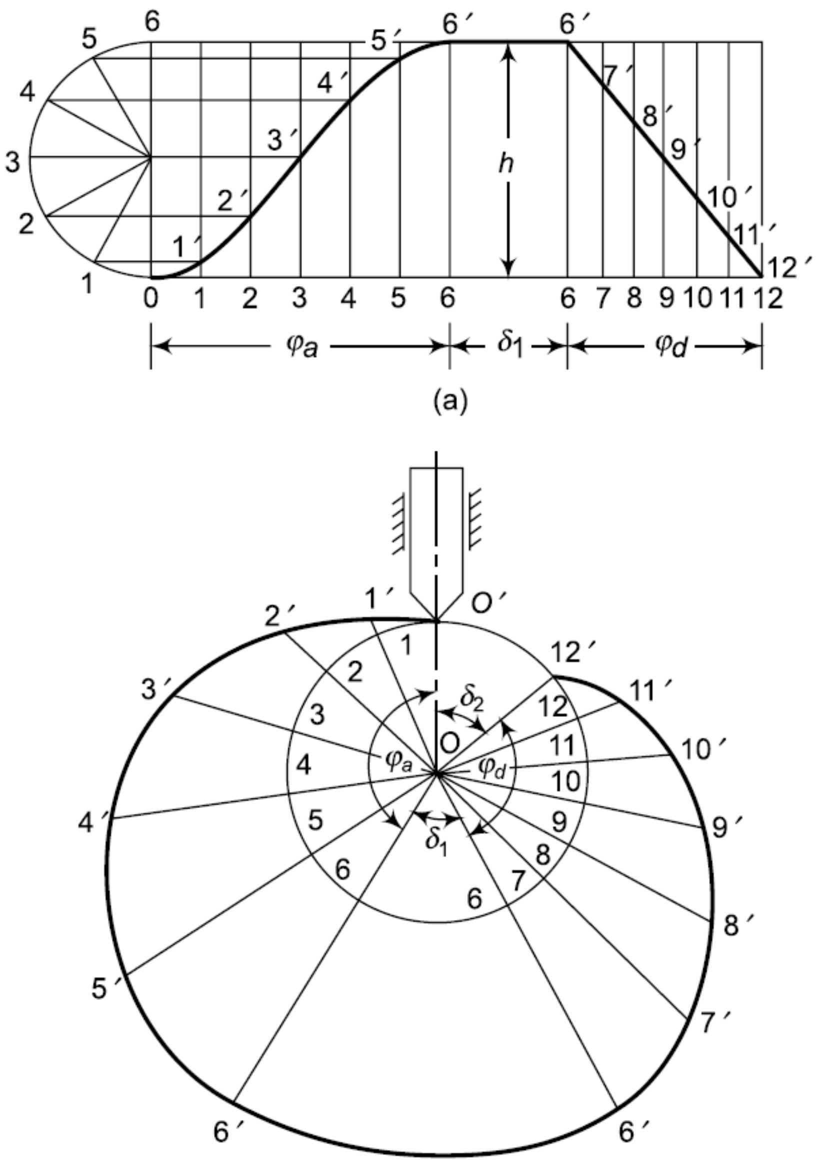


Fig. 4.10

A cam with a minimum radius of 25 mm is to be designed for a knife-edge follower with the following data:

To raise the follower through 35 mm during 60^0 rotation of the cam

Dwell for next 40^0 of the cam rotation

Descending of the follower during the next 90^0 of the cam rotation

Dwell during the rest of the cam rotation

Draw the profile of cam if the ascending and descending of the cam with simple harmonic motion and the line of stroke of the follower is offset 10 mm from the axis of the cam shaft.

What is the maximum velocity and acceleration of the follower during the ascent and the descent if the cam rotates at 150 rpm?

– S = 35 mm :$Ø_a = 60^0$; N = 150 rpm ;

–$\delta_1 = 40°$; r_c = 25 mm : $Ø_d = 90^0$; x = 10 mm

– **During ascent:**

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 5\pi \ \frac{rad}{s}$$

$$v_{max} = \frac{\pi \times \omega \times s}{2\,\theta_0} = \frac{\pi \times 5\pi \times 35}{\times 150 \times \frac{\pi}{180}} = 827.7\ mm/s2$$

$$a_{max} = \frac{\pi^2 \times \omega^2 \times s}{2 \times ()^2_0} = \frac{\pi^2 \times 5\pi^2 \times 35}{2 \times (150 \times \frac{\pi}{180})^2} = 38.882\ {}^{m}/_{s^2}$$

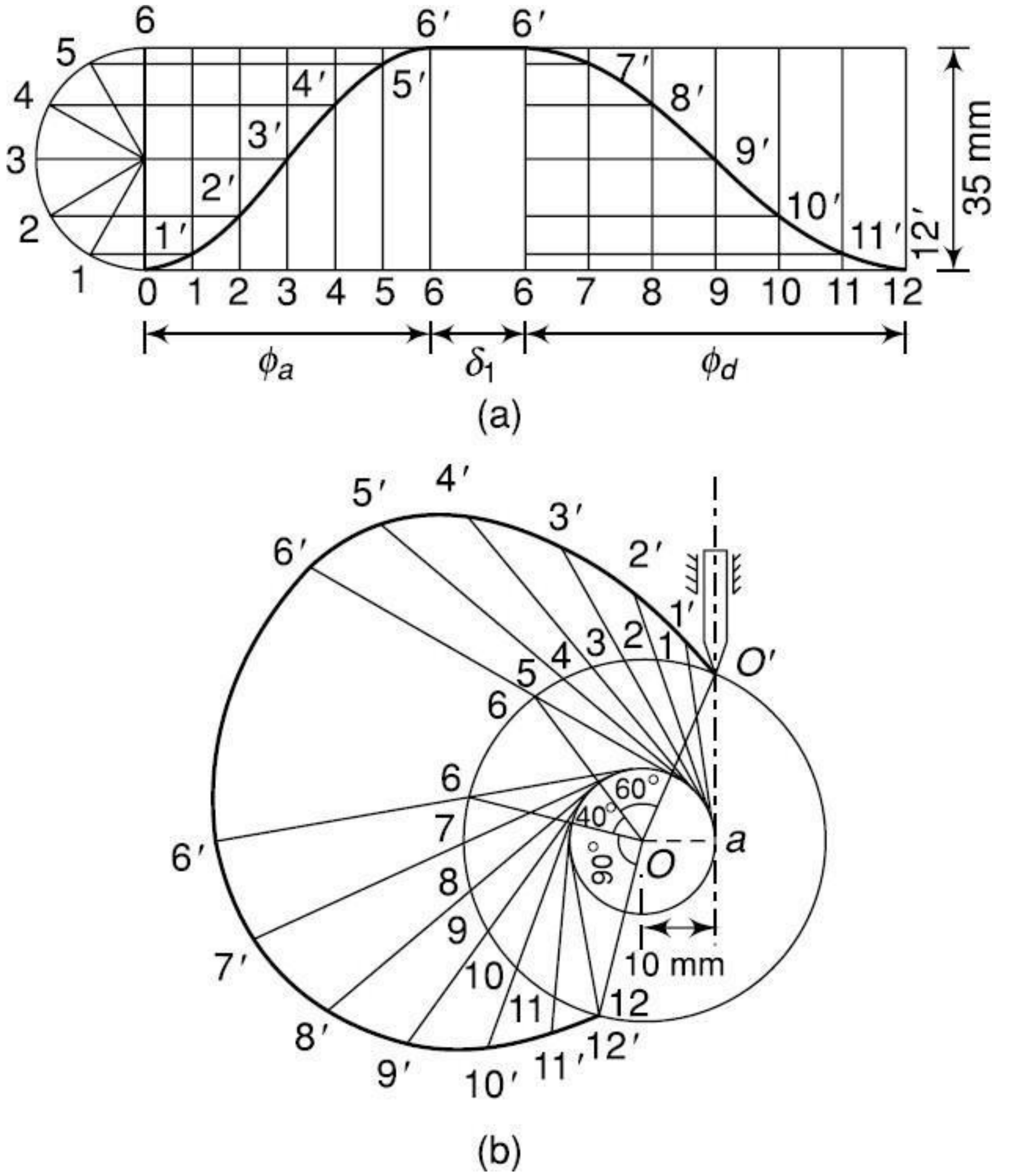


Fig. 7.11

– **Duringdescent:**

$$v_{max} = \frac{\pi \times \times s2}{\theta_0} = \frac{\pi \times 5 \times 35\ 2}{\times 90 \times \frac{\pi}{180}} = 549.80\ mm/s$$

$$a_{max} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta^0)^2} = \frac{\pi^2 \times 5\pi^2 \times 35}{2 \times (90 \times \frac{\pi}{180})^2} = 17.272 \; m/s^2$$

A cam is to give the following motion to the knife-edged follower:

To raise the follower through 30 mm with uniform acceleration and deceleration during 120⁰ rotation of thecam

Dwell for the next 30⁰ of the cam rotation

To lower the follower with simple harmonic motion during the next 90⁰ rotation of the cam

Dwell for the rest of the cam rotation

The cam has minimum radius of 30 mm and rotates counter-clockwise at a uniform speed of 800 rpm. Draw the profile of the cam if the line of stroke of the follower passes through the axis of the camshaft.

– S = 30 mm :Øa = 120⁰ ; N = 800 rpm ;

–$\delta_1 = 30°$; r_c = 30 mm : $Ø_d$ =90⁰;

– **Duringascent:**

$$\omega = \frac{2\,\pi\,N}{60} = \frac{2\,\pi \times 840}{60} = 88\,\frac{rad}{s}$$

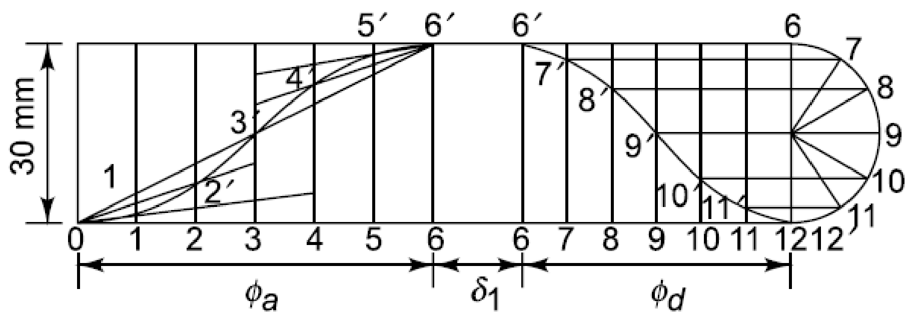
$$v_{max} = \frac{2 \times 88 \times 0.03}{120 \times \frac{\pi}{180}} = 2.52\; m/s$$

$$a_0 = \frac{4\,\omega^2.S}{(\;)^2} = \frac{4\;88^2 \times 0.03}{(120 \times \frac{\pi}{180})^2} = 211.9 \; m/s^2$$

– **Duringdescent:**

$$v_{max} = \frac{\pi \times \times s2}{\theta_0} = \frac{\pi \times 88 \times 0.03}{2 \times 90 \times \frac{\pi}{180}} = 2.64\; mm/s$$

$$a_{max} = \frac{\pi^2 \times \omega^2 \times s}{2 \times (\theta_0)^2} = \frac{\pi^2 \times 88^2 \times 0.03}{2 \times (90 \times \frac{\pi}{180})^2} = 467.6 \; m/s^2$$



(a)

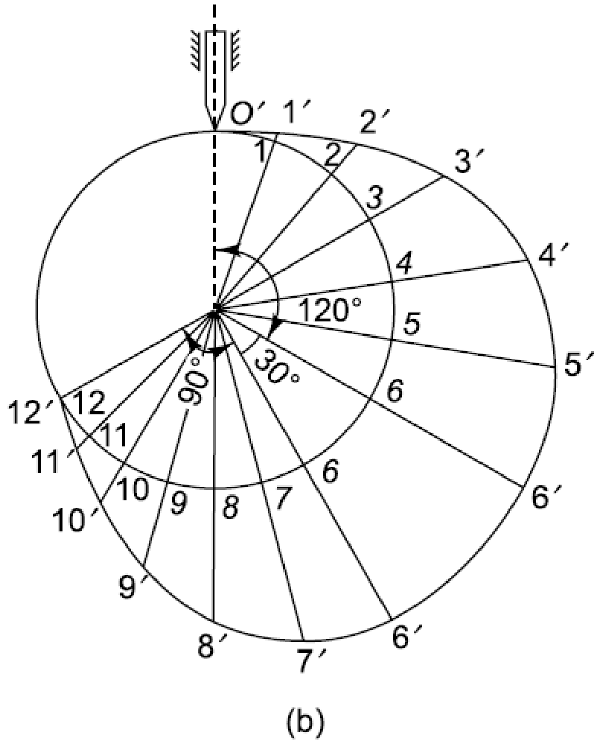


Fig. 7.12

Draw the profile of a cam operating a roller reciprocating follower and with the followingdata:

Minimum radius of cam = 25 mm

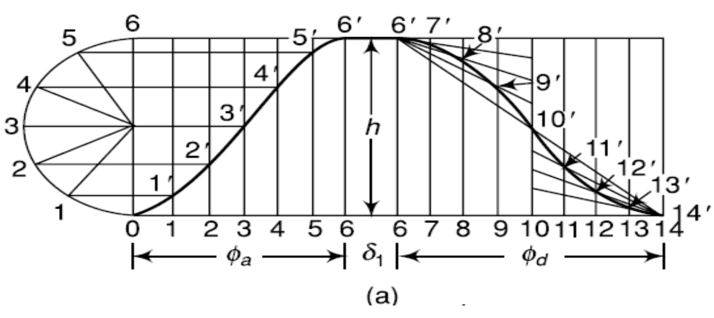
Lift = 30 mm

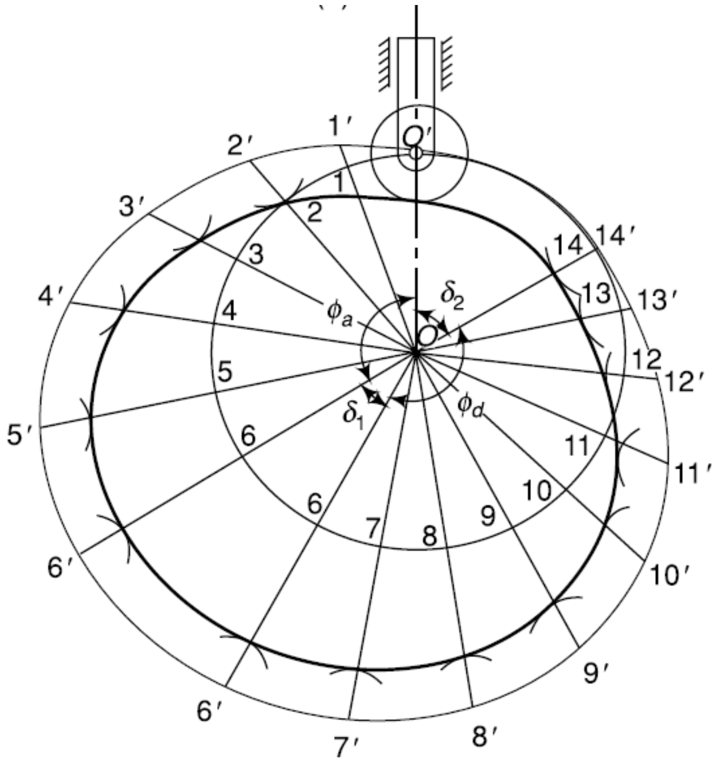
Roller diameter = 15 mm

The cam lifts the follower for 120º with SHM followed by a dwell period of 30º. Then the follower lowers down during 150º of the cam rotation with uniform acceleration and deceleration followed by dwell period. If the cam rotates at a uniform speed of 150 rpm. Calculate the maximum velocity and acceleration of the follower during the descent period.

– S = 30 mm :Øa = 120º ; N = 150 rpm ;$Ø_d$ = 150º

–$\delta_1 = 30°$; r_c = 25 mm :$\delta_2 = 60°$; r_r= 7.5 mm





(b)

Fig. 7.13

$$v_{max} = \frac{2 \times s \times \omega}{\varphi_d}$$

$$v_{max} = \frac{2 \times 30 \times \frac{2\times\pi\times150}{60}}{150 \times \frac{\pi}{180}} = 360 m/s$$

$$f_{max} = \frac{4 \times S \times \omega^2}{(\varphi_d)^2}$$

$$f_{max} = \frac{4 \times 30 \times \left(\frac{2\times\pi\times150}{60}\right)^2}{\left(150 \times \frac{\pi}{180}\right)^2} = 4320\ mm/s^2$$

The following data relate to a cam profile in which the follower moves with uniform acceleration and deceleration during ascent and descent.

Minimum radius of cam = 25 mm

Roller diameter = 7.5 mm

Lift = 28 mm

Offset of follower axis = 12 mm towards right

Angle of ascent = 60⁰

Angle of descent = 90⁰

Angle of dwell between ascent and descent = 45⁰

Speed of cam = 200 rpm

Draw the profile of the cam and determine the maximum velocity and the uniform acceleration of the follower during the outstroke and the return stroke.

- S = 28 mm :Øa = 60º ; N = 200 rpm ;Ød =90º
- $\delta_1 = 45°$; r_c = 25 mm :$\delta_2 = 165°$; r_r= 7.5 mm; x = 12mm

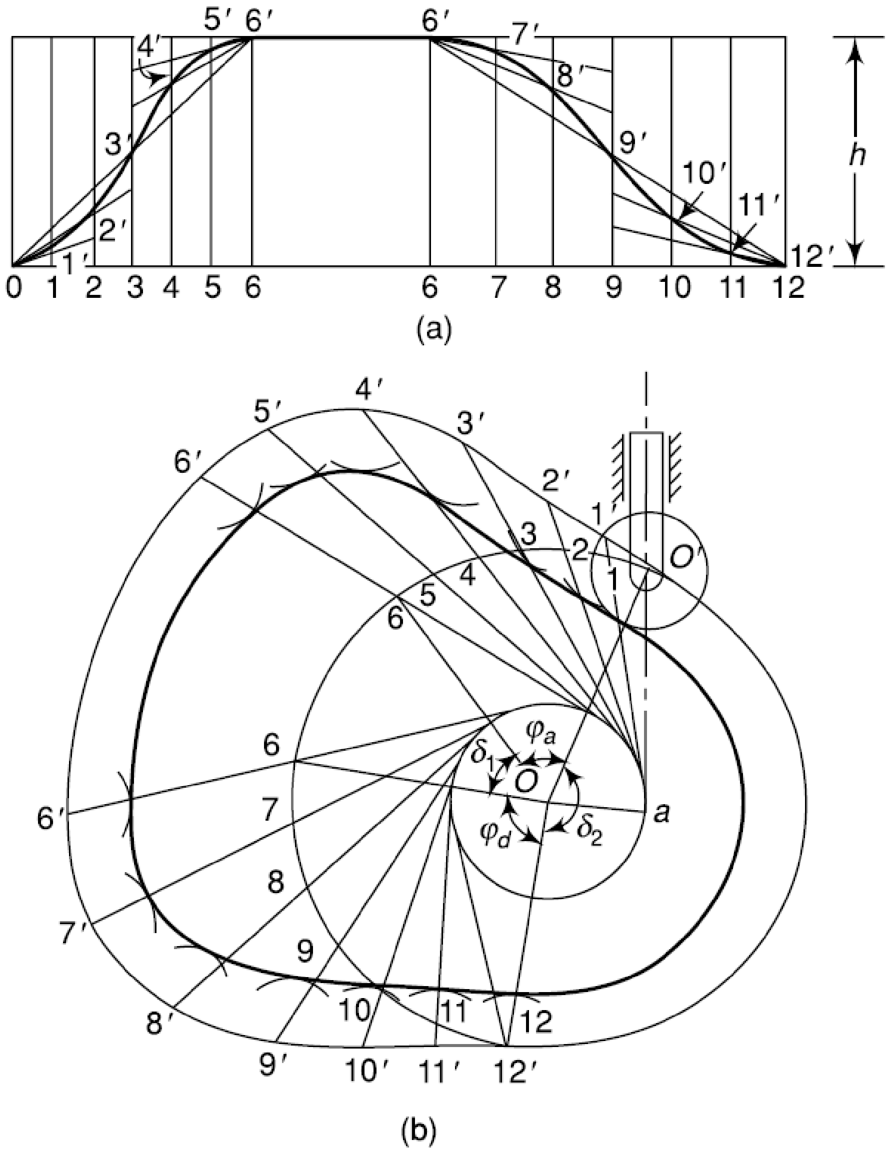


Fig. 7.14

- **During outstroke:**

$$v_{max} = \frac{2 \times s \times \omega}{\varphi_d}$$

$$v_{max} = \frac{2 \times 28 \times 20.94}{60 \times \frac{\pi}{180}} = 1.12 m/s$$

$$f_{max} = \frac{4 \times S \times \omega^2}{(\varphi_d)^2}$$

$$f_{max} = \frac{4 \times 30 \times (20.94)^2}{(60 \times \frac{\pi}{180})^2} = 44800 mm/s^2$$

- **During Return stroke:**

$$v_{max} = \frac{2 \times s \times \omega}{\varphi_d}$$

$$v_{max} = \frac{2 \times 28 \times 20.94}{90 \times \frac{\pi}{180}} = 0.747\ m/s$$

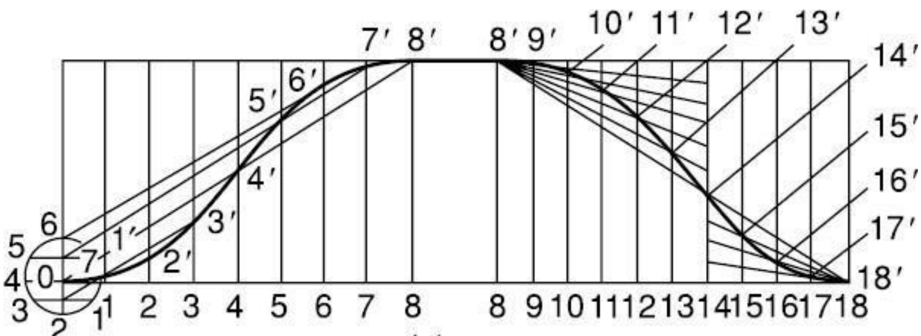
$$f_{max} = \frac{4 \times S \times \omega^2}{(\varphi_d)^2}$$

$$f_{max} = \frac{4 \times 30 \times (20.94)^2}{(90 \times \frac{\pi}{180})^2} = 19900 mm/s^2$$

A flat-faced mushroom follower is operated by a uniform rotating cam. The follower is raised through a distance of 25 mm in 120° rotation of the cam, remains at rest for next 30° and is lowered during further 120° rotation of the cam. The raising of the follower takes place with cycloidal motion and the lowering with uniform acceleration and deceleration. However, the uniform acceleration is 2/3 of the uniform deceleration. The least radius of the cam is 25 mm which rotates at 300 rpm.

Draw the cam profile and determine the values of the maximum velocity and maximum acceleration during rising and maximum velocity and uniform acceleration and deceleration during lowering of the follower.

- S = 30 mm : $Øa = 60°$; N = 200 rpm ; $Ø_d = 90°$
- $\delta_1 = 45°$; r_c = 25 mm : $\delta_2 = 165°$; r_r = 7.5 mm; x = 12 mm
-



(a)

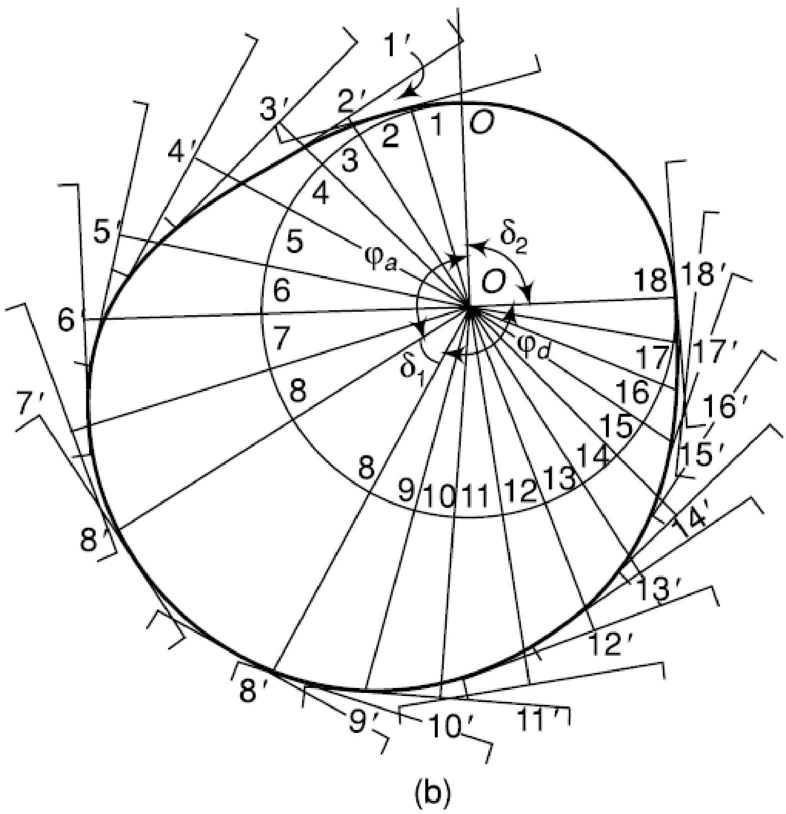


Fig. 7.15

– **During ascent:**

$$v_{max} = \frac{2 \times s \times \omega}{\varphi_a}$$

$$v_{max} = \frac{2 \times 25 \times 31.4}{120 \times \frac{\pi}{180}} = 0.75 \frac{m}{s}$$

$$f_{max} = \frac{4 \times S \times \omega^2}{(\varphi_a)^2}$$

$$f_{max} = \frac{4 \times 30 \times (31.4)^2}{\left(120 \times \frac{\pi}{180}\right)^2} = 35310 \frac{mm}{s^2}$$

The following data relate to a cam operating an oscillating an oscillating roller follower:

Minimum radius of cam = 44
mm Dia.Ofroller = 14 mm
Length of the arm =40
mm Distance from fulcrum
Centre from cam center = 50
mm Angleofascent = 75⁰
Angleofdescent = 105⁰
Angle of dwellin

Highestposition = 60^{0}
Angle of oscillation of
Follower =28^{0}
Draw the profile of the cam if the ascent and descent both take place with SHM.

- S = 19.5 mm :$Ø_a$ = 75^{0} ;$Ø_d$ =105^{0}
- $\delta_1 = 60°$; r_c = 22 mm :$\delta_2 = 120°$; r_r= 7.5mm;

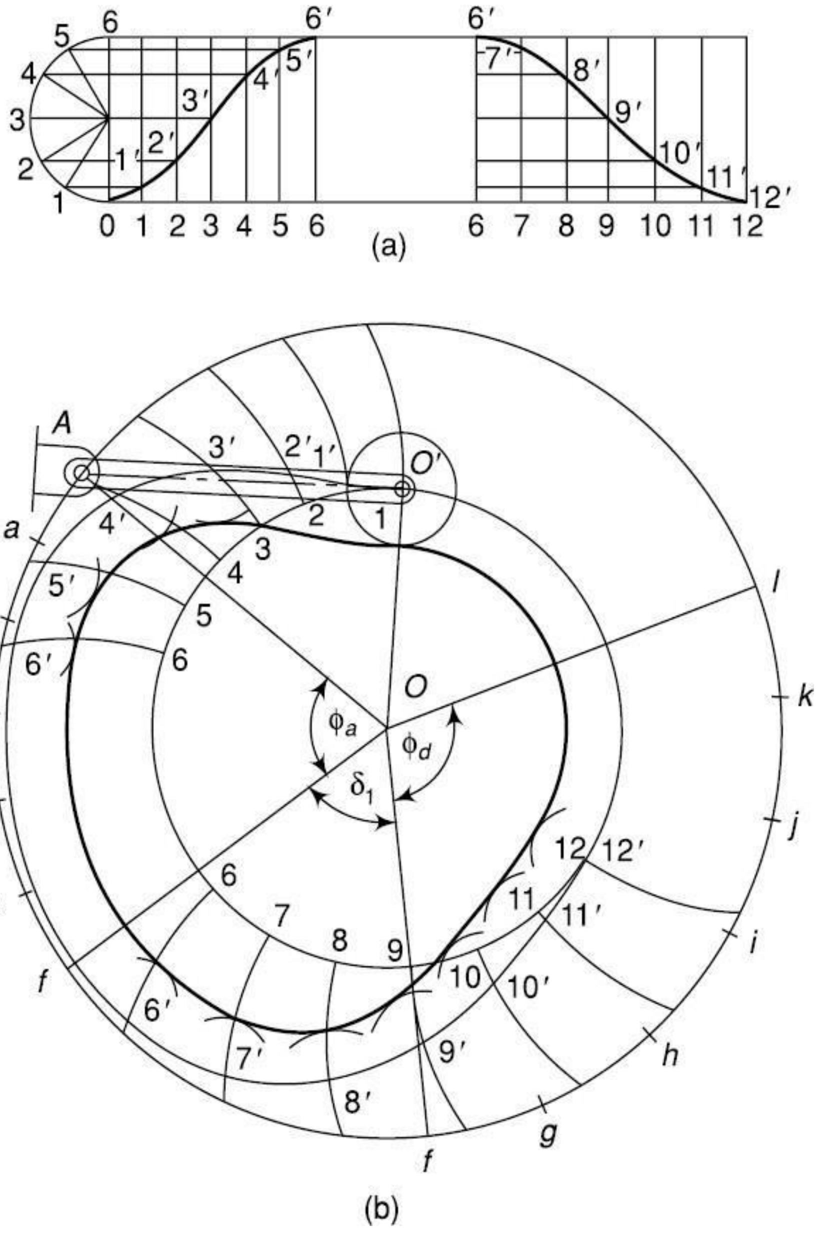


Fig. 4.16

UNIT 3

FRICTION AND FRICTION DRIVE

Course Objectives:

To understand the working principles of different type brakes and clutches.

Course Outcomes:

Understand basics related to friction and its practical application in mechanical engineering.

Introduction:

Then the system can be treated as **static**, which permits application of. Techniques of **static force analysis**. **Dynamic force analysis** is the evaluation of input **forces** or torques and joint **forces**. Considering motion of members. Evaluation of the inertia **force** /torque is explained first.

The inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position. It is numerically equal to the accelerating force in magnitude, but opposite in direction. Mathematically,

Inertia force = – Accelerating force = – m.a

Where m = Mass of the body, and

a = Linear acceleration of the centre of gravity of the body.

Similarly, the inertia torque is an imaginary torque, which when applied upon the rigid body, brings it in equilibrium position. It is equal to the accelerating couple in magnitude but opposite in direction.

Resultant Effect of a System of Forces Acting on a Rigid Body:

Consider a rigid body acted upon by a system of forces. These forces may be reduced to a single resultant force F whose line of action is at a distance h from the centre of gravity G. Now let us assume two equal and opposite forces (of magnitude F) acting through G, and parallel to the resultant force, without influencing the effect of the resultant force F, as shown in Fig. 15.1. A little consideration will show that the body is now subjected to a couple (equal to F × h) and a force; equal and parallel to the resultant force F passing through G. The force F through G causes linear acceleration of the c.g. and the moment of the couple (F × h) causes angular acceleration of the body about an axis passing through G and perpendicular to the point in which the couple acts.

α = Angular acceleration of the rigid body due to couple,

h = Perpendicular distance between the force and centre of gravity of the body,

m = Mass of the body,

k = Least radius of gyration about an axis through G, and

I = Moment of inertia of the body about an axis passing through its centre of gravity and perpendicular to the point in which the couple acts = m.k2

We know that Force,

F = Mass × Acceleration = m.a ...(i) and

$F.h = m.k^2.\alpha = I.\alpha$

D-Alembert's Principle

Consider a rigid body acted upon by a system of forces. The system may be reduced to a single resultant force acting on the body whose magnitude is given by the product of the mass of the body and the linear acceleration of the centre of mass of the body. According to Newton's second law of motion.

F = m.a ...(i)

Where F = Resultant force acting on the body,

m = Mass of the body, and

a = Linear acceleration of the centre of mass of the body

The equation (i) may also be written as:

F – m.a = 0

A little consideration will show, that if the quantity – m.a be treated as a force, equal, opposite and with the same line of action as the resultant force F, and include this force with the system of forces of which F is the resultant, then the complete system of forces will be in equilibrium. This principle is known as D-Alembert's principle. The equal and opposite force – m.a is known as reversed effective force or the inertia force (briefly written as FI). The equation (ii) may be written as

F + FI = 0 ... (iii)

Thus, D-Alembert's principle states that the resultant force acting on a body together with the reversed effective force (or inertia force), are in equilibrium. This principle is used to reduce a dynamic problem into an equivalent static problem.

Friction in Machine Elements

Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as ***external threads***. But if the threads are cut on the internal surface of a hollow rod, these are known as ***internal threads***. The screw threads are mainly of two types *i.e.* V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. In general, the V threads are used for the purpose of tightening pieces together *e.g.* bolts and nuts etc. But the square threads are used in screw jacks, vice screws etc. The following terms are important for the study of screw

Helix. It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced by a particle while moving along a screw thread.

Pitch. It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw

Lead. It is the distance; a screw thread advances axially in one turn.

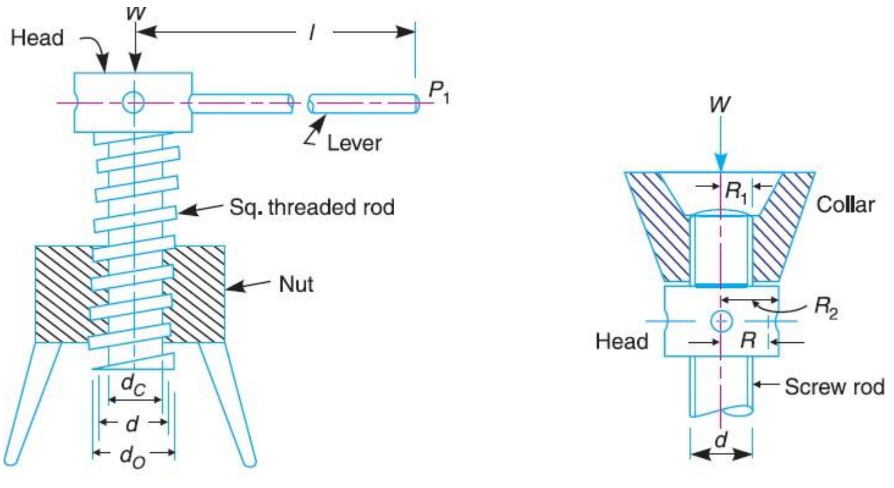
Depth of thread. It is the distance between the top and bottom surfaces of a thread (also known as **crest** and **root** of a thread).

Single-threaded screw. If the lead of a screw is equal to its pitch. it is known as single threaded screw.

Lead = Pitch × Number of threads

Helix angle. It is the slope or inclination of the thread with the horizontal.

The screw jack is a device, for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works, is similar to that of an inclined plane.



(*a*) Screw jack. (*b*) Thrust collar.

Fig (*a*) shows a common form of a screw jack, which consists of a square threaded rod (also called screw rod or simply screw) which fits into the inner threads of the nut. The load, to be raised or lowered, is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

Torque Required to Lifting the Load by a Screw Jack :

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig (*a*).

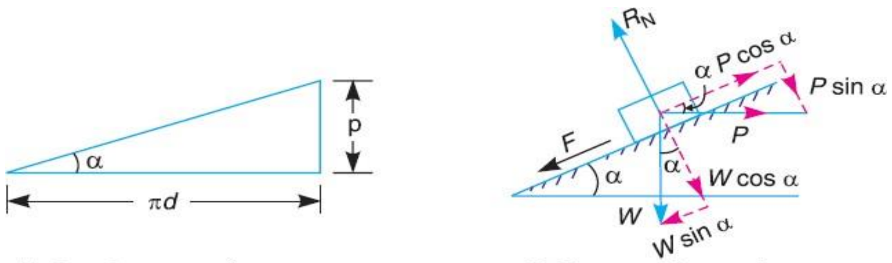
Let p = Pitch of the screw,

d = Mean diameter of the screw, α= Helix angle,

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut = tan ø, Where φ is the friction angle.



(*a*) Development of a screw. (*b*) Forces acting on the screw.

From the geometry of the Fig(*a*), we find that

$$\tan \alpha = p/\pi d$$

Since the principle on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the lever of a screw jack may be considered to be horizontal as shown in Fig(*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the screw are shown in Fig(b). Resolving the forces along the plane,

$P \cos \alpha = W \sin \alpha + F = W \sin\alpha + \mu.R_N$ ***(i)***

and resolving the forces perpendicular to the plane,

$R_N = P \sin \alpha + W \cos\alpha$ ***(ii)*** Substituting this value of R_N in equation(i),

$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$

$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$

$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$

$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$

$$P = W \times \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}$$

Substituting the value of $\mu = \tan \varphi$ in the above equation, we get

$$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos\varphi$,

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi} = W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}$$

$= W \tan (\alpha + \varphi)$

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

Torque required to overcoming friction between the screw and nut,

When the axial load is taken up by a thrust collar or a flat surface, as shown in Fig (b),so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \mu_1 .W \left(\frac{R_1 + R_2}{2} \right) = \mu_1 .W.R$$

R_1 and R_2 = Outside and inside radii of the collar,

R = Mean radius of the collar, and

μ_1 = Coefficient of friction for the collar.

$$T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1 .W.R$$

Total torque required to overcome friction ($i.e.$ to rotate the screw), If an effort $P1$ is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of the lever, *i.e.*

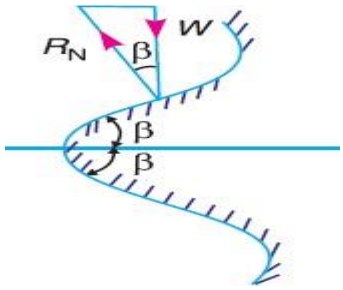
$$T = P \times \frac{d}{2} = P_1.l$$

Friction of a V-thread

The normal reaction in case of a square threaded screw is

$R_N = W \cos\alpha$, where α= Helix angle.

But in case of V-thread (or acme or trapezoidal threads), the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load



W, as shown in Fig.

Let 2□= Angle of the V-thread, and

□= Semi-angle of the V-thread.

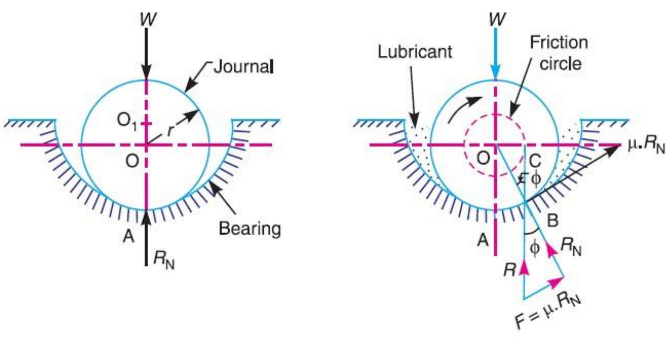
$$R_N = \frac{W}{\cos\beta}$$

$$\text{frictional force, } F = \mu.R_N = \mu \times \frac{W}{\cos\beta} = \mu_1.W$$

$$\frac{\mu}{\cos\beta} = \mu_1, \text{ known as virtual coefficient of friction.}$$

Friction in Journal Bearing-Friction Circle

A journal bearing forms a turning pair as shown in Fig (*a*). The fixed outer element of a turning pair is called a ***bearing*** and that portion of the inner element (*i.e.* shaft) which fits in the bearing is called a ***journal.*** The journal is slightly less in diameter than the bearing, in order to permit the free movement of the journal in a bearing.



(*a*) (*b*)

When the bearing is not lubricated (or the journal is stationary), then there is a line contact between the two elements as shown in Fig (a). The load W on the journal and normal reaction R_N (equal to W) of the bearing acts through the centre. The reaction RN acts vertically upwards at point A. This point A is known as **seat** or **point of pressure.**

Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig(*b*). The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction. Therefore, the reaction *R* does not act vertically upward, but acts at another point of pressure *B*. This is due to the fact that when shaft rotates, a frictional force $F = \mu R_N$ acts at the circumference of the shaft which has a tendency to rotate the shaft in opposite direction of motion and this shifts the point *A* to point *B*.In order that the rotation may be maintained, there must be a couple rotating the shaft.

Let ø= Angle between *R* (resultant of *F* and R_N) and R_N,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in meters.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$R = W$, and $T = W \times OC = W \times OB \sin\varphi = W.r\sin\varphi$

Since ø is very small, therefore substituting $\sin\varphi = \tan\varphi$

$T = W.r\tan\phi = \mu.W.r$ ($\mu = \tan\phi$)

If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

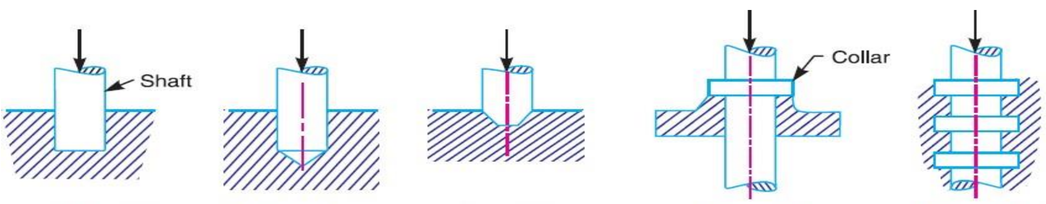
$P = T\omega = T \times 2\pi N/60$ watts Where *N* = Speed of the shaft in r.p.m.

Friction of Pivot and Collar Bearing

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft. The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust.

The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface as shown in Fig. 10.16 (a) and (b) respectively. When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig (c).

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar, as shown in Fig (*d*) or several collars along the length of a shaft, as shown in Fig(*e*) in order to reduce the intensity of pressure.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

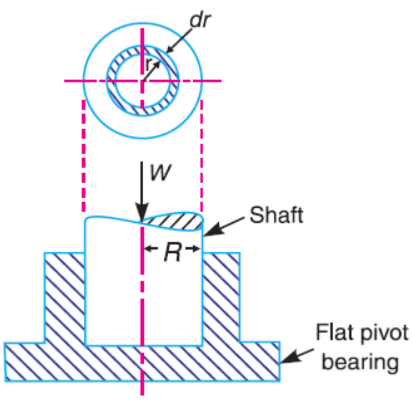
In modern practice, ball and roller thrust bearings are used when power is being transmitted and when thrusts are large as in case of propeller shafts of ships.

A little consideration will show that in a new bearing, the contact between the shaft and bearing may be good over the whole surface. In other words, we can say that the pressure over the rubbing surfaces is uniformly distributed. But when the bearing becomes old, all parts of the rubbing surface will not move with the same velocity, because the velocity of rubbing surface increases with the distance from the axis of the bearing. This means that wear may be different at different radii and this causes to alter the distribution of pressure. Hence, in the study of friction of bearings, it is assumed hat
The pressure is uniformly distributed throughout the bearing surface, and
The wear is uniform throughout the bearing surface.

Flat Pivot Bearing :

When a vertical shaft rotates in a flat pivot bearing (known as **foot step bearing**), as shown in Fig., the sliding friction will be along the surface of contact between the shaft and the bearing.

Let W =Load transmitted over the bearing surface,



R =Radius of bearing surface,

p =Intensity of pressure per unit area of bearing Surface between rubbing surfaces, and

μ =Coefficient of friction.

We will consider the following two cases:

1. When there is a uniform pressure
2. When there is a uniform wear

Considering uniform pressure

$$p = \frac{W}{\pi R^2}$$

When the pressure is uniformly distributed over the bearing area, then

Consider a ring of radius r and thickness $dr$of the bearing area. Area of bearing surface, $A = 2\pi r.dr$

Load transmitted to the ring,

$\delta W = p \times A = p \times 2\pi r.dr$ **(*i*)**

Frictional resistance to sliding on the ring acting tangentially at radius r, $F_r = \mu.\delta W = \mu\ p \times 2\pi\ r.dr = 2\pi\mu.p.r.dr$

Frictional torque on the ring,

$T_r = F_r \times r = 2\pi\ \mu\ p\ r.dr \times r = 2\ \pi\ \mu\ p r^2 dr$ **(*ii*)**

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

$$\therefore \text{ Total frictional torque, } T = \int_0^R 2\pi\mu\, p r^2\, dr = 2\pi\mu\, p \int_0^R r^2\, dr$$

$$= 2\pi\mu\, p \left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R \qquad \ldots\left(\because\ p = \frac{W}{\pi R^2}\right)$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi N/60 \qquad \ldots(\because\ \omega = 2\pi N/60)$$

N = Speed of shaft in r.p.m.

Considering uniform wear

We have already discussed that the rate of wear depends upon the intensity of pressure and the velocity of rubbing surfaces (v). It is assumed that the rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surfaces (*i.e.* $p.v.$). Since the velocity of rubbing surfaces increases with the distance (*i.e.* radius r) from the axis of the bearing, therefore for uniform Wear

$$p.r = C \text{ (a constant)} \quad \text{or} \quad p = C / r$$

ınd the load transmitted to the ring,

$$\delta W = p \times 2\pi r.dr \qquad \text{...[From equation } (i)]$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi\, C.dr = 2\pi\, C\,[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu\, p\, r^2\ dr = 2\pi\mu \times \frac{C}{r} \times r^2\ dr \qquad \text{...}\left(\because\ p = \frac{C}{r}\right)$$

$$= 2\pi\ \mu.C.r\ \ dr \qquad \text{...}(iii)$$

∴ Total frictional torque on the bearing,

$$T = \int_0^R 2\pi\ \mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\mu.C \times \frac{R^2}{2} = \pi\mu.C.R^2$$

$$= \pi\mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \times \mu.W.R \qquad \text{...}\left(\because\ C = \frac{W}{2\pi R}\right)$$

PROBLEMS

Example 1.A vertical shaft 150 mm in diameter rotating at 100 r.p.m. rests on a flat end foot step bearing. The shaft carries a vertical load of 20 kN. Assuming uniform pressure distribution and coefficient of friction equal to 0.05, estimate power lost in friction.

Solution. Given :D = 150 mm or R = 75 mm = 0.075 m ; N = 100 r.p.m or ω= 2 δ × 100/60= 10.47 rad/s ; W = 20 kN = 20 × 103 N ; μ = 0.05

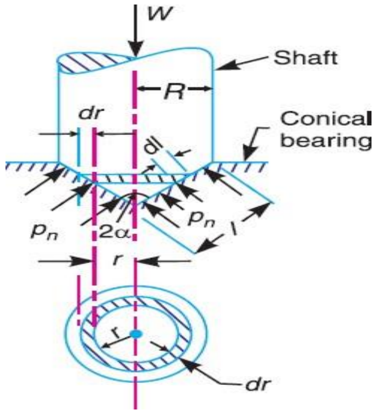
We know that for uniform pressure distribution, the total frictional torque,

$$T = \frac{2}{3} \times \mu W.R = \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 0.075 = 50 \text{ N-m}$$

∴ Power lost in friction,

$$P = T.\omega = 50 \times 10.47 = 523.5 \text{ W} \quad \textbf{Ans.}$$

Conical Pivot Bearing



The conical pivot bearing supporting a shaft carrying a load W is shown in Fig. Let

P_n = Intensity of pressure normal to the cone,

α = Semi angle of the cone,

μ = Coefficient of friction between the shaft and the bearing,

R = Radius of the shaft.

Consider a small ring of radius r and thickness dr.

Let dl is the length of ring along the cone, such that

$dl = dr\operatorname{cosec}\alpha$

Area of the ring, $A = 2\pi r.dl = 2\pi r.dr\operatorname{cosec}\alpha$ $\quad (dl = dr\operatorname{cosec}\alpha)$

Considering uniform pressure

We know that normal load acting on the ring, δW_n= Normal pressure ×Area

$= p_n \times 2\pi r.dr\operatorname{cosec}\alpha$ vertical load acting on the ring,

δW= Vertical component of $\delta W_n = \delta W_n.\sin\alpha$ Total vertical load transmitted to the bearing,

$$W = \int_0^R p_n \times 2\pi r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_0^R = 2\pi\, p_n \times \frac{R^2}{2} = \pi R^2.p_n$$

$$p_n = W / \pi R^2$$

$$T_r = F_r \times r = 2\pi\mu.p_n.\operatorname{cosec}\alpha.r.dr \times r = 2\pi\mu.p_n\ \operatorname{cosec}\alpha.r^2.dr$$

We know that frictional force on the ring acting tangentially at radius r,

The vertical load acting on the ring is also given by δW= Vertical component of p_n× Area of the ring

$$= p_n \sin\alpha \times 2\pi r.dr.\text{cosec}\,\alpha = p_n \times 2\pi r.dr$$

Integrating the expression within the limits from 0 to R for the total frictional torque on the conical pivot bearing.

Total frictional torque:

$$T = \int_0^R 2\,\pi\,\mu.p_n\ \text{cosec}\ \alpha.r^2\ dr = 2\,\pi\mu.p_n.\text{cosec}\ \alpha\left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu.p_n.\text{cosec}\ \alpha \times \frac{R^3}{3} = \frac{2\pi\ R^3}{3} \times \mu.p_n.\text{cosec}\ \alpha \qquad \ldots(i)$$

Substituting the value of p_n in equation (i),

$$T = \frac{2\pi R^3}{3} \times \pi \times \frac{W}{\pi R^2} \times \text{cosec}\ \alpha = \frac{2}{3} \times \mu W.R.\ \text{cosec}\ \alpha$$

Considering uniform wear

In Fig. let p_r be the normal intensity of pressure at a distance r from the central axis. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$p_r.r = C$ (a constant) or $p_r = C/r$

$$\delta W = p_r \times 2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

The load transmitted to the ring,

Total load transmitted to the bearing,

$$W = \int_0^R 2\pi C.dr = 2\pi C\ [r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu.p_r.\text{cosec}\ \alpha.r^2.dr = 2\pi\mu \times \frac{C}{r} \times \text{cosec}\,\alpha.r^2.dr$$

$$= 2\pi\mu.C.\text{cosec}\ \alpha.r.dr$$

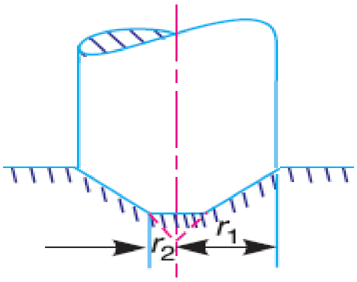
Total frictional torque acting on the bearing,

$$T = \pi\mu \times \frac{W}{2\pi R} \times \text{cosec } \alpha . R^2 = \frac{1}{2} \times \mu . W . R \text{ cosec } \alpha = \frac{1}{2} \times \mu . W . l$$

Substituting the value of C, we have

Trapezoidal or Truncated Conical Pivot Bearing

If the pivot bearing is not conical, but a frustum of a cone with $r1$ and $r2$, the external and internal



Area of the bearing surface,

$$A = \pi[(r_1)^2 - (r_2)^2]$$

∴ Intensity of uniform pressure,

$$p_n = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \quad \ldots(i)$$

radius respectively as shown in Fig, then

Considering uniform pressure

The total torque acting on the bearing is obtained by integrating the value of Tr, within the limits $r1$ and $r2$.

Total torque acting on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu . p_n \text{ cosec } \alpha . r^2 . dr = 2\pi\mu . p_n . \text{cosec } \alpha \left[\frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$= 2\pi\mu . p_n . \text{cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3} \right]$$

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3} \right]$$

$$= \frac{2}{3} \times \mu . W . \text{cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

Substituting the value of p_n from equation ***(i)***,

Considering uniform wear the load transmitted to the ring, $\delta W = 2\pi C . dr$

Total load transmitted to the ring,

$$W = \int_{r_2}^{r_1} 2\pi\, C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the torque acting on the ring, considering uniform wear, is Total torque acting on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\, \mu.C\, cosec\, \alpha.r.dr = 2\pi\, \mu.C.cosec\, \alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= \pi\, \mu.C.cosec\, \alpha\left[(r_1)^2 - (r_2)^2\right]$$

Substituting the value of *C* from equation ***(ii)*****,** weget

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times cosec\, \alpha\, [(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu.W\,(r_1 + r_2)\; cosec\; \alpha = \mu.W.R\; cosec\; \alpha$$

$$R = \text{Mean radius of the bearing} = \frac{r_1 + r_2}{2}$$

PROBLEMS

Example 1.A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm2. The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Solution. Given: W = 20 kN = 20 × 103 N ; 2 α= 120º or α= 60º ; p_n= 0.3 N/mm^2 ;N = 200 r.p.m. or ω= 2π× 200/60 = 20.95 rad/s ; μ =0.1

Outer and inner radii of the bearing surface.

Let r_1 and r_2 = Outer and inner radii of the bearing surface, in mm. Since the external diameter is twice the internal diameter, therefore

$r_1 = 2\, r_2$

$$0.3 = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{20 \times 10^3}{\pi[(2r_2)^2 - (r_2)^2]} = \frac{2.12 \times 10^3}{(r_2)^2}$$

$(r_2)^2 = 2.12 \times 10^3 / 0.3 = 7.07 \times 10^3$ or $r_2 = 84$ mm **Ans.**

$r_1 = 2\, r_2 = 2 \times 84 = 168$ mm **Ans.**

We know that intensity of normal pressure (pn),

Power absorbed in friction

$$T = \frac{2}{3} \times \mu.W.\text{cosec}\ \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

$$= \frac{2}{3} \times 0.1 \times 20 \times 10^3 \times \text{cosec}\ 60^\circ = \left[\frac{(168)^3 - (84)^3}{(168)^2 - (84)^2} \right] \text{N-mm}$$

$$= 301760 \text{ N-mm} = 301.76 \text{ N-m}$$

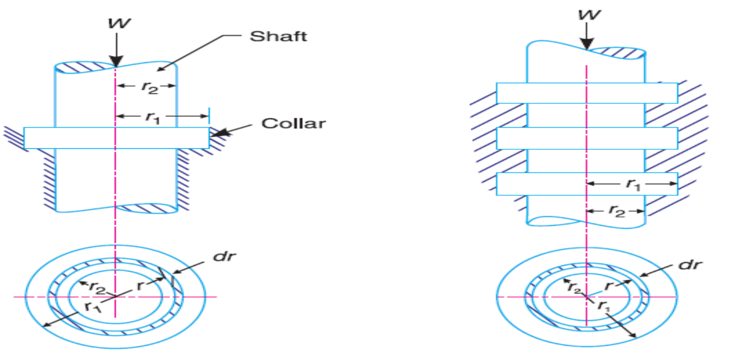
We know that total frictional torque (assuming uniform pressure),

Power absorbed in friction

$P = T.\omega = 301.76 \times 20.95 = 6322$ W $= 6.322$ kW

Flat Collar Bearing

We have already discussed that collar bearings are used to take the axial thrust of the rotating hafts. There may be a single collar or multiple collar bearings as shown in Fig.(*a*) and (*b*)respectively. The collar bearings are also known as ***thrust bearings.*** The friction in the collar bearings may be found as discussed below:



(*a*) Single collar bearing (*b*) Multiple collar bearing.

Consider a single flat collar bearing supporting a shaft as shown in Fig(*a*).

Let $r1$ = External radius of the collar,

$r2$ = Internal radius of the collar.

Area of the bearing surface,

$$A = \pi\,[(r_1)^2 - (r_2)^2]$$

Considering uniform pressure

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

The frictional torque on the ring of radius *r* and thickness *dr*,

$$T_r = 2\pi\mu.p.r^2.dr$$

Integrating this equation within the limits from $r2$ to $r1$ for the total frictional torque on the collar.

Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Substituting the value of *p* from equation ***(i)*,**

Considering uniform wear

The load transmitted on the ring, considering uniform wear is,

$$\delta W = p_r.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

Total load transmitted to the collar,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)} \qquad \ldots(ii)$$

We also know that frictional torque on the ring; we also know that frictional torque on the ring,

$$T_r = \mu.\delta W.r = \mu \times 2\pi C.dr.r = 2\pi\mu.C.r.dr$$

Total frictional torque on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu\, C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2]$$

Substituting the value of C from equation ***(ii)*,**

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W(r_1 + r_2)$$

PROBLEMS

Example 1.A thrust shaft of a ship has 6collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine90 r.p.m., find the power absorbed in friction at the thrust block, assuming.

1. Uniform pressure

2. Uniform wear.

Solution. Given: n = 6 ; d_1 = 600 mm or r_1 = 300mm ;d_2 = 300 mm or r_2 = 150 mm ;

W = 100 kN= 100×10^3 N ;

μ = 0.12 ; N = 90 r.p.m. or ω= 2 $\pi\times$ 90/60 = 9.426 rad/s

Power absorbed in friction, assuming uniform pressure

We know that total frictional torque transmitted,

$$T = \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{2}{3} \times 0.12 \times 100 \times 10^3\left[\frac{(300)^3 - (150)^3}{(300)^2 - (150)^2}\right] = 2800 \times 10^3 \text{ N-mm}$$

$$= 2800 \text{ N-m}$$

Power absorbed in friction,

$P = T\omega$= 2800 × 9.426 = 26 400 W = 26.4 kW

Power absorbed in friction assuming uniform wear

We know that total frictional torque transmitted,

$$T = \frac{1}{2} \times \mu.W(r_1 + r_2) = \frac{1}{2} \times 0.12 \times 100 \times 10^3 \ (300 + 150) \text{ N-mm}$$

$$= 2700 \times 10^3 \text{ N-mm} = 2700 \text{ N-m}$$

Power absorbed in friction,

$P = T.\omega = 2700 \times 9.426 = 25\,450$ W = 25.45 kW

Example 2.A shaft has a number of a collars integral with it. The external diameter of the collars is 400 mm and the shaft diameter is 250 mm. If the intensity of pressure is 0.35 N/mm^2(uniform) and the coefficient of friction is 0.05, estimate power absorbed when the shaft runs at 105 r.p.m. carrying a load of 150 kN. Number of collars required.

Solution. Given: d1 = 400 mm or r_1 = 200 mm ; d2 = 250 mm or r_2 = 125 mm ; p = 0.35N/mm^2 ; μ = 0.05 ; N = 105 r.p.m or

$\omega = 2\pi \times 105/60 = 11$ rad/s ; W = 150 kN = 150×10^3 N

Power absorbed

We know that for uniform pressure, total frictional torque transmitted

$$T = \frac{2}{3} \times \mu.W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right] = \frac{2}{3} \times 0.05 \times 150 \times 10^3 \left[\frac{(200)^3 - (125)^3}{(200)^2 - (125)^2} \right] \text{N-mm}$$

$$= 5000 \times 248 = 1240 \times 10^3 \text{ N-mm} = 1240 \text{ N-m}$$

Power absorbed,

$P = T\omega = 1240 \times 11 = 13640$ W = 13.64 kW

Number of collarsrequired

Let n = Number of collarsrequired.

We know that the intensity of uniform pressure (p),

$$0.35 = \frac{W}{n.\pi[(r_1)^2 - (r_2)^2]} = \frac{150 \times 10^3}{n.\pi[(200)^2 - (125)^2]} = \frac{1.96}{n}$$

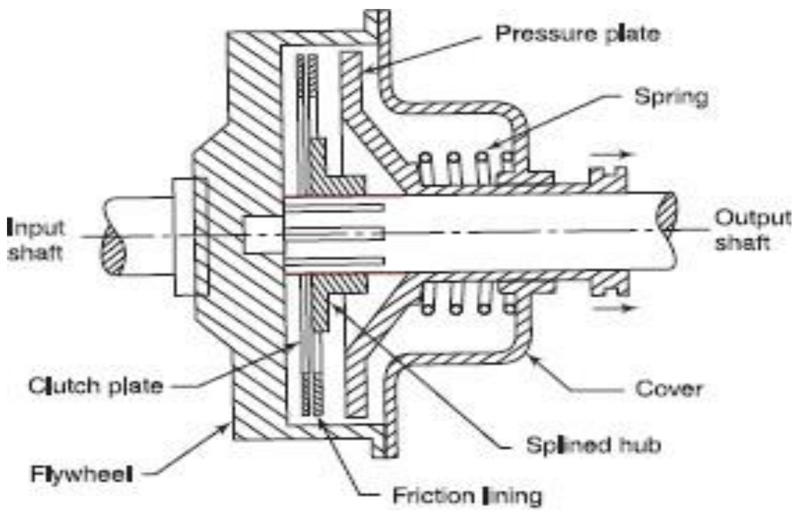
$$n = 1.96/0.35 = 5.6 \text{ say } 6 \textbf{ Ans.}$$

FRICTION CLUTCHES :

A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident. In friction clutches, the connection of the engine shaft to the gear box shaft is affected by friction between two or more rotating concentric surfaces. The surfaces can be pressed firmly against one another when engaged and the clutch tends to rotate as a single unit.

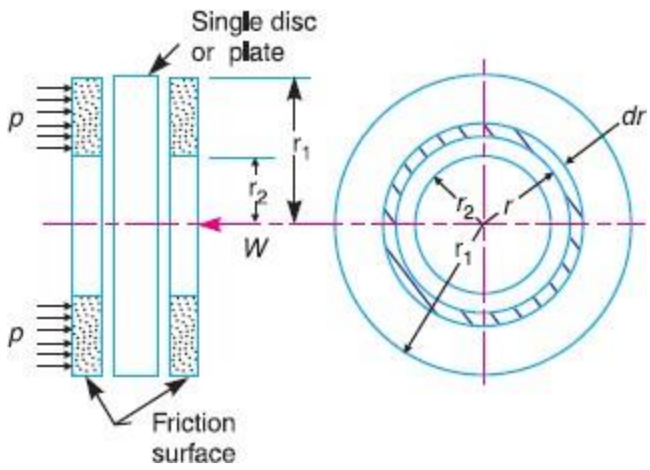
SINGLE PLATE CLUTCH (DISC CLUTCH)

A disc clutch consists of a clutch plate attached to a splined hub which is free to slide axially on splines cut on the driven shaft. The clutch plate is made of steel and has a ring of friction lining on each side. The engine shaft supports a rigidly fixed flywheel. A spring-loaded pressure plate presses the clutch plate firmly against the flywheel when the clutch is engaged. When disengaged, the springs press against a cover attached to the flywheel. Thus, both the flywheel and the pressure plate rotate with the input shaft. The movement of the clutch pedal is transferred to the pressure plate through a thrust bearing. Figure 8.13 shows the pressure plate pulled back by the release levers and the friction linings on the clutch plate are no longer in contact with the pressure plate or the flywheel. The flywheel rotates without driving the clutch plate and thus, the driven shaft.



When the foot is taken off the clutch pedal, the pressure on the thrust bearing is released. As a result, the springs become free to move the pressure plate to bring it in contact with the clutch plate. The clutch plate slides on the splined hub and is tightly gripped between the pressure plate and the fly wheel. The friction between the linings on the clutch plate, and the flywheel on one side and the pressure plate on the other, cause the clutch plate and hence, the driven shaft to rotate. In case the resisting torque on the drive shaft exceeds the torque at the clutch, clutch slip will occur.

Torque transmitted by plate or disc clutch



The following notations are used in the derivation T= Torque transmitted by the clutch
P= intensity of axial pressure

r1&r2=external and internal radii of friction faces

μ= co-efficient of friction

Consider an elemental ring of radius r and thickness dr Friction surface = 2πrdr
Axial force on the dw= pressure *area

= P*2πrdr

Frictional force acting on the ring tangentially at radius r Fr= μdw=μ*p*2πrdr

Frictional torque acting on the ring $T_r=F_r*r=\mu p*2\pi r*dr*r=2\pi\mu pr^2dr$

Considering uniform pressure

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity ofpressure,

$P=W/\pi[(r^2-(r)^2]$ (i)

1 2

Where *W* = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius *r* and thickness *dr*is

$T_r = 2\ \pi\mu.p.r^2 dr$

Integrating this equation within the limits from r_2 to r_1 for the total frictional

torque.

Therefore Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.p.r^2.dr = 2\ \pi\mu\ p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu\ p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (i),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. Considering uniform wear

Let p be the normal intensity of pressure at a distance r from the axis of the Clutch. Since the intensity of pressure varies inversely with the distance, therefore

$p.r. = C$ (a constant) or $p = C/r$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr = 2\pi C.dr$$

$\therefore$ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C\,dr = 2\pi C\left[r\right]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p\,r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu.W(r_1 + r_2) = \mu.W.R$$

R = Mean radius of the friction surface = $(r_1 + r_2)/2$

Multiple plate clutch

In a multi-plate clutch, the number of frictional linings and the metal plates is

Increased which increases the capacity of the clutch to transmit torque. Figure8.14 show a simplified diagram of a multi-plate clutch. The friction rings are splined on their outer circumference and engage with corresponding splines on the flywheel. They are free to slide axially.

The Friction material thus, rotates with the flywheel and the engine shaft. The Number of friction rings

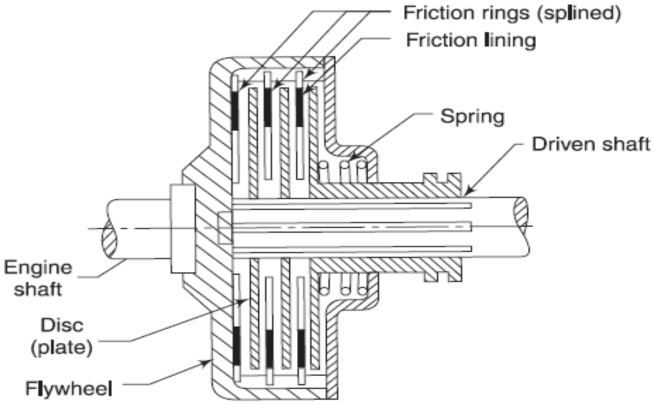


Fig. 8.14

depends upon the torque to be transmitted.

The driven shaft also supports discs on the splines which rotate with the driven shaft and can slide axially. If the actuating force on the pedal is removed, a spring presses the discs into contact with the friction rings and the torque is transmitted between the engine shaft and the driven shaft. If *n* is the total number of plates both on the driving and the driven members, the number of active surfaces will be $n - 1$.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

Number of pairs of contact surfaces,

$n = n_1 + n_2 - 1$

And total frictional torque acting on the friction surfaces or on the clutch,

$T = n.\mu.W.R$

Where R = Mean radius of the frictionsurfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{r_1 + r_2}{2}$$

PROBLEMS

Example1. Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.

Solution. Given: W = 4 kN = 4 × 103 N, r_2 = 50 mm ;r_1 = 100mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$p_{max} \times r2 = C$ or $C = 50\ p_{max}$

We know that the total force on the contact surface (W),

$4 \times 103 = 2\ \pi\ C\ (r_1 - r_2) = 2\ \pi \times 50\ p_{max}\ (100 - 50) = 15\ 710\ p_{max}$

$P_{max} = 4 \times 103/15\ 710 = 0.2546$ N/mm^2

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), Therefore $P_{min} \times r_1 = C$ or $C = 100\ p_{min}$

We know that the total force on the contact surface (W),

$4 \times 103 = 2\,\pi\, C\,(r_1 - r_2) = 2\pi \times 100\, p_{min}\,(100 - 50) = 31\,420\, p_{min}$

$P_{min} = 4 \times 103/31\,420 = 0.1273$ N/mm^2

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2$$

Example2. A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surfaceis not to exceed 0.1 N/mm2.Ifthe coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500r.p.m.

Solution.Given: $d_1 = 300$ mm or $r_1 = 150$ mm ;$d_2 = 200$ mm or $r_2 = 100$ mm ;

$p = 0.1$ N/mm^2; $\mu = 0.3$;N= 2500 r.p.m. or $\omega = 2\pi \times 2500/60 = 261.8$ rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform

$p.r_2 = C$ or $C = 0.1 \times 100 = 10$ N/mm

We know that the axial thrust,

$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 10\,(150 - 100) = 3142$ N

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

Mean radius of the friction surfaces for uniform wear,

We know that torque transmitted,

$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65$ N-m

Power transmitted by a clutch,

$P = T * \omega = 235.65 \times 261.8 = 61\,693$ W = 61.693 kW

CONE CLUTCH

A cone clutch, as shown in Fig. 10.24, was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch

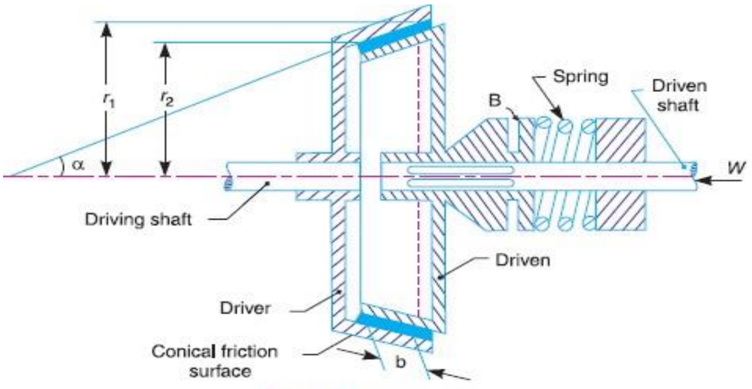


Fig. 10.24. Cone clutch.

It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of thedriven.

The driven member resting on the feather key in the driven shaft, maybe shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member islined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.*contact surfaces) depends upon the allowable normal pressure and the coefficient offriction.

Consider a pair of friction surface as shown in Fig. Since the area of contact of a pair of friction surface is a frustum of a cone, therefore the torque transmitted by the cone clutch maybe determined in the similar manner as discussed.

Let p_n = Intensity of pressure with which the conical friction surfaces areheld together (*i.e.* normal pressure between contactsurfaces),

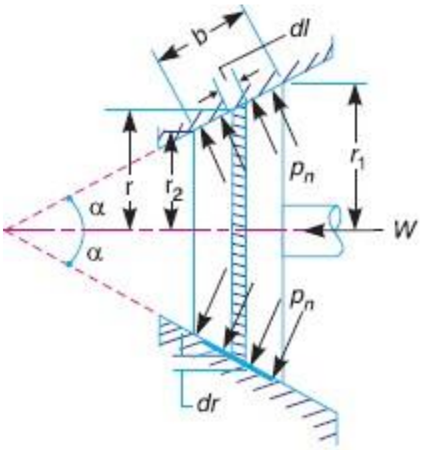
r_1 and r_2 = Outer and inner radius of friction surfaces respectively

R = Mean radius of the friction surface=$(r_1 + r_2)/2$

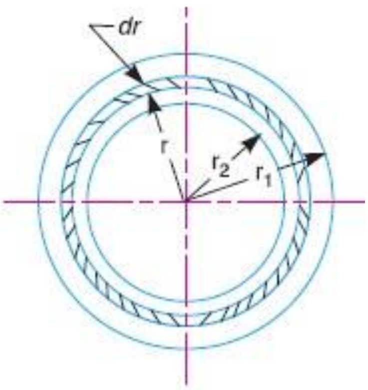
α= Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).



(a)



(b)

Consider a small ring of radius r and thickness dr, as shown in Fig. 10.25 (b).

Let dl is length of ring of the friction surface, such that

$dl = dr.\text{cose}\ \alpha$ Area of the ring= $A = 2\pi\ r.dl = 2\pi r.dr \text{cosec}\ \alpha$

We shall consider the following two cases :

When there is a uniform pressure,and

When there is a uniformwear.

Considering uniform pressure

We know that normal load acting on the ring,

δW_n = Normal pressure × Area of ring = $p_n \times 2\ \pi r.dr.\text{cosec}\ \alpha$ The axial load acting on the ring,

δW= Horizontal component of δW_n (*i.e.* in the direction of W)

$= \delta W_n \times \sin\alpha = p_n \times 2\pi\ r.dr.\ \text{cosec}\ \alpha \times \sin\alpha = 2\pi \times p_n .r.dr$

Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi\, p_n.r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n \left[(r_1)^2 - (r_2)^2\right]$$

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

We know that frictional force on the ring acting tangentially at radius r, $F_r = \mu.\delta W_n = \mu.p_n \times 2\ \pi\ r.dr.\text{cosec}\ \alpha$

Frictional torque acting on the ring,

$T_r = F_r \times r = \mu.p_n \times 2\,\pi r.dr.\ \text{cosec}\ \alpha.r = 2\,\pi\mu.p_n.\text{cosec}\ \alpha.r_2\,dr$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{cosec}\ \alpha.r^2.dr = 2\pi\mu\, p_n.\text{cosec}\ \alpha\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\ p_n.\text{cosec}\ \alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec}\ \alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec}\ \alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Considering uniform wear

In Fig. 10.25, let $p_r$be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with thedistance.

$P_r.r = C$ (a constant) or $p_r = C / r$

We know that the normal load acting on the ring,

δW_n = Normal pressure × Area of ring = $p_r \times 2\pi r.dr\text{cosec}\alpha$ The axial load acting on the ring ,

$\delta W = \delta W_n \times \sin\alpha = p_r.2\,\pi\, r.dr.\text{cosec}\ \alpha.\sin\alpha = p_r \times 2\pi\, r.dr$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that frictional force acting on the ring,

$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi r \times dr\text{cosec}\alpha$ Frictional torque acting on the ring,

$$= \mu \times \frac{C}{r} \times 2\pi r^2 .dr.\text{cosec}\ \alpha = 2\pi\mu.C\ \text{cosec}\ \alpha \times r\ dr$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.\text{cosec}\ \alpha.r\,dr = 2\pi\mu.C.\text{cosec}\ \alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C.\text{cosec}\ \alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation (*i*), we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \text{cosec}\ \alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \mu.W\ \text{cosec}\ \alpha\left(\frac{r_1 + r_2}{2}\right) = \mu.W.R\ \text{cosec}\ \alpha$$

$$R = \frac{r_1 + r_2}{2} = \text{Mean radius of friction surface}$$

$T_r = F_r \times r = \mu.p_r \times 2\pi\ r.dr.\text{cosec}\alpha \times r$

PROBLEMS

Example 1.An engine developing 45 kW at 1000 r.p.m. is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5º and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm 2. Determine: 1. the axial spring force necessary to engage to clutch, and 2. the face widthrequired.

Solution.Given :P = 45 kW = 45 × 103 W ; N = 1000 r.p.m. or ω= 2π× 1000/60 = 104.7rad/s ; α= 12.5º ; D = 500 mm or R = 250 mm = 0.25 m ; μ = 0.2 ;

p_n=0.1N/mm^2

Axial spring force necessary to engage theclutch

First of all, let us find the torque (T) developed by the clutch and the normal load (W_n) acting on the friction surface.

We know that power developed by the clutch (P),

45 × 103 = $T\omega$= T × 104.7 or T = 45 × 10^3/104.7 = 430 N-m

We also know that the torque developed by the clutch (T), 430 = $\mu.W_n.R$ = 0.2 × W_n× 0.25 = 0.05 W_n

W_n = 430/0.05 = 8600 N

Axial spring force necessary to engage the clutch,

$W_e = W_n(\sin\alpha + \mu\cos\alpha)$

$= 8600 (\sin 12.5° + 0.2 \cos 12.5°) = 3540$ N

Face widthrequired

Let b = Face widthrequired

We know that normal load acting on the friction surface (W_n), $8600 = p_n \times 2\pi R.b = 0.1 \times 2\pi \times 250 \times b = 157\, b$

$b = 8600/157 = 54.7$ mm

Example 2.A conical friction clutch is used to transmit 90 kW at 1500 r.p.m. The semi cone angle is 20º and the coefficient of friction is 0.2. If the mean diameter of the bearing surface is375 mm and the intensity of normal pressure is not to exceed 0.25 N/mm2, find the dimensions of the conical bearing surface and the axial load required.

Solution.Given: P = 90 kW = 90×10^3 W ;N = 1500 r.p.m. or $\omega = 2\pi \times 1500/60 = 156$rad/s ; $\alpha = 20°$; $\mu = 0.2$; D = 375 mm or R = 187.5 mm ; p_n = 0.25 N/mm^2

Dimensions of the conical bearing surface

Let r_1 and r_2 = External and internal radii of the bearing surface respectively,

b = Width of the bearing surface in mm, and

T = Torque transmitted.

We know that power transmitted (P), $90 \times 103 = T\omega = T \times 156$

$T = 90 \times 103/156 = 577$ N-m = 577×10^3 N-mm

The torque transmitted (T),

$577 \times 103 = 2\pi\mu\, p_n.R^2.b = 2\pi \times 0.2 \times 0.25\,(187.5)^2 b = 11\,046\, b$

$b = 577 \times 10^3/11\,046 = 52.2$ mm

We know that $r_1 + r_2 = 2R = 2 \times 187.5 = 375$ mm i

$r1 - r2 = b \sin\alpha = 52.2 \sin 20° = 18$ mm ii

From equations (i) and (ii),

r_1 = 196.5 mm, and r_2 = 178.5 mm

Axial load required

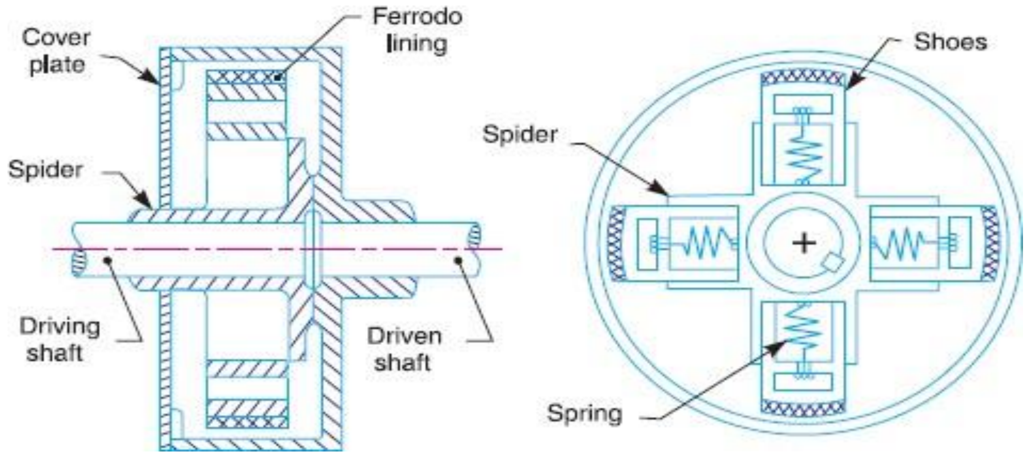
Since in case of friction clutch, uniform wear is considered and the intensity of pressure is maximum at the minimum contact surface radius ($r2$), therefore

$p_n.r_2 = C$ (a constant) or $C = 0.25 \times 178.5 = 44.6$ N/mm We know that the axial load required,

$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 44.6\,(196.5 - 178.5) = 5045$ N

Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 10.28. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held



Against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to betransmitted.

In order to determine the mass and size of the shoes, the following procedure is adopted:

Mass of the shoes

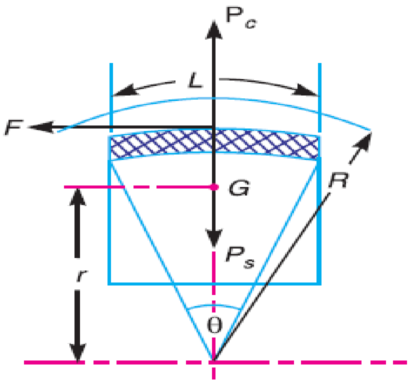


Fig. 10.29. Forces on a shoe of centrifugal clutch.

Consider one shoe of a centrifugal clutch as shown in Fig Let m = Mass of eachshoe,

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad/s

= $2\pi N/60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

∝ = Coefficient of friction between the shoe and rim.

We know that the centrifugal force acting on each shoe at the running speed,

$*P_c = m\,.\omega^2.r$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place,

$P_s = m\,(\omega_1)^2 r$

∴ The net outward radial force (*i.e.* centrifugal force) with which

The shoe presses against the rim at the running speed

$= P_c - P_s$

The frictional force acting tangentially on each shoe,

F = ∝ $(P_c - P_s)$

∴ Frictional torque acting on each shoe,

$= F \times R = \propto (P_c - P_s) R$

Total frictional torque transmitted,

$T = \propto (P_c - P_s) R \times n = n.F.R$

From this expression, the mass of the shoes (m) may be

evaluated.

Size of the shoes

Let l = Contact length of theshoes, b = Width of theshoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reason-able life, the intensity of pressure may be taken as 0.1 N/mm^2.

We know that $\theta = l/R$ rad or $l = \theta.R$

∴ Area of contact of the shoe,

$A = l.b$

The force with which the shoe presses against the rim

$A \times p = l.b.p$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$l.b.p = P_c - P_s$

PROBLEMS

Example 1.A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine: 1. Mass of the shoes, and 2. Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60º and the pressure exerted on the shoes is 0.1N/mm^2.

Solution.Given :P = 15 kW = 15 × 10^3 W ; N = 900 r.p.m. or ω = 25 × 900/60 = 94.26 rad/s ; n = 4 ; R = 150 mm = 0.15 m ; r = 120 mm = 0.12 m ; ∝ = 0.25

Since the speed at which the engagement begins (*i.e.* ω_1) is 3/4th of the running speed (*i.e.* ω), therefore

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Let T = Torque transmitted at the running speed.

We know that power transmitted (P),

$= T.\omega = T \times 94.26$ or $T = 15 \times 10^3/94.26 =$

=159 N-m

15
×
10

Mass of theshoes

Let m = Mass of the shoes in kg.

We know that the centrifugal force acting on each shoe,

$P_c = m.\omega^2.r = m\,(94.26)^2 \times 0.12 = 1066\,m$ N

the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed ω_1,

$P_s = m\,(\omega_1)^2 r = m\,(70.7)^2 \times 0.12 = 600\,m$ N

∴ Frictional force acting tangentially on each shoe,

F= $\propto (P_c - P_s) = 0.25\,(1066\,m - 600\,m)\ 116.5\,m$ N

We know that the torque transmitted (T),

$159 = n.F.R = 4 \times 116.5\,m \times 0.15 = 70\,m$ or $m = 2.27$ kg

Size of theshoes

Let l = Contact length of shoes in mm,

b = Width of the shoes in mm,

θ Angle subtended by the shoes at the centre of the spider in radians

= 60º = π/3 rad, and

p = Pressure exerted on the shoes in N/mm^2 = 0.1 N/mm^2

We know that $l = \theta . R = \frac{\pi}{3} \times 150 = 157.1$ mm

$$l.b.p = P_c - P_s = 1066\, m - 600\, m = 466\, m$$

$\therefore$ $157.1 \times b \times 0.1 = 466 \times 2.27 = 1058$

b = 1058/157.1 × 0.1 = 67.3 mm

Example2. A centrifugal clutch has four shoes which slide radially in a spider keyed to the driving shaft and make contact with the internal cylindrical surface of a rim keyed to the driven shaft. When the clutch is at rest, each shoe is pulled against a stop by a spring so as to leave a radial clearance of 5 mm between the shoe and the rim. The pull exerted by the spring is then 500 N. The mass centre of the shoe is 160 mm from the axis of theclutch.

If the internal diameter of the rim is 400 mm, the mass of each shoe is 8 kg, the stiffness of each spring is 50 N/mm and the coefficient of friction between the shoe and the rim is 0.3 ;find the power transmitted by the clutch at 500r.p.m.

Solution.Given : n = 4 ; c = 5 mm ; S = 500 N ; r = 160 mm ; D = 400 mm or R = 200 mm=0.2 m ; m = 8 kg ; s = 50 N/mm ; $\propto$ = 0.3 ; N = 500 r.p.m. or $\omega = 2\pi \times 500/60 = 52.37$ rad/s

We know that the operating radius,

$r_1 = r + c = 160 + 5 = 165$ mm = 0.165 m

Centrifugal force on each shoe,

$P_c = m\,.\omega^2.r_1 = 8\,(52.37)^2 \times 0.165 = 3620$ N

The inward force exerted by the spring,

$P_4 = S + c.s = 500 + 5 \times 50 = 750$ N

$\therefore$ Frictional force acting tangentially on each shoe, $F = \propto (P_c - P_s) = 0.3\,(3620 - 750) = 861$ N

We know that total frictional torque transmitted by the clutch,

$T = n.F.R = 4 \times 861 \times 0.2 = 688.8$ N-m

$\therefore$ Power transmitted,

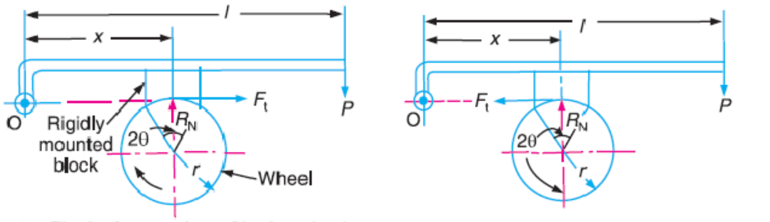
$P = T.\omega = 688.8 \times 52.37 = 36\,100$ W = 36.1 kW

BRAKES

A *brake* is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc

Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 19.1. The other end of the lever is pivoted on a fixed fulcrum *O*.



(*a*) Clockwise rotation of brake wheel (*b*) Anticlockwise rotation of brake wheel.

Let P = Force applied at the end of thelever

R_N= Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$F_t = \mu . R_N$ ...(*i*)

The braking torque, $T_B = F_t . r = \mu . R_N . r$ ...(*ii*)

Let us now consider the following three cases:

Case1. When the line of action of tangential braking force (*Ft*) passes through the fulcrum *O* of the lever, and the brake wheel rotates clockwise as shown in Fig. 19.1(*a*), then for equilibrium, taking moments about

$$R_N \times x = P \times l \text{ or } R_N = \frac{P \times l}{x}$$

Braking torque,

$$T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

the fulcrum *O*, wehave

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 19.1 (*b*), then the braking torque is same, *i.e.*

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case2. When the line of action of the tangential braking force (*Ft*) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig.

(*a*), then for equilibrium, taking moments about the fulcrum*O*,

$$R_N \times x + F_t \times a = P.l \quad \text{or} \quad R_N \times x + \mu R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

$$T_B = \mu R_N.r = \frac{\mu.p.l.r}{x + \mu.a}$$

(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

When the brake wheel rotates anticlockwise, as shown in Fig. 19.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$$

$$R_N (x - \mu.a) = P.l \quad \text{or} \quad R_N = \frac{P.l}{x - \mu.a}$$

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

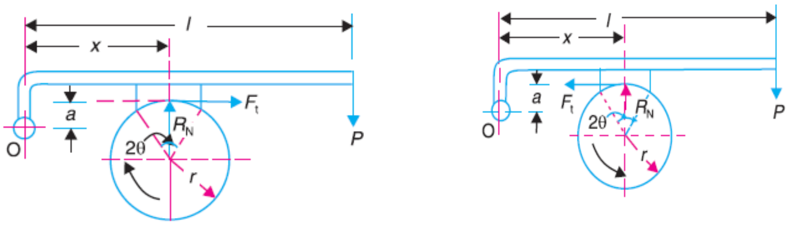
Case 3.When the lie of action of the tangential braking force (F_t) passes through a distance '*a*' above the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig.

(*a*), then fore equilibrium, taking moments about the fulcrum *O*, wehave

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$$

$R_N(x - \mu.a) = P.l$

$$R_N = \frac{P.l}{x - \mu.a}$$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

When the brake wheel rotates anticlockwise as shown in Fig. 19.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$R_N \times x + F_t \times a = P.l$ $R_N \times x + \mu.R_N \times a = P.l$

$$R_N = \frac{P.l}{x + \mu.a} \qquad T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre.

Instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque or a pivoted block

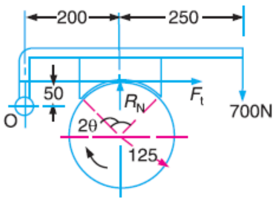
$$T_B = F_t \times r = \mu'.R_N.r$$

$$\mu' = \text{Equivalent coefficient of friction} = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta},$$

$$\mu = \text{Actual coefficient of friction.}$$

PROBLEMS

Example1. A single block brake is shown in Fig. 19.5.The diameter of the drum is 250 mm and the angle of contact is90°. If the operating force of 700 N is applied at the end of alever and the coefficient of friction between the drum and the lining is0.35,Determine the torque that may be transmitted by the blockbrake.



All dimensions in mm.

Fig. 19.5

Solution.Given :d = 250 mm or r = 125 mm ; $2\theta = 90° = \pi / 2$ rad ; P = 700 N ; μ = 0.35

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi / 2 + \sin 90°} = 0.385$$

R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.R_N$

Taking moments about the fulcrum O, we have

$$700(250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\, F_t$$

520 F_t – 50F_t = 700 × 450 or F_t = 700 × 450/470 = 670 N

We know that torque transmitted by the blockbrake,

$T_B = F_t \times r$ = 670 × 125 = 8 3750 N-mm =83.75N-m

Example 2.A bicycle and rider of mass 100 kg are travelling at the rate of 16 km/h on a level road. A brake is applied to the rear wheel which is 0.9 m in diameter and this is the only resistance acting. How far will the bicycle travel and how many turns will it make before it comes to rest? The pressure applied on the brake is 100 N and $\mu = 0.05$.

Solution.Given: $m = 100$ kg, $v = 16$ km / h = 4.44 m / s ;$D = 0.9$ m ; R

N = 100 N; $\mu = 0.05$

Distance travelled by the bicycle before it comes to rest

Let x = Distance travelled (in meters) by the bicycle before it comes to rest.

We know that tangential braking force acting at the point of contact of the brake wheel,

$F_t = \mu.R_N = 0.05 \times 100 = 5$ N

$= F_t \times x = 5 \times x = 5x$ N-m (i)

We know that kinetic energy of the bicycle

$$= \frac{m.v^2}{2} = \frac{100(4.44)^2}{2}$$
$$= 986 \text{ N-m} \qquad \ldots (ii)$$

In order to bring the bicycle to rest, the work done against friction must be equal to kinetic energy of the bicycle. Therefore equating equations (*i*) and (*ii*),

$5x = 986$ or $x = 986/5 = 197.2$ m

Number of revolutions made by the bicycle before it comes to rest

Let N = Required number of revolutions.

We know that distance travelled by the bicycle (x), $197.2 = \pi DN = \pi \times 0.9N = 2.83N$

$N = 197.2 / 2.83 = 70$

Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to the normal force (RN). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as shown in Fig. 19.9, is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force P is produced by an electromagnet or solenoid.

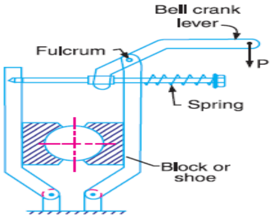


Fig. 19.9. Double block or shoe brake.

In a double block brake, the braking action is doubled by the use of two blocks and these blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$T_B = (F_{t1} + F_{t2})\, r$

Where F_{t1} and F_{t2} are the braking forces on the two blocks.

Internal Expanding Brake

An internal expanding brake consists of two shoes $S1$ and $S2$ as shown in Fig.

19.24. The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum $O1$ and $O2$ and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 19.24. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.

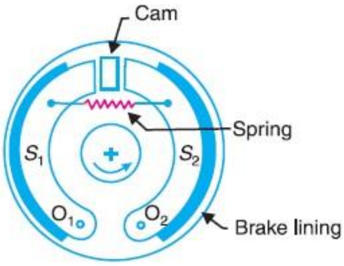


Fig. 19.24. Internal expanding brake.

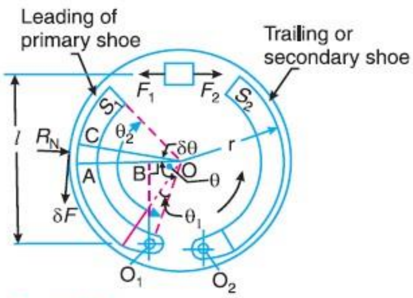


Fig. 19.25. Forces on an internal expanding brake.

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. 19.25. It may be noted that for the anticlockwise direction, the left hand shoe is known as *leading* or *primary shoe* while the right hand shoe is known as *trailing* or *secondaryshoe*.

Let r = Internal radius of the wheelrim,

b = Width of the brake lining,

p_1 = Maximum intensity of normal pressure,

p_N= Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Consider a small element of the brake lining

AC subtending an angle $\delta\theta$ at the centre. Let OA makes an angle θ with OO_1 as shown in Fig. 19.25. It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about$O1$, therefore the rate of wear of the shoe lining at A will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from $O_1$to OA, *i.e.*

O_1B. From the geometry of the figure,

$O_1B = OO_1 \sin\theta$

normal pressure at A,

$$p_N \propto \sin\theta \quad \text{or} \quad p_N = p_1 \sin\theta$$

∴ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N(b.r.\delta\theta) = p_1 \sin\theta(b.r.\delta\theta)$$

and braking or friction force on the element,

$$\delta F = \mu \times \delta R_N = \mu.p_1 \sin\theta(b.r.\delta\theta)$$

∴ Braking torque due to the element about O,

$$\delta T_B = \delta F \times r = \mu.p_1 \sin\theta(b.r.\delta\theta)r = \mu.p_1 b\, r^2(\sin\theta.\delta\theta)$$

and total braking torque about O for whole of one shoe,

$$T_B = \mu\, p_1 b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\, p_1 b\, r^2 [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1B = \delta R_N (OO_1 \sin\theta)$$

$$= p_1 \sin\theta(b.r.\delta\theta)(OO_1 \sin\theta) = p_1 \sin^2\theta(b.r.\delta\theta)OO_1$$

∴ Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta(b.r.\delta\theta)OO_1 = p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

$$= p_1 .b.r.OO_1 \int_{\theta_1}^{\theta_2} \frac{1}{2}(1-\cos 2\theta)d\theta \qquad \ldots\left[\because \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)\right]$$

$$= \frac{1}{2} p_1 .b.r.OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2} p_1 .b.r.OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2} p_1 .b.r.OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F(r - OO_1 \cos\theta) \qquad \ldots(\because AB = r - OO_1 \cos\theta)$$

$$= \mu\, p_1 \sin\theta (b.r.\delta\theta)\,(r - OO_1 \cos\theta)$$

$$= \mu . p_1 .b.r(r\sin\theta - OO_1 \sin\theta\cos\theta)\delta\theta$$

$$= \mu . p_1 .b.r\left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right)\delta\theta \qquad \ldots(\because 2\sin\theta\cos\theta = \sin 2\theta$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu\, p_1 b\, r \int_{\theta_1}^{\theta_2} \left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right) d\theta$$

$$= \mu\, p_1 b\, r \left[-r\cos\theta + \frac{OO_1}{4}\cos 2\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b\, r \left[-r\cos\theta_2 + \frac{OO_1}{4}\cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4}\cos 2\theta_1\right]$$

$$= \mu\, p_1 b\, r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

When the brakes are applied to all the fourwheels

Since the vehicle moves on a level road, therefore retardation of the car,

$a = g.\mu = 9.81 \times 0.6 = 5.886$ m/s2

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2 \times 5.886} = 16.4 \text{ m}$$

Example2. A vehicle moving on a rough plane inclined at 10° with the horizontal at a speed of 36 km/h has a wheel base 1.8 metres. The centre of gravity of the vehicle is 0.8 metre from the rear wheels and 0.9 metre above the inclined plane. Find the distance travelled by the vehicle before coming to rest and the time taken to do so when
The vehicle moves up the plane,and
The vehicle moves down theplane.
The brakes are applied to all the four wheels and the coefficient of friction is 0.5.

Solution.
Given : α= 10°; u = 36 km / h = 10 m / s ; L = 1.8 m ; x = 0.8 m ; h = 0.9 m ; μ = 0.5 Let s = Distance travelled by the vehicle before coming to rest,and

t = Time taken by the vehicle in coming to rest.

When the vehicle moves up the plane and brakes are applied to all the four wheel

Since the vehicle moves up the inclined plane, therefore retardation of the vehicle,

$a = g\,(\mu \cos\alpha + \sin\alpha)$

$= 9.81\,(0.5\cos 10° + \sin 10°) = 9.81(0.5 \times 0.9848 + 0.1736) = 6.53$ m/s^2

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2 \times 6.53} = 7.657 \text{ m}$$

and final velocity of the vehicle (v),

$0 = u + a.t = 10 - 6.53t$ (Minus sign due toretardation)

$t = 10 / 6.53 = 1.53$

When the vehicle moves down the plane and brakes are applied to all the four wheels

Since the vehicle moves down the inclined plane, therefore retardation of the vehicle,

$a = g\,(\mu \cos\alpha - \sin\alpha)$

$= 9.81(0.5\cos 10° - \sin 10°) = 9.81(0.5 \times 0.9848 - 0.1736) = 3.13\ m/s^2$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2 \times 3.13} = 16\ m$$

and final velocity of the vehicle (*v*),

$0 = u + a.t = 10 - 3.13t$ . . . (Minus sign due toretardation)

$t = 10/3.13 = 3.2$ s

UNIT 4

BELT, ROPES AND CHAINS

Course objectives:

1. Able to learn about the working of Belt, Rope and Chains.

Course Outcomes:

1. Knowledge acquired about belt, rope and chain for various applications.

Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.

2. The tension under which the belt is placed on the pulleys.

3. The arc of contact between the belt and the smaller pulley.

4. The conditions under which the belt is used. It may be noted that

(a) The shafts should be properly in line to insure uniform tension across the belt section.

(b) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

(c) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.

(d) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.

(e) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

(f) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 meters and the minimum should not be less than 3.5 times the diameter of the larger pulley. Selection of a Belt Drive

Various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,

2. Speed reduction ratio,

3. Power to be transmitted,

4. Centre distance between the shafts,

5. Positive drive requirements,

6. Shafts layout,

7. Space available, and 8. Service conditions.

Types of Belt Drives

The belt drives are usually classified into the following three groups:

1. Light drives. These are used to transmit small powers at belt speeds up to about 10 m/s as in agricultural machines and small machine tools.

2. Medium drives. These are used to transmit medium powers at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

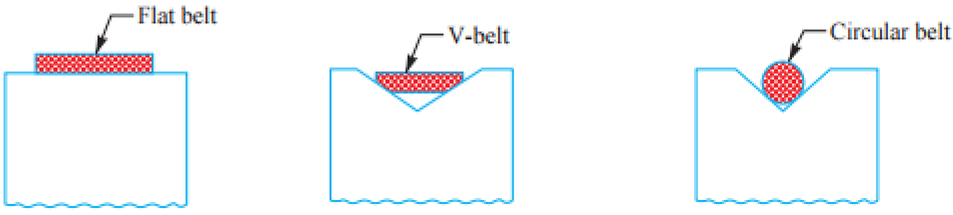
3. Heavy drives. These are used to transmit large powers at belt speeds above 22 m/s as in Compressors and generators.

Types of Belts

Though there are many types of belts used these days, yet the following are important from the

1. Flat belt.

The flat belt as shown in Fig. 18.1 (a), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

2. V- belt. The V-belt as shown in Fig. (b) is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. Circular belt or rope. The circular belt or rope as shown in Fig. (c) Is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart. If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

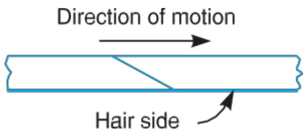
MATERIAL USED FORBELTS

The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows

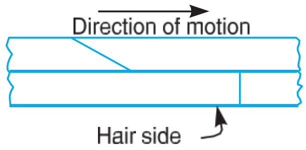
LEATHER BELTS.

The most important material for the belt is leather. The best leather belts are made from 1.2 metres to 1.5 metres long strips cut from either side of the back bone of the top grade steer hides. The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibres on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons, the hair side of a belt should be in contact with the pulley surface, as shown in Fig. This gives a more intimate contact between the belt and the pulley and places the greatest tensile strength of the belt section on the outside, where the tension is maximum as the belt passes over the pulley.

(a) Single layer belt.



(b) Double layer belt



2. The leather may be either oak-tanned or mineral salt tanned e.g. chrome tanned. In order to increase the thickness of belt, the strips are cemented together. The belts are specified according to the number of layers e.g. single, double or triple ply and according to the thickness of hides used e.g. light, medium or heavy.

   The leather belts must be periodically cleaned and dressed or treated with a compound or dressing containing neat foot or other suitable oils so that the belt will remain soft and flexible.

3. **COTTON OR FABRIC BELTS**: Most of the fabric belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belts water proof and to prevent injury to the fibres. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in farm machinery, belt conveyor etc.

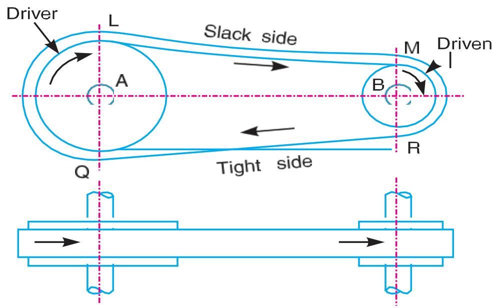
4. **RUBBER BELT.** The rubber belts are made of layers of fabric impregnated with rubber com- position and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principal advantages of these belts is that they may be easily made endless. These belts are found suitable for saw mills, paper mills where they are exposed to moisture.

5. **BALATA BELTS.** These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not effected animal oils or alkalies. The balata belts should not be at temperatures above 40° C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 per cent higher than rubber belts.

TYPES OF FLAT BELT DRIVES

The power from one pulley to another may be transmitted by any of the following types of belt drives:

OPEN BELT DRIVE. The open belt drive, as shown in Fig. 3.3, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (i.e. lower side RQ)



and delivers it to the other side (i.e. upper side L M). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as tight side whereas the upper side belt (because of less tension) is known as slack side, as shown in Fig.

CROSSED OR TWIST BELT DRIVE : The crossed or twist belt drive, as shown in Fig. is used with shafts arranged parallel and rotating in the opposite directions.

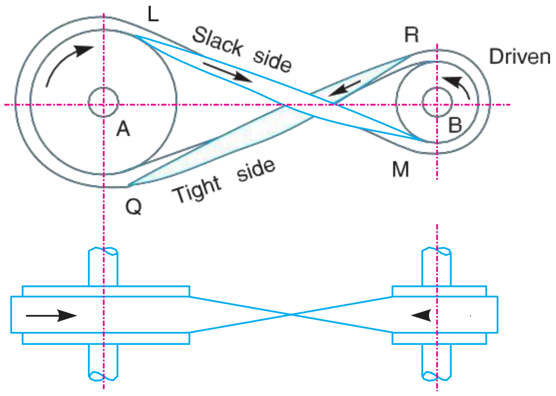
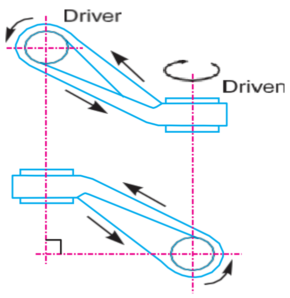


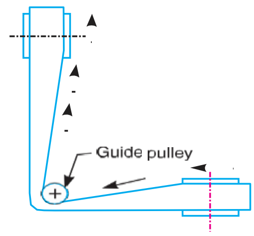
Fig. Crossed or twist belt drive.

In this case, the driver pulls the belt from one side (i.e. RQ) and delivers it to the other side (i.e. L M). Thus the tension in the belt RQ will be more than that in the belt L M. The belt RQ (because of more tension) is known as tight side, whereas the belt LM (because of less tension) is known as slack side, as shown in Fig. A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 b, where b is the width of belt and the speed of the belt should be less than 15m/s.

QUARTER TURN BELT DRIVE. The quarter turn belt drives also known as right angle belt drive, as shown in Fig. (a), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to b, where b is the width of belt.

In case the pulleys cannot be arranged, as shown in Fig.(a), or when the reversible motion is desired, then a quarter turn belt drive with guide pulley, as shown in Fig.(b), may be used.

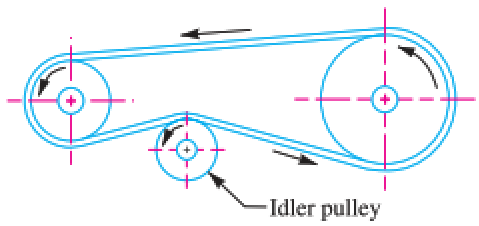


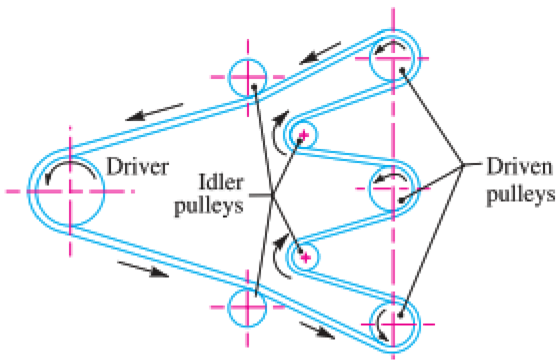


(a) Quarter turns belt drive. (b) Quarter turn belt drive with guide pulley

4. BELT DRIVE WITH IDLER PULLEYS.

A belt drive with an idler pulley (also known as jockey pulley drive) as shown in Fig. 18.7, is used with shafts arranged parallel and when an open belt drive can't be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension can't be obtained by other means. When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig.





COMPOUND BELT DRIVE.

A compound belt drive, as shown in Fig. is used when power is transmitted from one shaft to another through a number of pulleys.

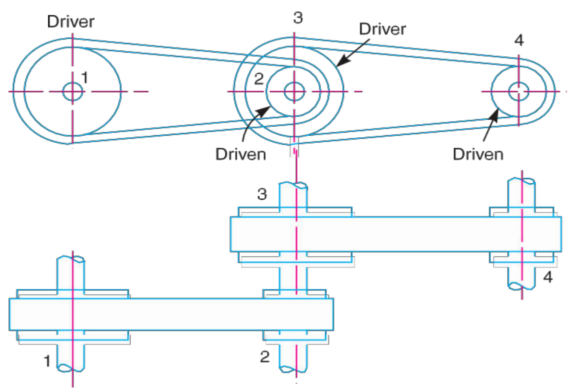
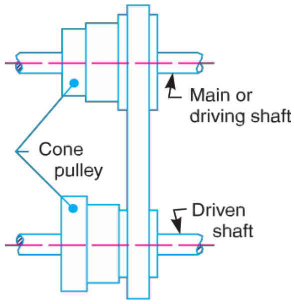


Fig: Compound belt drive.

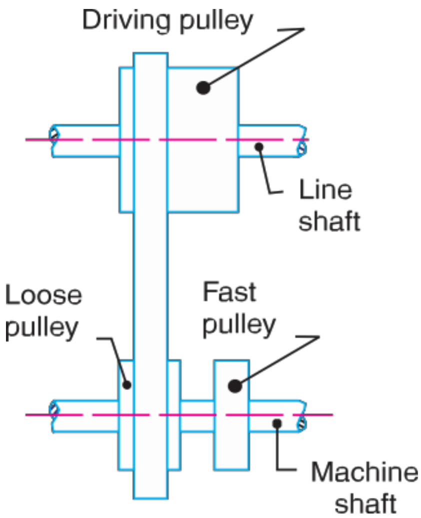
STEPPED OR CONE PULLEY DRIVE.

A stepped or cone pulley drive, as shown in Fig. is used for changing the speed of the driven shaft while the



main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

7. Fast and loose pulley drive. A fast and loose pulley drive, as shown in Fig. is used when the driven or



machine shaft is to be started or stopped whenever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

VELOCITY RATIO OF A BELT DRIVE:

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m.,

N_2 = Speed of the follower in r.p.m.,

∴ Length of the belt that passes over the driver, in one minute

$$= \pi d_1 N_1$$

Similarly, length of the belt that passes over the follower, in one minute

$$= \pi d_2 N_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\because \quad \pi d_1 N_1 = \pi d_2 N_2$$

and velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

When thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes : 1. The velocity ratio of a belt drive may also be obtained as discussed below:

We know that the peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi d_1 N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi d_2 N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

2. In case of a compound belt drive as shown in Fig. 18.7, the velocity ratio is given by

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

SLIP OF THE BELT

In the previous articles we have discussed the motion of belts and pulleys assuming a firm Frictional grip between the belts and the pulleys. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called slip of the belt and is generally expressed as a percentage.

s1 % = Slip between the driver and the belt, and

s2 % = Slip between the belt and follower,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

CREEP OF BELT:

When the belt passes from slack side to the tight side, certain of the belt extends and it contracts again when the belt passes from the tight side to the slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep. The total effect of creep is reducing slightly the speed of the driven pulley or follower. Considering creep, velocity ratio is given by

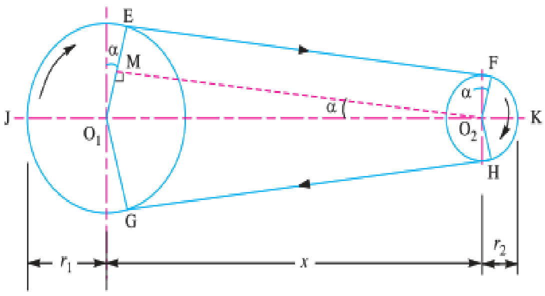
$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

Where σ_1 & σ_2 =stress in the belt on the tight and slack side

E = young's modulus for the material of the belt

Note: since the effect of creep is very small, therefore it is generally neglected.

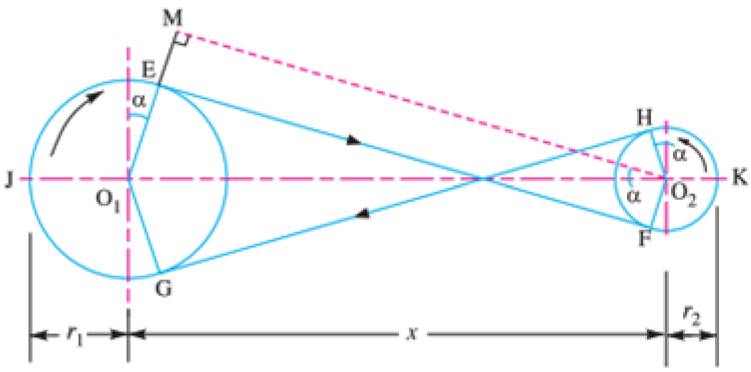
Length of Open Belt Drive:



$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

Length of a Cross Belt Drive



$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

Power Transmitted by a Belt:

T1 and T2 = Tensions in the tight side and slack side of the belt respectively in Newton's,

r1 and r2 = Radii of the driving and driven pulleys respectively in meters,

v = Velocity of the belt in m/s.

$$\mathbf{P= (T_1\text{-}T_2)\ V\ \frac{N-m}{sec}}$$

CENTRIFUGAL TENSION:

When the belt runs at lower speed, the initial tension given to the belt will be sufficient to keep the belt on the pulley with required grip, on the other hand, if the belt speed increases, due to centrifugal action, the belt will try to fly off from the pulley. At the same time, the tensions at the tight side and slack side will increase. The force applied on the shaft due to centrifugal action is called as centrifugal tension.

Let T_1 = Tension in the tight side

T_2 = Tension in the slack side

Centrifugal tension

$$\mathbf{T_c = mv^2}$$

Note: It is known that, the total tensions at tight side and slack side are given by

$$T_{t1} = T_1 + T_c \quad \text{and} \quad T_{t2} = T_2 + T_c$$

Since the centrifugal tension depends on the belt velocity, at low speeds the centrifugal action and its tension may be neglected. But for the higher speeds, the centrifugal tension will be taken into account.

$T_{t1} = T_1$ and $T_{t2} = T_2$ at low speeds, and $T_{t1} = T_1 + T_c$ and $T_{t2} = T_2 + T_c$ high speeds.

Also since the centrifugal force tries to pull the belt away from the pulley resulting the decrease of power transmitting capacity, the linear velocity of the belt is limited to 17.5 to 22.5 m/s, in order to control the centrifugal tension. If μ is the coefficient of friction between the belt and pulley and θ is the angle of contact for driving pulley in radians, then it is found that the ratio of driving tensions is

$$\mathbf{2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta}$$

$$\left(\frac{T_1}{T_2}\right) = e^{\mu\theta}$$

when the centrifugal tension (T_c) is neglected.

$$\frac{T_1 - T_c}{T_{2-T_c}} = e^{\mu\theta}$$

When the centrifugal tension (T_c) is considered.
Maximum Tension in the Belt

σ = Maximum safe stress,

b = Width of the belt, and

t = Thickness of the belt.

T = Maximum stress × Cross-sectional area of belt **= σ.b.t**

When centrifugal tension is neglected, then

T (or Tt1) =T1, i.e. Tension in the tight side of the belt.

When centrifugal tension is considered, then

T (or Tt1) =T1 + TC

Condition for the Transmission of Maximum Power

1. We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$$T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

From equation (*iv*), we find that the velocity of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

Initial Tension in the Belt

the belt is subjected to some tension, called initial tension

T_0= Initial tension in the belt,

T_1= Tension in the tight side of the belt,

T_2= Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

$$T_0 = \frac{T_1 - T_2}{2} \text{ (Neglecting centrifugal tension)}$$

$$T_0 = \frac{T_1 + T_2 + 2T_c}{2} \text{ (Considering centrifugal tension)}$$

Problems:

1. In a horizontal belt drive for a centrifugal blower, the blower is belt driven At 600 r.p.m. by a 15 kW, 1750 r.p.m. electric motor. The centre distance is twice the diameter of the larger pulley. The density of the belt material = 1500 kg/m; maximum allowable stress = 4 MPa;µ1 = 0.5 (motor pulley); µ2= 0.4 (blower pulley); peripheral velocity of the belt = 20 m/s. Determine the following:

1. Pulley diameters, 2. Belt length, 3. Cross-sectional area of the belt;

4. Minimum initial tension for operation without slip; and 5. Resultant force in the plane of the blower when operating with an initial tension 50 per cent greater than the minimum value.

Solution.

Solution.

N2 = 600 r.p.m. ;

P = 15 kW = 15 × 10 W;

N1 = 1750 r.p.m . ;ρ = 1500 kg/m^3

σ = 4 MPa = 4 × 10^6 N/m^2 ,

µ1 = 0.5 ; µ2 = 0.4 ;

v = 20 m/s

Fig. Shows a horizontal belt drive. Suffix 1 refers to a motor pulley and suffix 2 refers to a blower pulley.

1. *Pulley diameters*

Let d_1 = Diameter of the motor pulley, and

d_2 = Diameter of the blower pulley.

We know that peripheral velocity of the belt (v),

$$20 = \frac{\pi\, d_1 N_1}{60} = \frac{\pi d_1 \times 1750}{60} = 91.64\, d_1$$

∴ $d_1 = 20 / 91.64 = 0.218 \text{ m} = 218 \text{ mm}$ **Ans.**

We also know that $\frac{N_2}{N_1} = \frac{d_1}{d_2}$

∴ $$d_2 = \frac{d_1 \times N_1}{N_2} = \frac{218 \times 1750}{600} = 636 \text{ mm}$$ **Ans.**

2. *Belt length*

Since the centre distance (x) between the two pulleys is twice the diameter of the larger pulley (*i.e.* 2 d_2), therefore centre distance,

$$x = 2\, d_2 = 2 \times 636 = 1272 \text{ mm}$$

We know that length of belt,

$$L = \frac{\pi}{2}(d_1 + d_2) + 2\,x + \frac{(d_1 - d_2)^2}{4x}$$

$$= \frac{\pi}{2}(218 + 636) + 2 \times 1272 + \frac{(218 - 636)^2}{4 \times 1272}$$

$$= 1342 + 2544 + 34 = 3920 \text{ mm} = 3.92 \text{ m}$$ **Ans.**

3. *Cross-sectional area of the belt*

Let a = Cross-sectional area of the belt.

First of all, let us find the angle of contact for both the pulleys. From the geometry of the figure, we find that

$$\sin \alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2\,x} = \frac{636 - 218}{2 \times 1272} = 0.1643$$

$\therefore \quad \alpha = 9.46°$

We know that angle of contact on the motor pulley,

$$\theta_1 = 180° - 2\alpha = 180 - 2 \times 9.46 = 161.08°$$
$$= 161.08 \times \pi / 180 = 2.8 \text{ rad}$$

and angle of contact on the blower pulley,

$$\theta_2 = 180° + 2\alpha = 180 + 2 \times 9.46 = 198.92°$$
$$= 198.92 \times \pi / 180 = 3.47 \text{ rad}$$

Since both the pulleys have different coefficient of friction (μ), therefore the design will refer to a pulley for which $\mu.\theta$ is small.

$\therefore$ For motor pulley,

$$\mu_1.\theta_1 = 0.5 \times 2.8 = 1.4$$

and for blower pulley, $\quad \mu_2.\theta_2 = 0.4 \times 3.47 = 1.388$

Since $\mu_2.\theta_2$ for the blower pulley is less then $\mu_1.\theta_1$, therefore the design is based on the blower pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that power transmitted (P),

$$15 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 20$$

$$\therefore \quad T_1 - T_2 = 15 \times 10^3 / 20 = 750 \text{ N} \qquad \text{...}(i)$$

We also know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu_2.\theta_2 = 0.4 \times 3.47 = 1.388$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{1.388}{2.3} = 0.6035 \quad \text{or} \quad \frac{T_1}{T_2} = 4 \qquad \text{...}(ii)$$

... (Taking antilog of 0.6035)

From equations (*i*) and (*ii*),

$$T_1 = 1000 \text{ N}\ ; \text{ and } \ T_2 = 250 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = a \times l \times \rho$$
$$= a \times 1 \times 1500 = 1500\, a \text{ kg / m}$$

$\therefore$ Centrifugal tension,

$$T_C = m.v^2 = 1500\, a\, (20)^2 = 0.6 \times 10^6\, a \text{ N}$$

We know that maximum or total tension in the belt,

$$T = T_1 + T_C = 1000 + 0.6 \times 10^6\, a \text{ N} \qquad \text{...}(iii)$$

We also know that maximum tension in the belt,

$$T = \text{Stress} \times \text{area} = \sigma \times a = 4 \times 10^6\, a \text{ N} \qquad \text{...}(iv)$$

4. *Minimum initial tension for operation without slip*

We know that centrifugal tension,

$$T_C = 0.6 \times 10^6\, a = 0.6 \times 10^6 \times 294 \times 10^{-6} = 176.4 \text{ N}$$

∴ Minimum initial tension for operation without slip,

$$T_0 = \frac{T_1 + T_2 + 2T_C}{2} = \frac{1000 + 250 + 2 \times 176.4}{2} = 801.4 \text{ N} \quad \textbf{Ans.}$$

5. *Resultant force in the plane of the blower when operating with an initial tension 50 per cent greater than the minimum value*

We have calculated above that the minimum initial tension,

$$T_0 = 801.4 \text{ N}$$

∴ Increased initial tension,

$$T_0' = 801.4 + 801.4 \times \frac{50}{100} = 1202 \text{ N}$$

Let T_1' and T_2' be the corresponding tensions in the tight side and slack side of the belt respectively.

We know that increased initial tension (T_0'),

$$1202 = \frac{T_1' + T_2' + 2T_C}{2} = \frac{T_1' + T_2' + 2 \times 176.4}{2}$$

$$\therefore \quad T_1' + T_2' = 1202 \times 2 - 2 \times 176.4 = 2051.2 \text{ N} \qquad ...(v)$$

Since the ratio of tensions will be constant, *i.e.* $\frac{T_1'}{T_2'} = \frac{T_1}{T_2} = 4$, therefore from equation (*v*), we have

$$4T_2' + T_2' = 2051.2 \quad \text{or} \quad T_2' = 2051.2 / 5 = 410.24 \text{ N}$$

and $$T_1' = 4\, T_2' = 4 \times 410.24 = 1640.96 \text{ N}$$

∴ Resultant force in the plane of the blower

$$= T_1' - T_2' = 1640.96 - 410.24 = 1230.72 \text{ N} \quad \textbf{Ans.}$$

2. A belt 100 mm wide and 10 mm thick is transmitting power at 1000 meters/min. The net driving tension is 1.8 times the tension on the slack side. If the safe permissible stress on the belt section in 1.6 MPa, calculate the maximum power that can be transmitted at this speed. Assume density of the leather as 1000 kg/m^3.Calculate the absolute maximum power that can be transmitted by this belt and the speed at which this can be transmitted.

Solution. Given : b = 100 mm = 0.1 m ; t = 10 mm = 0.01 m ; v = 1000 m/min = 16.67 m/s ; $T_1 - T_2 = 1.8\, T_2$; σ = 1.6 MPa = 1.6 N/mm^2 ; ρ = 1000 kg/m^3

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the maximum tension in the belt,

$$T = \sigma.b.t = 1.6 \times 100 \times 10 = 1600 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= 0.1 \times 0.01 \times 1 \times 1000 = 1 \text{ kg/m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1\,(16.67)^2 = 278 \text{ N}$$

We know that

$$T_1 = T - T_C = 1600 - 278 = 1322 \text{ N}$$

and $T_1 - T_2 = 1.8\,T_2$

$$\therefore \quad T_2 = \frac{T_1}{2.8} = \frac{1322}{2.8} = 472 \text{ N}$$

We know that the power transmitted.

$$P = (T_1 - T_2)\,v = (1322 - 472)\,16.67 = 14\,170 \text{ W} = 14.17 \text{ kW} \textbf{ Ans.}$$

Speed at which absolute maximum power can be transmitted

We know that the speed of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{1600}{3 \times 1}} = 23.1 \text{ m/s} \textbf{ Ans.}$$

Absolute maximum power

We know that for absolute maximum power, the centrifugal tension,

$$T_C = T/3 = 1600/3 = 533 \text{ N}$$

∴ Tension in the tight side,

$$T_1 = T - T_C = 1600 - 533 = 1067 \text{ N}$$

and tension in the slack side,

$$T_2 = \frac{T_1}{2.8} = \frac{1067}{2.8} = 381 \text{ N}$$

∴ Absolute maximum power transmitted,

$$P = (T_1 - T_2)\,v = (1067 - 381)\,23.1 = 15\,850 \text{ W} = 15.85 \text{ kW} \textbf{ Ans.}$$

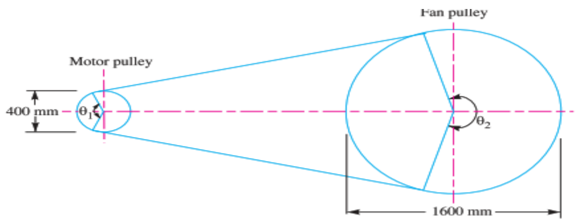
An electric motor drives an exhaust fan. Following data are provided :

	Motor pulley	*Fan pulley*
Diameter	*400 mm*	*1600 mm*
Angle of warp	*2.5 radians*	*3.78 radians*
Coefficient of friction	*0.3*	*0.25*
Speed	*700 r.p.m.*	—
Power transmitted	*22.5 kW*	—

Calculate the width of 5 mm thick flat belt. Take permissible stress for the belt material as 2.3 MPa.

Solution. Given : $d_1 = 400$ mm or $r_1 = 200$ mm ; $d_2 = 1600$ mm or $r_2 = 800$ mm ; $\theta_1 = 2.5$ rad ; $\theta_2 = 3.78$ rad ; $\mu_1 = 0.3$; $\mu_2 = 0.25$; $N_1 = 700$ r.p.m. ; $P = 22.5$ kW $= 22.5 \times 10^3$ W ; $t = 5$ mm $= 0.005$ m ; $\sigma = 2.3$ MPa $= 2.3 \times 10^6$ N/m^2

Fig. 18.19 shows a system of flat belt drive. Suffix 1 refers to motor pulley and suffix 2 refers to fan pulley.



We have discussed in Art. 18.19 (Note 2) that when the pulleys are made of different material [*i.e.* when the pulleys have different coefficient of friction (μ) or different angle of contact (θ), then the design will refer to a pulley for which $\mu.\theta$ is small.

$\therefore$ For motor pulley, $\mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$

and for fan pulley, $\mu_2.\theta_2 = 0.25 \times 3.78 = 0.945$

Since $\mu_1.\theta_1$ for the motor pulley is small, therefore the design is based on the motor pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the velocity of the belt,

$$v = \frac{\pi\, d_1.N_1}{60} = \frac{\pi \times 0.4 \times 700}{60} = 14.7 \text{ m/s} \quad \text{... } (d_1 \text{ is taken in metres})$$

and the power transmitted (P),

$$22.5 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.7$$

$$\therefore \quad T_1 - T_2 = 22.5 \times 10^3 / 14.7 = 1530 \text{ N} \quad \text{...}(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.75}{2.3} = 0.3261 \quad \text{or} \quad \frac{T_1}{T_2} = 2.12 \quad \text{...}(ii)$$

... (Taking antilog of 0.3261)

From equations (*i*) and (*ii*), we find that

$$T_1 = 2896 \text{ N}; \quad \text{and} \quad T_2 = 1366 \text{ N}$$

Let b = Width of the belt in metres.

Since the velocity of the belt is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m³.

$\therefore$ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= b \times 0.005 \times 1 \times 1000 = 5\, b \text{ kg/m}$$

and centrifugal tension, $T_C = m.v^2 = 5\, b\, (14.7)^2 = 1080\, b$ N

We know that the maximum (or total) tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

or $2896 + 1080\, b = 2.3 \times 10^6\, b \times 0.005 = 11\,500\, b$

$\therefore$ $11\,500\, b - 1080\, b = 2896$ or $b = 0.278$ say 0.28 m or 280 mm **Ans.**

3. A belt is required to transmit 18.5 kW from a pulley of 1.2 m diameter running at 250rpm to another pulley which runs at 500 rpm. The distance between the centers of pulleys is 2.7 m. The following data refer to an open belt drive, = 0.25. Safe working stress for leather is 1.75 N/mm2. Thickness of belt = 10mm. Determine the width and length of belt taking centrifugal tension into account. Also find the initial tension in the belt and absolute power that can be transmitted by this belt and the speed at which this can be transmitted.

Data:

Open belt drive; $N = 18.5$ kW; $n_1 = 500$ rpm = Speed of smaller pulley;

$d_2 = 1.2$ m $= 1200$ mm $= D$ = Diameter of larger pulley; $n_2 = 250$ rpm = Speed of larger pulley;

$C = 2.7$ m $= 2700$ mm; $\mu = 0.25$; $\sigma_1 = 1.75$ N/mm²; $t = 10$ mm

(i) Diameter of smaller pulley

$$n_1 d_1 = n_2 d_2$$

$$500 \times d_1 = 250 \times 1200$$

$\therefore$ Diameter of smaller pulley $\mathbf{d_1} = \mathbf{600\ mm = d}$

(ii) Velocity

$$v = \frac{\pi(D+t)n_2}{60{,}000} = \frac{\pi(1200+10)250}{60{,}000} = 15.839 \text{ m/sec.}$$

(iii) Centrifugal stress

$$\sigma_c = \frac{wv^2}{g} \times 10^6$$

Assume specific weight of leather as 10×10^{-6} N/mm³

$$\therefore\ \sigma_c = \frac{10 \times 10^{-6}}{9810} \times 15.839^2 \times 10^6 = 0.25573 \text{ N/mm}^2$$

(iv) Capacity

Since coefficient of friction is same for both smaller and larger pulleys, capacity $= e^{\mu\theta_s}$

$$\text{i.e., } e^{\mu\theta} = e^{\mu\theta_s}$$

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

$$= \pi - \left\{2\sin^{-1}\left(\frac{1200-600}{2 \times 2700}\right)\right\}\frac{\pi}{180} = 2.92 \text{ radians}$$

$$\therefore\ e^{\mu\theta} = e^{0.25 \times 2.92} = 2.075$$

(v) Constant

$$k = \frac{e^{\mu\theta}-1}{e^{\mu\theta}} = \frac{2.075-1}{2.075} = 0.52$$

(vi) Width of belt

Power transmitted per mm² area $= \dfrac{(\sigma_1 - \sigma_c)kv}{1000}$

$$= \frac{(1.75-0.25573)0.52 \times 15.839}{1000} = 0.01231 \text{ kW}$$

(ix) Absolute power

For maximum power transmission

$$\sigma_c = \frac{\sigma_1}{3} = \frac{1.75}{3} = 0.5833 \text{ N/mm}^2$$

$$\text{Also } \sigma_c = \frac{w}{g} v^2 \times 10^6$$

$$\therefore 0.5833 = \frac{10 \times 10^{-6}}{9810} \times v^2 \times 10^6$$

$$\therefore \mathbf{v = 23.92 \text{ m/sec}}$$

$$\therefore \text{Power transmitted \textbackslash mm}^2 = \frac{(\sigma_1 - \sigma_C)kv}{1000}$$

$$= \frac{(1.75 - 0.5833)0.52 \times 23.92}{1000}$$

$$= 0.0145 \text{ kW}$$

$$\therefore \text{Total absolute power} = \text{Area of c/s of belt} \times \text{power per mm}^2$$

$$= 1503.18 \times 0.0145 = 21.7961 \text{ kW}$$

$$\therefore \text{Absolute power} = 21.8 \text{ kW.}$$

4. Select a V-belt drive to transmit 10 kW of power from a pulley of 200 mm diameter mounted on an electric motor running at 720 rpm to another pulley mounted on compressor running at 200 rpm. The service is heavy duty varying from 10 hours to 14 hours per day and centre distance between centres of pulleys is 600 mm.

Data:

$$\mathbf{N = 10\,kW;\; d_1 = 200\text{ mm} = d;\; n_1 = 720\text{ rpm};\; n_2 = 200\text{ rpm};\; C = 600\text{ mm}}$$

Heavy duty 10 hours to 14 hours per day.

Solution :

i. Diameter of larger pulley

$$\mathbf{n_1 d_1 = n_2 d_2}$$

$$720 \times 200 = 200 \times d_2$$

$$\therefore \mathbf{d_2 = 720\text{ mm} = D} = \text{diameter of larger pulley}$$

ii. Select the cross-section of belt

Equivalent Pitch diameter of smaller pulley $d_e = d_p F_b$ where $d_p = d_1 = 200$ mm

$$\frac{n_1}{n_2} = \frac{720}{200} = 3.6$$

From Table when $\frac{n_1}{n_2} = 3.6$

Smaller diameter factor $F_b = 1.14$

$$\therefore d_e = 200 \times 1.14 = 228 \text{ mm.}$$

iii. Velocity

$$v = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 200 \times 720}{60000} = 7.54 \text{ m/sec}$$

iv. Power capacity

For 'C' cross-section belt

$$N^* = v\left[\frac{1.47}{V^{0.09}} - \frac{143.27}{d_e} - \frac{2.34v^2}{10^4}\right]$$

$$= 7.54\left[\frac{1.47}{7.54^{0.09}} - \frac{143.27}{228} - \frac{2.34 \times 7.54^2}{10^4}\right]$$

$$N^* = 4.4 \text{ kW}$$

Number of bolts:

$$i = \frac{NF_a}{N^* F_c . F_d}$$

for heavy duty 10 – 14 hours/day correction factor for service $F_a = 1.3$

$$L = 2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C}$$

$$= 2 \times 600 + \frac{\pi}{2}(720 + 200) + \frac{(720 - 200)^2}{4 \times 600} = 2757.8 \text{ mm}$$

The nearest standard value of nominal pitch length for the selected C- cross section belt L = 2723 mm ,Nominal inside length = 2667 mm, For nominal inside length = 2667 mm, and C-cross section belt, correction factor for length Fe = 0.94

$$\text{Angle of contact } \theta = 2\cos^{-1}\left(\frac{D - d}{2C}\right)$$

$$= 2\cos^{-1}\left(\frac{720 - 200}{2 \times 600}\right) = 128.64°$$

From Table when $\theta = 128.64°$

Correction factor for angle of contact $F_d = 0.86$ (Assume V-V belt)

$$\therefore i = \frac{10 \times 1.3}{4.4 \times 0.94 \times 0.86} = 3.655$$

$\therefore$ Number of V belts **i = 4**

Types of Pulleys for Flat Belts:

Following are the various types of pulleys for flat belts:

1. Cast iron pulleys, 2. Steel pulleys, 3. Wooden pulleys, 4. Paper pulleys and 5. Fast and loose pulleys.

Design of Cast Iron Pulleys

1. Dimensions of pulley

(i) The diameter of the pulley (D) may be obtained either from velocity ratio consideration or centrifugal stress consideration. We know that the centrifugal stress induced in the rim of the pulley,

$$\sigma_t = \rho . v^2$$

where

ρ = Density of the rim material

= 7200 kg/m^3 for cast iron

v = Velocity of the rim = πDN / 60, D being the diameter of pulley and

N is speed of the pulley.

The following are the diameter of pulleys in mm for flat and V-belts.

20, 22, 25, 28, 32, 36, 40, 45, 50, 56, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224,250, 280, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120, 1250, 1400, 1600, 1800,2000, 2240, 2500, 2800, 3150, 3550, 4000, 5000, 5400.

The first six sizes (20 to 36 mm) are used for V-belts only.

The first six sizes (20 to 36 mm) are used for V-belts only.

B = 1.25 b ; where b = Width of belt.

(iii) The thickness of the pulley rim (t) varies from

$\frac{D}{300} + 2$ mm to $\frac{D}{300} + 3$ for single belt

$\frac{D}{300} + 6$ mm for double belt.

The diameter of the pulley (D) is in mm.

2. Dimensions of arms

(i) The number of arms may be taken as 4 for pulley diameter from 200 mm to 600 mm and 6 for diameter from 600 mm to 1500 mm.

(ii) The cross-section of the arms is usually elliptical with major axis (a1) equal to twice the minor axis (b1). The cross-section of the arm is obtained by considering the arm as cantilever i.e. fixed at the hub end and carrying a concentrated load at the rim end. The length of the cantilever is taken equal to the radius of the pulley. It is further assumed that at any given time, the power is transmitted from the hub to the rim or vice versa, through only half the total number of arms.

T = Torque transmitted,

R = Radius of pulley, and

n = Number of arms,

n = Number of arms,

$\therefore$ Tangential load per arm,

$$W_T = \frac{T}{R \times n / 2} = \frac{2\,T}{R \cdot n}$$

Maximum bending moment on the arm at the hub end,

$$M = \frac{2\,T}{R \times n} \times R = \frac{2\,T}{n}$$

and section modulus,

$$Z = \frac{\pi}{32} \times b_1\,(a_1)^2$$

Now using the relation,

σ_b or σ_t = M / Z, the cross-section of the arms is

(iii) The arms are tapered from hub to rim. The taper is usually

/48 to 1/32.

(iv) When the width of the pulley exceeds the diameter of the pulley, then two rows of arms are provided, as shown in Fig. 19.4. This is done to avoid heavy arms in one row.

3. Dimensions of hub

(i) The diameter of the hub (d1) in terms of shaft diameter (d) may be fixed by the following relation :

d1= 1.5 d + 25 mm

The diameter of the hub should not be greater than 2 d.

(ii) The length of the hub,

$$\mathbf{L} = \frac{\pi}{2} \times d$$

The minimum length of the hub is $\frac{2}{3}$ B but it should not be more than width of the pulley (B).

Advantages and Disadvantages of V-belt Drive over Flat Belt Drive

Advantages

1. The V-belt drive gives compactness due to the small distance between centres of pulleys.

2. The drive is positive, because the slip between the belt and the pulley groove is negligible.

3. Since the V-belts are made endless and there is no joint trouble, therefore the drive is smooth.

4. It provides longer life, 3 to 5 years.

5. It can be easily installed and removed.

6. The operation of the belt and pulley is quiet.

7. The belts have the ability to cushion the shock when machines are started.

8. The high velocity ratio (maximum 10) may be obtained.

9. The wedging action of the belt in the groove gives high value of limiting *ratio of tensions. Therefore the power transmitted by V-belts is more than flat belts for the same coefficient of friction, arc of contact and allowable tension in the belts.

10. The V-belt may be operated in either direction, with tight side of the belt at the top or bottom. The centre line may be horizontal, vertical or inclined.

Disadvantages

1. The V-belt drive cannot be used with large centre distances, because of larger weight per unit length.

2. The V-belts are not so durable as flat belts.

3. The construction of pulleys for V-belts is more complicated than pulleys of flat belts.

4. Since the V-belts are subjected to certain amount of creep, therefore these are not suitable for constant speed applications such as synchronous machines and timing devices.

5. The belt life is greatly influenced with temperature changes, improper belt tension and mismatching of belt lengths.

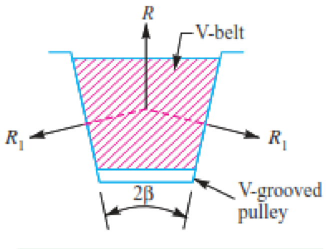
6. The centrifugal tension prevents the use of V-belts at speeds below 5 m / s and above 50 m / s.

Ratio of Driving Tensions for V-belt

R1= Normal reactions between belts and sides of the groove.

R = Total reaction in the plane of the groove.

μ = Coefficient of friction between the belt and sides of the groove.



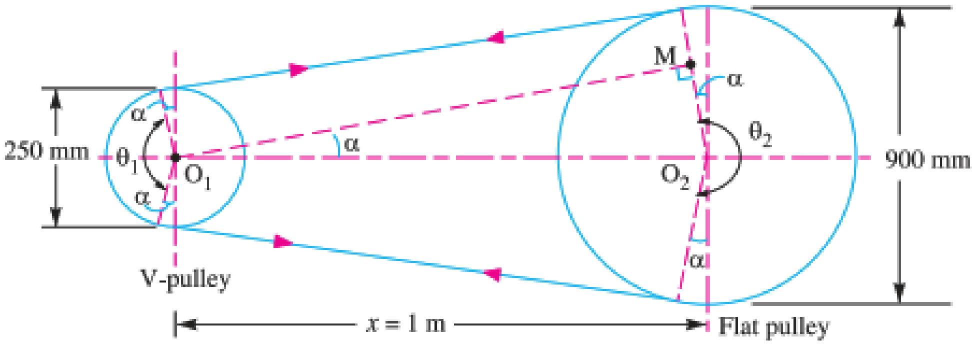
$$2.3 \log (T_1 / T_2) = \mu.\theta \operatorname{cosec} \beta$$

5. A V-belt is driven on a flat pulley and a V-pulley. The drive transmits 20 kW from a 250 mm diameter V-pulley operating at 1800 r.p.m. to a 900 mm diameter flat pulley. The centre distance is 1 m, the angle of groove 40° and μ = 0.2. If density of belting is 1110 kg / m and allowable stress is 2.1 MPa for belt material, what will be the number of belts required if C-size V-belts having 230 mm^3 cross-sectional areas are used.

Solution. Given : $P = 20$ kW ; $d_1 = 250$ mm $= 0.25$ m ; $N_1 = 1800$ r.p.m. ; $d_2 = 900$ mm $= 0.9$ m ; $x = 1$ m $= 1000$ mm ; $2\beta = 40°$ or $\beta = 20°$; $\mu = 0.2$; $\rho = 1110$ kg/m^3 ; $\sigma = 2.1$ MPa $= 2.1$ N/mm^2 ; $a = 230$ mm^2 $= 230 \times 10^{-6}$ m^2

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{900 - 250}{2 \times 1000} = 0.325$$

$$\alpha = 18.96°$$



We know that angle of contact on the smaller or V-pulley,

$$\theta_1 = 180° - 2\alpha = 180° - 2 \times 18.96 = 142.08°$$
$$= 142.08 \times \pi/180 = 2.48 \text{ rad}$$

and angle of contact on the larger or flat pulley,

$$\theta_2 = 180° + 2\alpha = 180° + 2 \times 18.96 = 217.92°$$
$$= 217.92 \times \pi / 180 = 3.8 \text{ rad}$$

We have already discussed that when the pulleys have different angle of contact (θ), then the design will refer to a pulley for which $\mu.\theta$ is small.

We know that for a smaller or V-pulley,

$$\mu.\theta = \mu.\theta_1 \operatorname{cosec}\beta = 0.2 \times 2.48 \times \operatorname{cosec} 20° = 1.45$$

and for larger or flat pulley,

$$\mu.\theta = \mu.\theta_2 = 0.2 \times 3.8 = 0.76$$

Since ($\mu.\theta$) for the larger or flat pulley is small, therefore the design is based on the larger or flat pulley.

We know that peripheral velocity of the belt,

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.25 \times 1800}{60} = 23.56 \text{ m/s}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = a \times l \times \rho$$
$$= 230 \times 10^{-6} \times 1 \times 1100 = 0.253 \text{ kg / m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 0.253\ (23.56)^2 = 140.4 \text{ N}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that maximum tension in the belt,

$$T = \text{Stress} \times \text{area} = \sigma \times a = 2.1 \times 230 = 483 \text{ N}$$

We also know that maximum or total tension in the belt,

$$T = T_1 + T_C$$

$$\therefore \quad T_1 = T - T_C = 483 - 140.4 = 342.6 \text{ N}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta_2 = 0.2 \times 3.8 = 0.76$$

$$\log\left(\frac{T_1}{T_2}\right) = 0.76 / 2.3 = 0.3304 \quad \text{or} \quad \frac{T_1}{T_2} = 2.14 \qquad \text{...(Taking antilog of 0.3304)}$$

and

$$T_2 = T_1 / 2.14 = 342.6 / 2.14 = 160 \text{ N}$$

∴ Power transmitted per belt

$$= (T_1 - T_2)\, v = (342.6 - 160)\ 23.56 = 4302 \text{ W} = 4.302 \text{ kW}$$

We know that number of belts required

$$= \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} = \frac{20}{4.302} = 4.65 \text{ say } 5 \textbf{ Ans.}$$

Rope Drives:

The ropes drives use the following two types of ropes :

1. Fibre ropes, and 2. *Wire ropes.

The fibre ropes operate successfully when the pulleys are about 60 metres apart, while the wire ropes are used when the pulleys are upto 150 metres apart.

Advantages of Fibre Rope Drives

The fibre rope drives have the following advantages :

1. They give smooth, steady and quiet service.

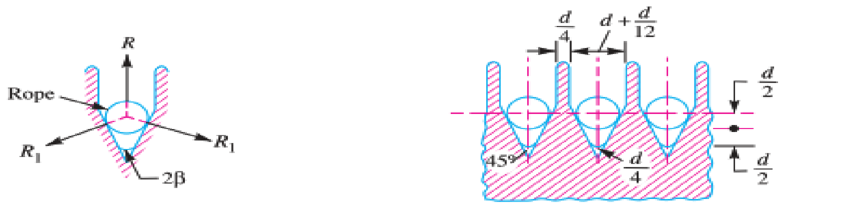
2. They are little affected by out door conditions.

3. The shafts may be out of strict alignment.

4. The power may be taken off in any direction and in fractional parts of the whole amount.

5. They give high mechanical efficiency.

Sheave for Fibre Ropes



(*a*) Cross-section of a rope. (*b*) Sheave (grooved pulley) for ropes.

Ratio of Driving Tensions for Fibre Rope

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu . \theta \operatorname{cosec} \beta$$

where μ, θ and β have usual meanings..

6. A pulley used to transmit power by means of ropes has a diameter of 3.6 metres and has 15 grooves of 45° angle. The angle of contact is 170° and the coefficient of friction between the ropes and the groove sides is 0.28. The maximum possible tension in the ropes is 960 N and the mass of the rope is 1.5 kg per metre length. Determine the speed of the pulley in r.p.m. and the power transmitted if the condition of maximum power prevail.

Solution. Given : d = 3.6 m ; n = 15 ; 2 β = 45° or β = 22.5° ; θ = 170° = 170 × π / 180 = 2.967 rad ; μ = 0.28 ; T = 960 N ; m = 1.5 kg / m

Solution. Given : d = 3.6 m ; n = 15 ; 2 β = 45° or β = 22.5° ; θ = 170° = 170 × π / 180 = 2.967 rad ; μ = 0.28 ; T = 960 N ; m = 1.5 kg / m

Speed of the pulley

Let N = Speed of the pulley in r.p.m.

We know that for maximum power, speed of the pulley,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{960}{3 \times 1.5}} = 14.6 \text{ m/s}$$

We also know that speed of the pulley (v),

$$14.6 = \frac{\pi d . N}{60} = \frac{\pi \times 3.6 \times N}{60} = 0.19 \, N$$

$\therefore$ N = 14.6 / 0.19 = 76.8 r.p.m. **Ans.**

Power transmitted

We know that for maximum power, centrifugal tension,

$$T_C = T / 3 = 960 / 3 = 320 \text{ N}$$

∴ Tension in the tight side of the rope,

$$T_1 = T - T_C = 960 - 320 = 640 \text{ N}$$

Let T_2 = Tension in the slack side of the rope.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta = 0.28 \times 2.967 \times \operatorname{cosec} 22.5° = 2.17$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{2.17}{2.3} = 0.9435 \quad \text{or} \quad \frac{T_1}{T_2} = 8.78 \quad \text{...(Taking antilog of 0.9435)}$$

and

$$T_2 = T_1 / 8.78 = 640 / 8.78 = 73 \text{ N}$$

∴ Power transmitted,

$$P = (T_1 - T_2)\, v \times n = (640 - 73)\, 14.6 \times 15 = 124\ 173 \text{ W}$$

$$= 124.173 \text{ kW}$$ **Ans.**

Wire Ropes:

When a large amount of power is to be transmitted over long distances from one pulley to another (i.e. when the pulleys are upto 150 metres apart), then wire ropes are used. The wire ropes are widely used in elevators, mine hoists, cranes, conveyors, hauling devices and suspension bridges. The wire ropes run on grooved pulleys but they rest on the bottom of the *grooves and are not wedged between the sides of the grooves

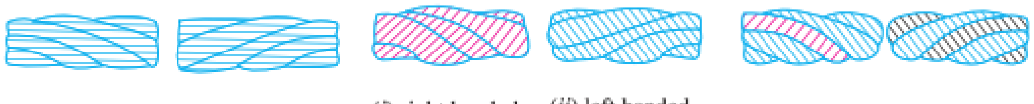
Advantages of Wire Ropes.

1. These are lighter in weight,
2. These offer silent operation,
3. These can withstand shock loads,
4. These are more reliable,
5. These are more durable,
6. They do not fail suddenly
7. The efficiency is high, and
8. The cost is low.

Classification of Wire Ropes:

1. Cross or regular lay ropes. In these types of ropes, the direction of twist of wires in the strands is opposite to the direction of twist of the stands, as shown in Fig. (a). Such type of ropes are most popular.

2. Parallel or lang lay ropes. In these type of ropes, the direction of twist of the wires in the strands is same as that of strands in the rope, as shown in Fig. (b). These ropes have better bearing surface but is harder to splice and twists more easily when loaded. These ropes are more flexible and resists wear more effectively. Since such ropes have the tendency to spin, therefore these are used in lifts and hoists with guide ways and also as haulage ropes.



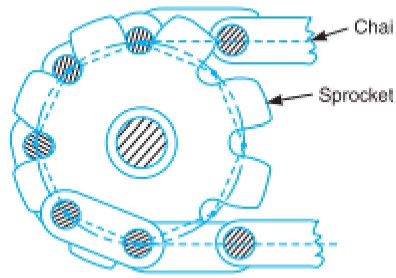
(a) Cross or regular lay ropes.

(b) Parallel or lang lay ropes.

(c) Composite or reverse laid ropes.

3. Composite or reverse laid ropes. In these types of ropes, the wires in the two adjacent strands are twisted in the opposite direction, as shown in Fig.

Chains: belt and rope drives that slipping may occur. In order to avoid slipping, steel chains are used. The chains are made up of rigid links which are hinged together in order to provide the necessary flexibility for warping around the driving and driven wheels. The wheels have projecting teeth and fit into the corresponding recesses, in the links of the chain as shown in Fig. 11.23. The wheels and the chain are thus constrained to move together without slipping and ensures perfect velocity ratio. The toothed wheels are known as sprocket wheels or simply sprockets. These wheels resemble to spur gears.



Advantages and Disadvantages of Chain Drive Over Belt or Rope Drive :

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.

2. Since the chains are made of metal, therefore they occupy less space in width than a belt or rope drive. 3. The chain drives may be used when the distance between the shafts is less.

4. The chain drive gives a high transmission efficiency (upto 98 per cent).

5. The chain drive gives less load on the shafts.

6. The chain drive has the ability of transmitting motion to several shafts by one chain only.

Disadvantages

1. The production cost of chains is relatively high.

2. The chain drive needs accurate mounting and careful maintenance.

3. The chain drive has velocity fluctuations especially when unduly stretched.

Terms Used in Chain Drive :

1. Pitch: It is the distance between the hinge centre of a link and the corresponding hinge centre of the adjacent link as shown in Fig.

2. Pitch circle diameter of the chain sprocket. It is the diameter of the circle on which the hinge centres of the chain lie.

Relation Between Pitch and Pitch Circle Diameter

Let d = Diameter of the pitch circle, and

T = Number of teeth on the sprocket.

From Fig. 11.25, we find that pitch of the chain,

$$p = AB = 2\,AO\sin\left(\frac{\theta}{2}\right) = 2 \times \frac{d}{2}\sin\left(\frac{\theta}{2}\right) = d\sin\left(\frac{\theta}{2}\right)$$

We know that $\theta = \frac{360^\circ}{T}$

$$\therefore \quad p = d\sin\left(\frac{360^\circ}{2T}\right) = d\sin\left(\frac{180^\circ}{T}\right)$$

$$d = p\,\text{cosec}\left(\frac{180^\circ}{T}\right)$$

Classification of Chains :

1. Hoisting and hauling (or crane) chains,

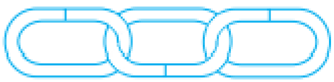
2. Conveyor (or tractive) chains, and

3. Power transmitting (or driving) chains.

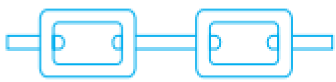
Hoisting and Hauling Chains:

These chains are used for hoisting and hauling purposes. The hoisting and hauling chains are of the following two types:

1. **Chain with oval links.** : The links of this type of chain are of oval shape,
2. **Chain with square links**. The links of this type of chain are of square shape, as shown in Fig.



(*a*) Chain with oval links.

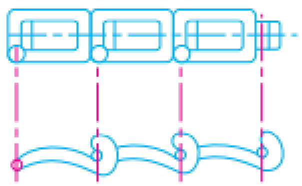


(*b*) Chain with square links.

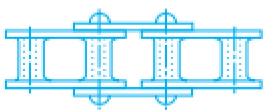
Conveyor Chains :

1. Detachable or hook joint type chain, as shown in Fig. and

2. Closed joint type chain, as shown in Fig.



(*a*) Detachable or hook joint type chain.



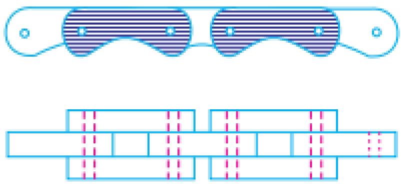
(*b*) Closed joint type chain.

Power Transmitting Chains

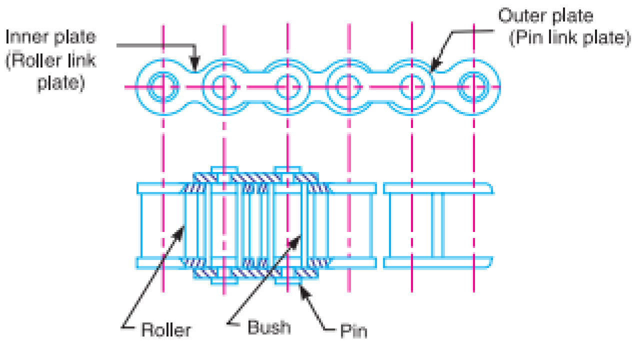
These chains are used for transmission of power, when the distance between the centres of shafts is short. These chains have provision for efficient lubrication.

The power transmitting chains are of the following three types.

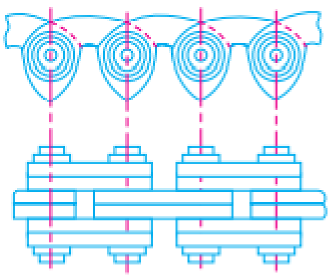
1. Block chain. A block chain, as shown in Fig. 11.31, is also known as bush chain. This type of chain was used in the early stages of development in the power transmission.



1. **Bush roller chain**. A bush roller chain, as shown in Fig. 11.32, consists of outer plates or pin link plates, inner plates or roller link plates, pins, bushes and rollers. A pin passes through the bush which is secured in the holes of the roller between the two sides of the chain. The rollers are free to rotate on the bush which protect the sprocket wheel teeth against wear.



2. **Inverted tooth or silent chain**. An inverted tooth or silent chain is shown in Fig. 11.33. It is designed to eliminate the evil effects caused by stretching and to produce noiseless running. When the chain stretches and the pitch of the chain increases, the links ride on the teeth of the sprocket wheel at a slightly increased radius. This automatically corrects the small change in the pitch. There is no relative sliding between the teeth of the inverted tooth chain and the sprocket wheel teeth. When properly lubricated, this chain gives durable service and runs very smoothly and quietly.



Length of Chain :

$$L = \frac{p}{2}(T_1 + T_2) + 2x + \frac{\left[\frac{p}{2}\operatorname{cosec}\left(\frac{180^\circ}{T_1}\right) - \frac{p}{2}\operatorname{cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{x}$$

If $x = m.p$, then

$$L = p\left[\frac{(T_1 + T_2)}{2} + 2m + \frac{\left[\operatorname{cosec}\left(\frac{180^\circ}{T_1}\right) - \operatorname{cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{4\,m}\right] = p.K$$

where K = Multiplying factor

$$= \frac{(T_1 + T_2)}{2} + 2m + \frac{\left[\operatorname{cosec}\left(\frac{180^\circ}{T_1}\right) - \operatorname{cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{4m}$$

PROBLEM: A chain drive is used for reduction of speed from 240 r.p.m. to 120 r.p.m. The number of teeth on the driving sprocket is 20. Find the number of teeth on the driven sprocket. If the pitch circle diameter of the driven sprocket is 600 mm and centre to centre distance between the two sprockets is 800 mm, determine the pitch and length of the chain.

Solution. Given : N_1 = 240 r.p.m ; N_2 = 120 r.p.m ; T_1 = 20 ; d_2 = 600 mm or r_2 = 300 mm = 0.3 m ; x = 800 mm = 0.8 m

Number of teeth on the driven sprocket

Let T_2 = Number of teeth on the driven sprocket.

We know that

$$N_1.T_1 = N_2.T_2 \quad \text{or} \quad T_2 = \frac{N_1.T_1}{N_2} = \frac{240 \times 20}{120} = 40 \quad \textbf{Ans.}$$

Pitch of the chain

Let p = Pitch of the chain.

We know that pitch circle radius of the driven sprocket (r_2),

$$0.3 = \frac{p}{2}\operatorname{cosec}\left(\frac{180^\circ}{T_2}\right) = \frac{p}{2}\operatorname{cosec}\left(\frac{180^\circ}{40}\right) = 6.37\,p$$

$\therefore$ p = 0.3 / 6.37 = 0.0471 m = 47.1 mm **Ans.**

Length of the chain

We know that pitch circle radius of the driving sprocket,

$$r_1 = \frac{p}{2}\operatorname{cosec}\left(\frac{180^\circ}{T_1}\right) = \frac{47.1}{2}\operatorname{cosec}\left(\frac{180^\circ}{20}\right) = 150.5\,\text{mm}$$

and $x = m.p$ or $m = x / p = 800 / 47.1 = 16.985$

We know that multiplying factor,

$$K = \frac{(T_1 + T_2)}{2} + 2m + \frac{\left[\operatorname{cosec}\left(\frac{180^\circ}{T_1}\right) - \operatorname{cosec}\left(\frac{180^\circ}{T_2}\right)\right]^2}{4m}$$

$$= \frac{(20 + 40)}{2} + 2 \times 16.985 + \frac{\left[\operatorname{cosec}\left(\frac{180^\circ}{20}\right) - \operatorname{cosec}\left(\frac{180^\circ}{40}\right)\right]^2}{4 \times 16.985}$$

$$= 30 + 33.97 + \frac{(6.392 - 12.745)^2}{67.94} = 64.56 \text{ say } 65$$

$\therefore$ Length of the chain,

$$L = p.K = 47.1 \times 65 = 3061.5 \text{ mm} = 3.0615 \text{ m} \quad \textbf{Ans.}$$

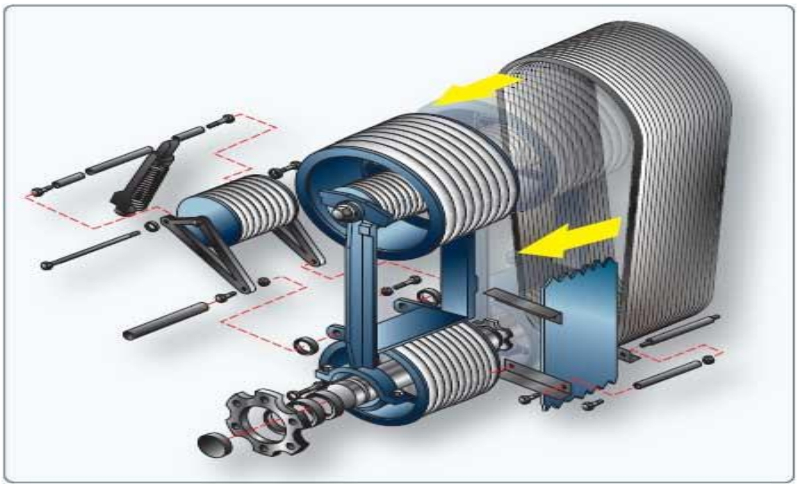
INDUSTRIAL APPLICATIONS

1. Belt and Rope Drives in Textile Industry

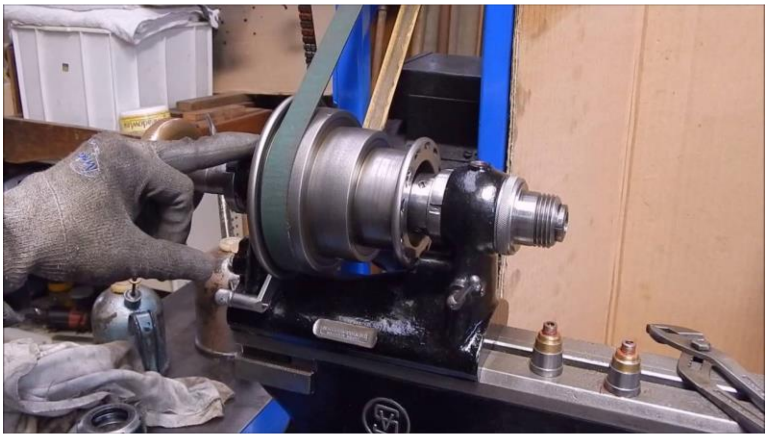
2. Agriculture

3. V belt drive are in automobiles to drive the accessories

4. Helicopter Transmission Systems – The Clutch



5. Lathe machine



TUTORIAL QUESTIONS:

1. Discuss about the various types of belt drives with neat sketches?
2. On what factors do the power transmitted by belts depends?
3. Name the type of stresses induced in the wire?
4. Under what circumstances a fibre rope and a wire rope is used? What are the advantages of a wire rope over fibre rope?
5. State the advantages and disadvantages of the chain drive over belt and rope drive.
6. Design a belt pulley for transmitting 10kW at 180 rpm. The velocity of the belt is not to exceed 10m/s and the maximum tension is not to exceed 15N/mm width. The tension on the slack side is one half that on the tight side. Determine all the principle dimensions of the pulley.
7. An overhung pulley transmits 35kW at 240rpm. The belt drive is vertical & the angle of wrap may be taken as 1800 . The distance of the pulley centre line from the nearest bearing is 350rpm. μ =0.25. The section of the arm may be taken as elliptical, the major axis being twice the minor axis. The following stress may be taken for design purpose: Shaft & Key: Tension & Compression-80MPa; Shear-50MPa Belt: Tension-2.5MPa Pulley rim: Tension-4.5MPa Pulley arms: Tension-15MPa Determine: a. Diameter of the pulley b. Diameter of the shaft.
8. A belt, 100 x 10mm is transmitting power at 15m/s. the angle of contact on the driver (smaller) pulley is 1650 , if the permissible stress for the belt material is 2N/mm2 ; determine the power that can be transmitted at this speed. Tale the density of leather as 1000kg/m3 and coefficient of friction as 0.3. Calculate the maximum power that can be transmitted.
9. The reduction of speed from 360 r.p.m. to 120 r.p.m. is desired by the use of chain drive. The driving sprocket has 10 teeth. Find the number of teeth on the driven sprocket. If the pitch radius of the driven sprocket is 250 mm and the centre to centre distance between the two sprocket is 400 mm, find the pitch and length of the chain

ASSIGNMENT QUESTIONS:

1. A belt, 102 x 11mm is transmitting power at 17m/s. the angle of contact on the driver (smaller) pulley is 1550, if the permissible stress for the belt material is 2N/mm^2 ; determine the power that can be transmitted at this speed. Tale the density of leather as 1000kg/m3 and coefficient of friction as 0.3. Calculate the maximum power that can be transmitted.
2. The layout of the leather belt drive transmitting 15 kW power is shown in Fig.1. The centre distance between the pulleys is twice the diameter of the big pulley. The belt should operate at a velocity of 20 m/s and the stresses in the belt should not exceed 2.25 MPa. The density of the leather belt is 0.95 g/cc and the coefficient of friction is 0.35. The thickness of the belt is 5 mm. Calculate: i) Diameter of the pulleys. ii) The length and width belts. iii) Belt tensions.
   Speeds are 1440 and 440.
3. Explian the classification of chains ?
4. A V-belt drive consists of three V-belts in parallel on grooved pulleys of the same size. The angle of groove is 30° and the coefficient of friction 0.12. The cross-sectional area of each belt is 800 mm2 and the permissible safe stress in the material is 3 MPa. Calculate the power that can be transmitted between two pulleys 400 mm in diameter rotating at 960 r.p.m
5. Explain what you understand by 'initial tension in a belt'.
6. The reduction of speed from 360 r.p.m. to 120 r.p.m. is desired by the use of chain drive. The driving sprocket has 10 teeth. Find the number of teeth on the driven sprocket. If the pitch radius of the driven sprocket is 250 mm and the centre to centre distance between the two sprocket is 400 mm, find the pitch and length of the chain

UNIT 5

GEARS AND GEAR TRAINS

Course Objectives:

To study the relative motion analysis and design of gears, gear trains.

Course Outcomes:

Evaluate gear tooth geometry and select appropriate gears for the required applications.

5

GEARS

Course Contents

Introduction
Advantages and Disadvantages of GearDrive
Classification ofGears
Terms Used inGears
Law of Gearing
Standard Tooth Profiles orSystems
Length of Path of Contact &Length of Arc ofContact
Interference in InvoluteGears
Minimum Number of Teeth on the Pinion in Order to Avoid Interference
Minimum Number of Teeth on the Wheel in Order to Avoid Interference
Minimum Number of Teeth on a Pinion for Involute Rack in Order to AvoidInterference
Comparison of Cycloidaland Involute toothforms
Helical and spiralgears
Examples

5.1 Introduction

- If power transmitted between two shafts is small, motion between them may be obtained by using two plain cylinders or discs 1 and 2 as shown infig.
- If there is no slip of one surface relative to the other, a definite motion of 1 can be transmitted to 2 and vice-versa. Such wheels are termed as "**friction wheels**". However, as the power transmitted increases, slip occurs between the discs and the motion no longer remainsdefinite.
- Assuming no slipping of the two surfaces, the following kinematic relationship exists for their linearvelocity:
- To transmit a definite motion of one disc to the other or to prevent slip between the surfaces, projection and recesses on the two discs can be made which can mesh with each other. This leads to formation of teeth on the discs and the motion between the surfaces changes from rolling to sliding. The discs with the teeth are known as **gears** or **gearwheels**.
- It is to be noted that if the disc I rotates in the clockwise direction, 2 rotates in the counter clockwise direction andvice-versa.

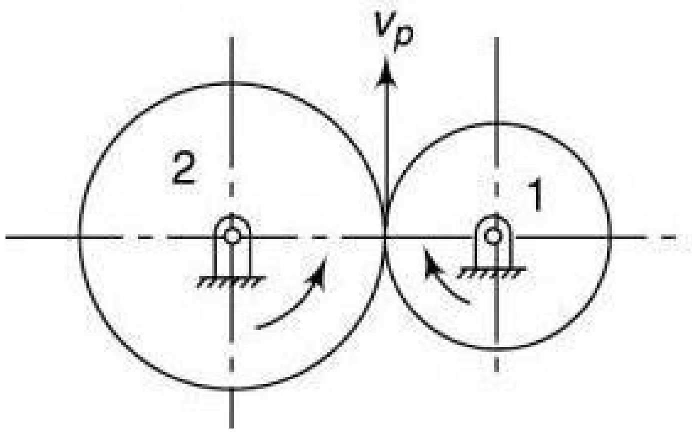


Fig. 5.1

5.2 Advantages and Disadvantages of GearDrive

Advantages

1. It transmits exact velocityratio.
2. It may be used to transmit largepower.
3. It has highefficiency.
4. It has reliableservice.
5. It has compactlayout.

Disadvantages

1. The manufacture of gears required special tools andequipment.
2. The error in cutting teeth may cause vibrations and noise duringoperation.
3. They arecostly.

5.3 Classification ofGears

5.3.1. According to the position of axes of theshafts

A. The axes of the two shafts between which the motion is to be transmitted, may be Parallelshaft,

B. Intersecting (Non parallel)shaft

C. Non-intersecting and non-parallelshaft.

A. Parallelshaft

- **Spurgear**

– The two parallel and co-planar shafts connected by the gears are called *spurgears.* These gears have teeth parallel to the axis of the wheel.

– They have straight teeth parallel to the axes and thus are not subjected to axial thrust due to toothload.

– At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axis of rotation. This results in sudden application of the load, high impact stresses and excessive noise at highspeeds.

– If the gears have external teeth on the outer surface of the cylinders, the shaft rotate in the oppositedirection.

– In an internal spur gear, teeth are formed on the inner surface of an annulus ring. An internal gear can mesh with an external pinion (smaller gear) only and the two shafts rotate in the samedirection.

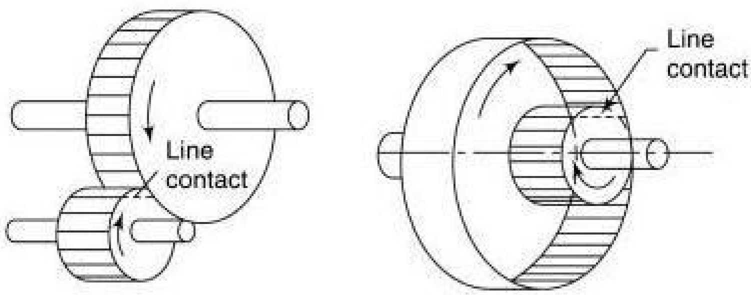


Fig.5.3 (a) Spur Gear

- *Spur rack andpinion*

– Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface isplane.

– The spur rack and pinion combination converts rotary motion intotranslator motion, orvice-versa.

– It is used in a lathe in which the rack transmits motion to thesaddle.

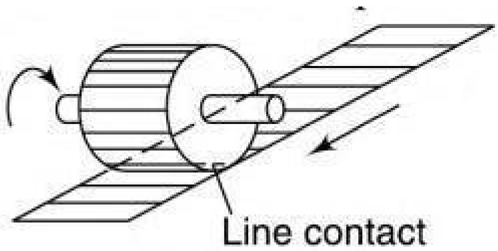


Fig. 5.3(b) Rack and pinion

- *Helical SpurGears*

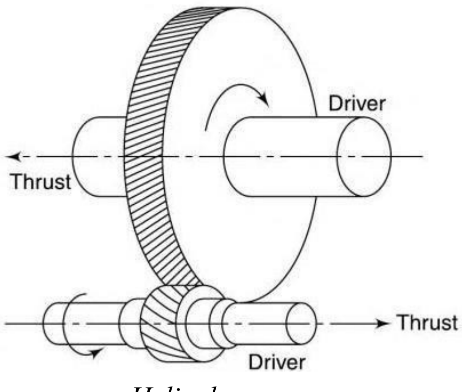
– In helical gears, the teeth are curved, each being helical in shape. Two mating gears have the same helix angle, but have teeth of oppositehands.

– At the beginning of engagement, contact occurs only at the point of leading edge of the curved teeth. As the gears rotate, the contact extends along a diagonal line across the teeth. Thus, the load application is gradual which results in low impact stresses and reduction in noise. Therefore, the helical gear can be used at higher velocities than the spur gears and have greater load-carryingcapacity.

– Helical gears have the **disadvantage** of having end thrust as there is a force component along the gear axis. The bearing and assemblies mounting the helical gears must be able to withstand thrustloads.

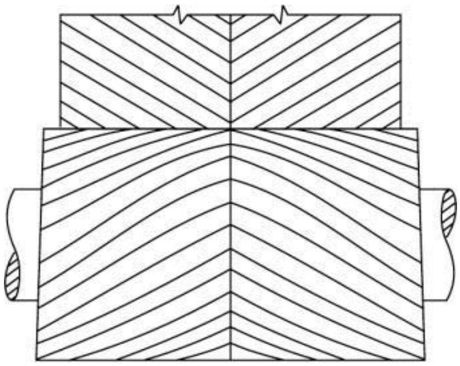
– **Double helical:** A double-helical gear is equivalent to a pair of helical gears secured together, one having a right hand helix and other left handhelix.
  - The teeth of two rows are separated by groove used for tool runout.
  - Axial thrust which occurs in case of single-helical gears is eliminated in double-helicalgears.
  - This is because the axial thrusts of the two rows of teeth cancel each other out. These can be run at high speeds with less noise andvibrations.

– **Herringbone gear**: If the left and the right inclinations of a double-helical gear meet at a common apex and there is no groove in between, the gear is known as Herringbone gear.



Helicalgear

(c)



Herringbone gear

(d)

Fig. 5.3

B. *IntersectingShafts*

– The two non-parallel or intersecting, but coplanar shafts connected by gearsare called **bevel gears**
– When teeth formed on the cones are straight, the gears are known as bevelgears when inclined, they are known as **spiral** or **helicalbevel**.

- **Straight Bevel Gears (http://www.bevelgear.co.za)**

– The teeth are straight, radial to the point of intersection of the shaft axes and vary in cross section throughout theirlength.
– Usually, they are used to connect shafts at right angles which run at lowspeeds
– Gears of the same size and connecting two shafts at right angles to each otherare known as "Mitre"gears.

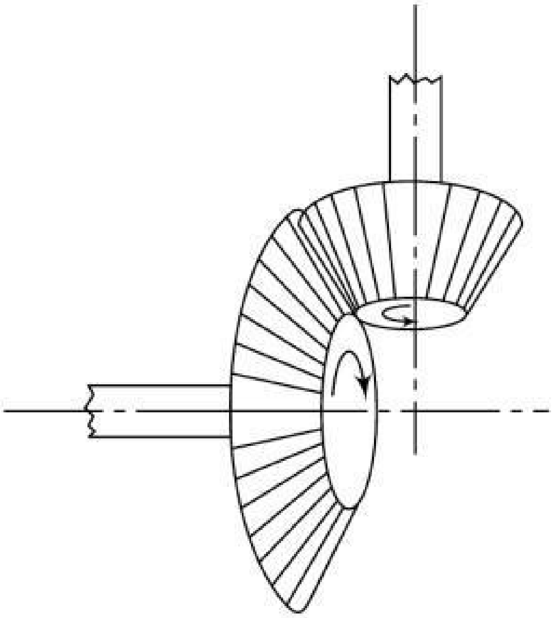


Fig. 5.3(e) Straight Bevel Gears

- *Spiral BevelGears*

– When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as spiral bevels or helicalbevels.
– They are smoother in action and quieter than straight tooth bevels as there is gradual load application and low impact stresses. Of course, there exists an axial thrust calling for stronger bearings and supportingassemblies.
– These are used for the drive to the differential of anautomobile.

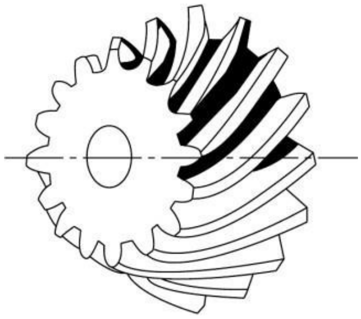


Fig. 5.3(f) Spiral Bevel Gear

- *Zero BevelGears*
- – Spiral bevel gears with curved teeth but with a zero degree spiral angle areknown as zero bevel gears.
- – Their tooth action and the end thrust are the same as that of straight bevel gears and, therefore, can be used in the samemountings.
- – However, they are quieter in action than the straight bevel type as the teeth are curved.

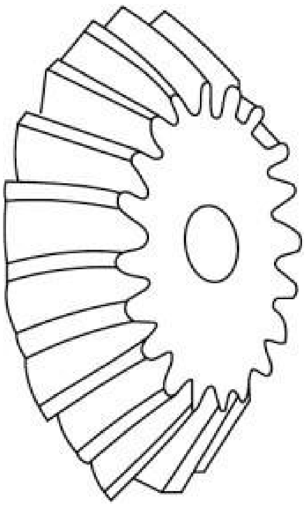


Fig. 5.3(g) Zero Bevel Gears

C. *Non-intersecting and non-parallel shaft(Skewshaft)*

- – The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiralgearing.
- – In these gears teeth have a pointcontact.
- – These gears are suitable for transmitting smallpower.
- – **Worm gear** is as special case of a spiral gear in which the larger wheel, usually, has a hollow shape such that a portion of the pitch diameter of the other gear is enveloped onit.

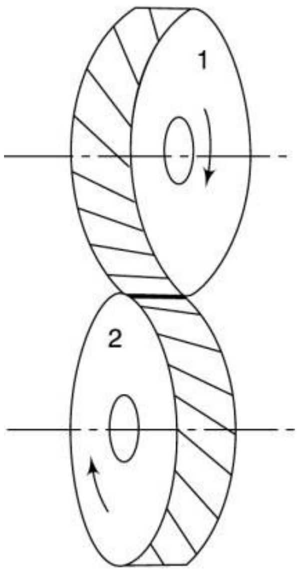


Fig.5.3 (h)Non-intersecting and non-parallel shaft

5.3.2. According to the peripheral velocity of thegears

(a) Lowvelocity V < 3m/sec

(b) Mediumvelocity 3 < V < 15m/sec

(c) Highvelocity V > 15m/sec

5.3.3. According to position of teeth on the gearsurface

(a) Straight,

(b) Inclined,and

(c) Curved.

5.4 Terms Used inGears

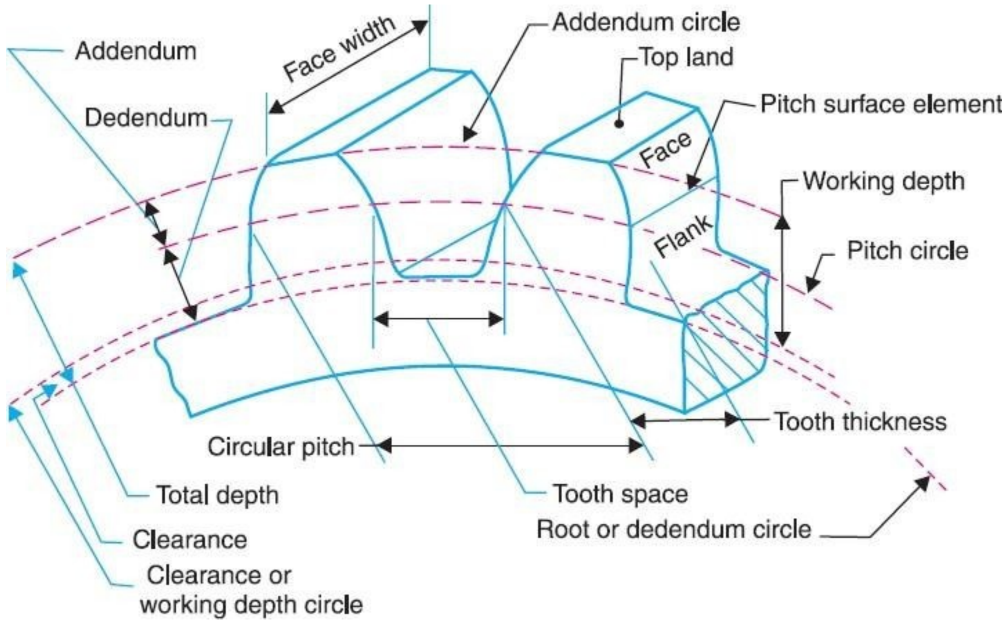


Fig.5.4 Terms used in gears.

1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actualgear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gearis usually specified by the pitch circle diameter. It is also known as **pitchdiameter**.

3. Pitch point. It is a common point of contact between two pitchcircles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitchcircle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitchpoint.

- For more power transmission lesser pressure on the bearing and pressureangle must be keptsmall.
- It is usually denoted byø.
- The standard pressure angles are 20° and 25°.Gears with pressure anglehasbecome obsolete.

6. Addendum. It is the radial distance ofa tooth from the pitch circle to the top of the tooth.

- Standard value = 1module

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

- Standard value = 1.157module

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitchcircle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called rootcircle.

10. Clearance. It is the radial difference between the addendum and the Dedendum of a tooth.

Addendum circle diameter =d+ 2m

Dedendum circle diameter = d−2×1.157m

Clearance = 1.157m−m

=0.157m

11. Full depth of Teeth It is the total radial depth of the toothspace.

Full depth= Addendum +Dedendum

12. Working Depth of Teeth The maximum depth to which a tooth penetrates intothe tooth space of the mating gear is the working depth ofteeth.

- Working depth = Sum of addendums of the twogears.

15. Working depth. It is the radial distance from the addendum circle to the clearancecircle. It is equal to the sum of the addendum of the two meshinggears.

16. Tooth thickness. It is the width of the tooth measured along the pitchcircle.

17. Tooth space.Itisthewidthofspacebetweenthetwoadjacentteethmeasured along the pitchcircle.

18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermalexpansion.

19. Face of tooth. It is the surface of the gear tooth above the pitchsurface.
20. Flank of tooth. It is the surface of the gear tooth below the pitchsurface.
21. Top land. It is the surface of the top of thetooth.
22. Face width. It is the width of the gear tooth measured parallel to itsaxis.

20. Fillet It is the curved portion of the tooth flank at the rootcircle.

21. ***Circular pitch***. It is the distance measured on the circumference of the pitch circle from point of one tooth to the corresponding point on the nexttooth.

- It is usually denoted by p_c.
  Mathematically,

$$\text{Circular pitch, } p_c = \frac{\pi d}{T}$$

Where *d* = Diameter of the pitch circle, and
T = Number of teeth on the wheel.

- The angle subtended by the circular pitch at the center of the pitch circle is knownas the **pitchangle**.

22. Module (m). It is the ratio of the pitch diameter in mm to the number of teeth.

$$m = \frac{d}{T}$$

$$\text{Also } p_c = \frac{\pi d}{T} = \pi m$$

- Pitch of two mating gear must besame.

23. Diametral Pitch (P) It is the number of teeth per unit length of the pitch circle diameter ininch.

OR

It is the ratio of no. of teeth to pitch circle diameter in inch.

$$P_d = \frac{T}{d}$$

- The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of secondchoice.

24. Gear Ratio (G). It is the ratio of the number of teeth on the gear to that on thepinion.

$$G = \frac{T}{t}$$ Where T= No of teeth on gear

t = No. of teeth on pinion

25. Velocity Ratio (VR) The velocity ratio is defined as the ratio of the angular velocityof the follower to the angular velocity of the drivinggear.

$$VR = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

26. Length of the path of contact. It is the length of the common normal cut-off by the Addendum circles of the wheel andpinion.

OR

The locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement is known as the contact.

a. **Path of Approach** Portion of the path of contact from the beginning ofthe engagement to the pitchpoint.
b. **Path ofRecess**Portionofthe pathofcontactfromthepitchpointtotheend ofengagement.

27. Arc of Contact The locus of a point on the pitch circle from the beginning to theend of engagement of two mating gears is known as the arc ofcontact.

a. **Arc of Approach** It is the portion of the arc of contact from the beginning of engagement to the pitchpoint.
b. **Arc of Recess** The portion of the arc of contact from the pitch point to the end of engagements the arc ofrecess.

28. Angle of Action ( ) It is the angle turned by a gear from the beginning of engagement to the end of engagement of a pair of teeth, i.e., the angle turned by arcs of contact ofrespective gearwheels.

$\delta=\alpha+\beta$ Where α=Angle of approach

β =Angle of recess

29. Contact ratio .It is the angle of action divided by the pitchangle

$$\text{Contact ratio} = \frac{\delta}{\gamma} = \frac{\alpha+\beta}{\gamma}$$

OR

$$\text{Contact ratio} = \frac{\text{Arcofcontact}}{\text{Circularpitch}}$$

5.5 Condition for Constant Velocity Ratio of Toothed Wheels –Law ofGearing

- To understand the theory consider the portions of two gear teeth gear 1 and gear 2as shown in figure1.5.
- The two teeth come in contact at point C and the direction of rotation of gear 1 is anticlockwise & gear 2 isclockwise.
- Let TT be the common tangent & NN be the common normal to the curve at the point of contact C. From points O, &O2, draw O1 A& O2 B perpendicular to common normalNN.
- When the point D is consider on gear 1, the point C moves in the direction of "CD" & when it is consider on gear 2. The point C moves in direction of"CE".
- The relative motion between tooth surfaces along the common normal NN must be equal to zero in order to avoidseparation.
- So, relativevelocity

$$V_1 \cos\alpha = V_2 \cos\theta$$

$$(\omega_1 \times O_1C)\cos\alpha = (\omega_2 \times O_2C)\cos\alpha \qquad (\because v = r\omega) \square (1)$$

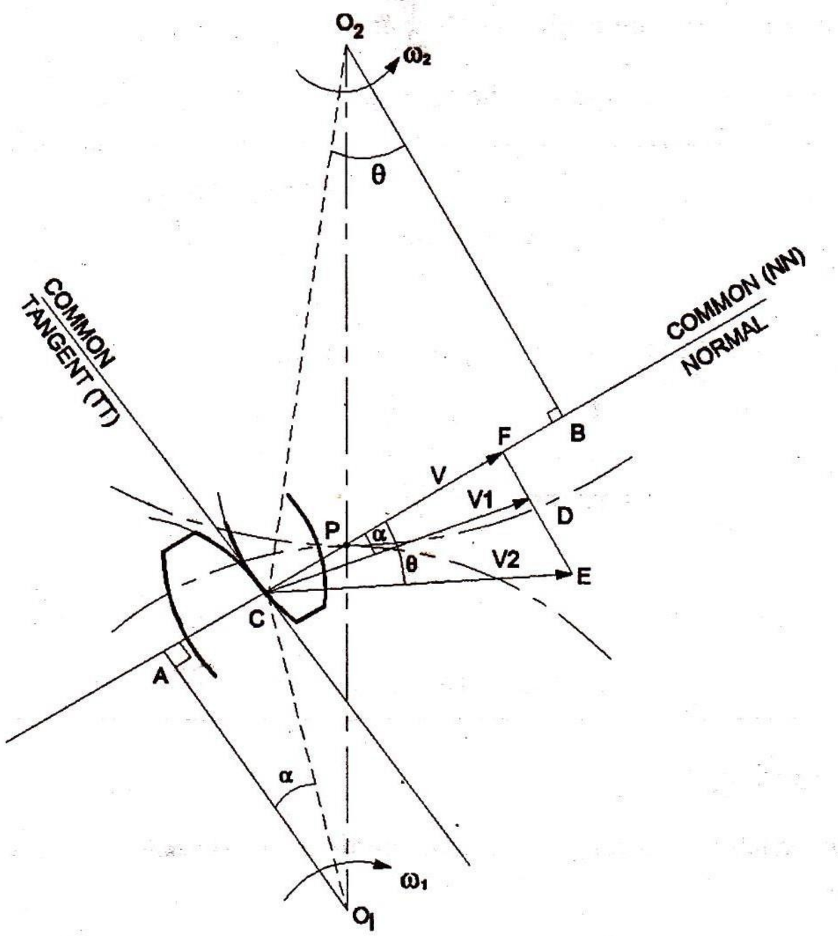


Fig.5.5 Law of gearing

- But from ΔO_1AC, $\cos\alpha = \frac{O_1A}{O_1C}$

  and from ΔO_2BC, $\cos\theta = \frac{O_2B}{O_2C}$

- Putting above value in equation (1) itbecome

$$(\omega_1 \times O_1C)\frac{O_1A}{O_1C} = (\omega_2 \times O_2C)\frac{O_2B}{O_2C}$$

$$\omega_1 \times O_1A = \omega_2 O_2B$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2B}{O_1A} \quad \text{...} \quad (2)$$

- From the similar triangle ΔO_1AP & ΔO_2BP

$$\frac{O_2B}{O_1A} = \frac{O_2P}{O_1P} \quad \text{...} \quad (3)$$

- Now equating equation (2) &(3)

$$\frac{\omega_1}{\omega_2} = \frac{O_2B}{O_1A} = \frac{O_2P}{O_1P} = \frac{PB}{AP}$$

- From the above we can conclude that the angular velocity ratio is inversely proportional tothe ratio of the distances of the point P from the central O1&O2.
- If it is desired that the angular velocities of two gear remain constant, the common normal at the point of contact of two teeth always pass through a fixed point P. This fundamental condition is called as law of gearing. Which must be satisfied while designing the profiles of teeth forgears.

5.6 Standard Tooth Profiles orSystems

Following four types of tooth profiles or systems are commonly used in practice for interchangeability:

a) $14\frac{1}{2}^\circ$ compositesystem.

b) $14\frac{1}{2}^\circ$ full depth involute system.

c) 20° full depth involutesystem.

d) 20° stub involutesystem.

a) $14\frac{1}{2}^{\circ}$ *composite system:*

- Thistypeofprofileismadewithcircular arcsattopandbottomportionandmiddle portion is a straight line as shown in Fig.1.6(a).
- The straight portion corresponds to the involute profile and the circular arcportion corresponds to the cycloidalprofile.
- Such profiles are used for general purpose gears.

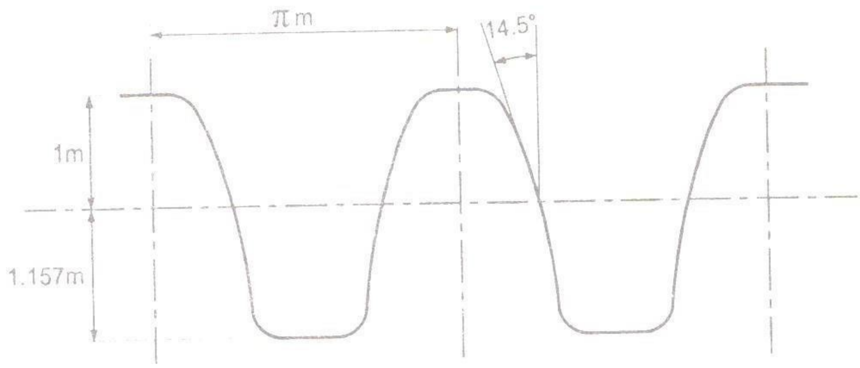


Fig.5.6(a) $14\frac{1}{2}^{\circ}$ *composite system*

b) $14\frac{1}{2}^{\circ}$ *full depth involute system:*

- This type of profile is made straight line except for the filletarcs.
- The whole profile corresponds to the involute profile. Therefore manufacturing ofsuch profile is easy but they have interfaceproblem.

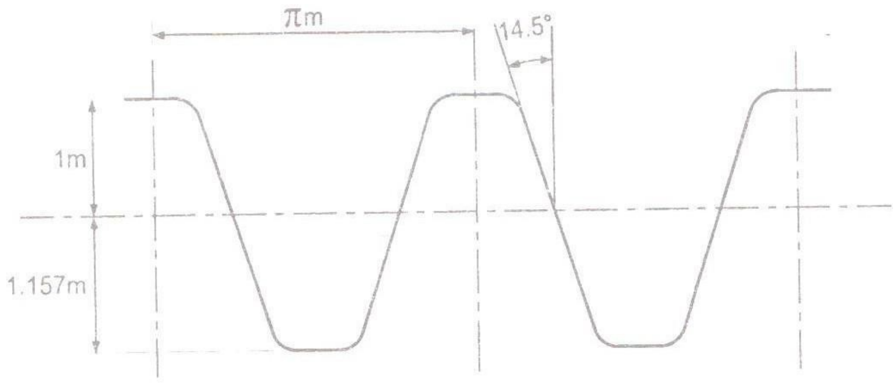


Fig.5.6(b) $14\frac{1}{2}^{\circ}$ *full depth involute system*

c) 20°full depth involutesystem:

- This type of profile is same as $14\frac{1}{2}^\circ$

- The increase of pressure angle from $14\frac{1}{2}^\circ$ to 20° results in a stronger tooth, since the tooth acting as a beam is wider at the base.
- This type of gears also have interference problem if number of teeth isless.

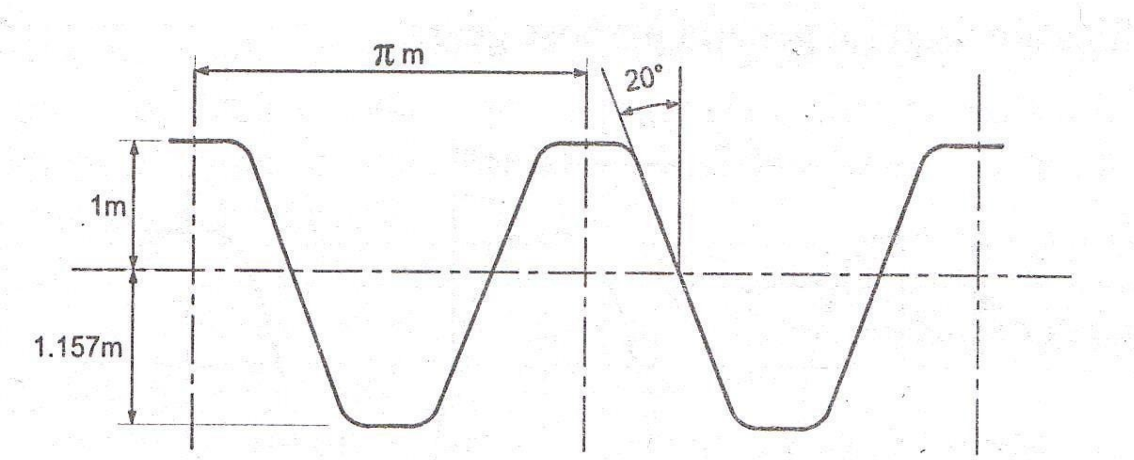


Fig.5.6(c) 20° full depth involute system

d) 20°stub involutesystem:

- The problem of interference in 20° full depth involute systemis minimized byremoving extra addendum of gear tooth which causesinterference.
- Such modified tooth profile is called "Stub toothprofile".
- This type of gears are used for heavyload.

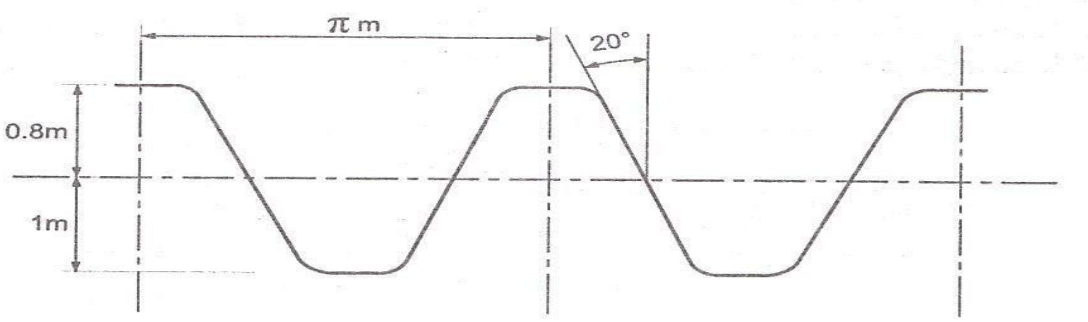


Fig.5.6(d) 20° *stub involute system*

5.7 Length of Path of Contact And Length of Arc ofContact

5.7.1 Length of Path ofContact

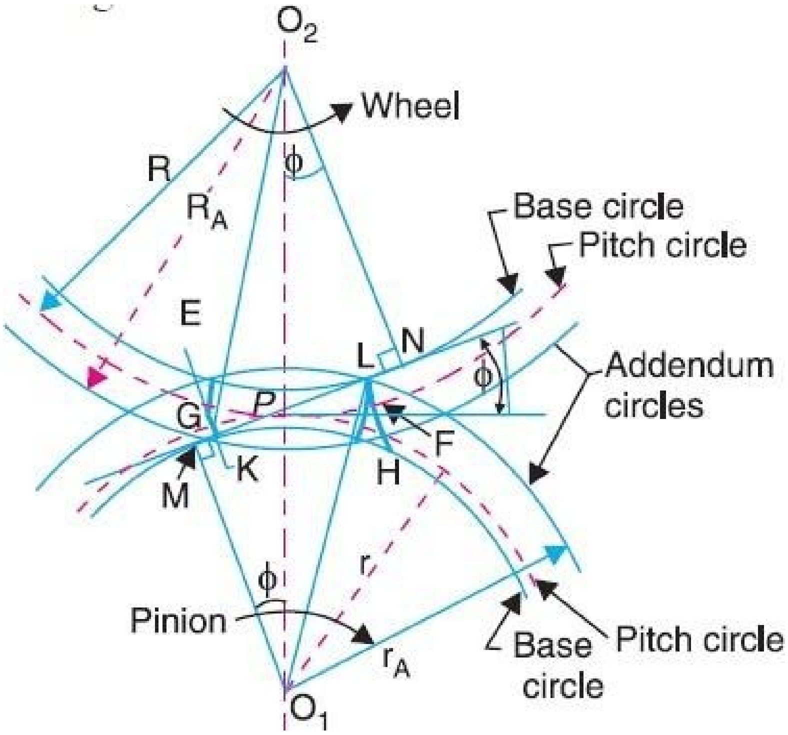


Fig.5.7 Length of path of contact

- When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at *K* (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at L (on the flank near the base circle ofwheel).
- MN is the common normal at the point of contacts and the common tangent to the base circles.
- The point K is the intersection of the addendum circle of wheel and the common tangent.
- The point L is the intersection of the addendum circle of pinion and commontangent.
- **Length of path of contact** is the length of common normal cutoff by the addendum circles of the wheel and thepinion.
- Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as **path of approach** and the part of the path of contact PL is known as **path ofrecess.**

L.P.C =KL

=KP+PL

Where, KP = path of approach

PL = path of recess

Let

$R = O_2P$ = pitch circle radius of wheel
$R_A = O_2K$ = addendum circle radius of wheel
$r = O_1P$ = pitch circle radius of pinion
$r_A = O_1L$ = addendum circle radius of pinion

Length of the path of contact = Path of approach + path of recess

$$= KP + PL$$

$$= (KN - PN) + (ML - MP)$$

$$= \left(\sqrt{O_2K^2 - O_2N^2} - PN\right) + \left(\sqrt{O_1L^2 - O_1M^2} - MP\right)$$

$$= \left(\sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi\right) + \left(\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi\right)$$

5.7.2 Length of Arc ofContact

- The arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair ofteeth.
- The arc of contact is EPF orGPH.
- Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH. The arc GP is known as **arc of approach** and the arc PH is called **arc ofrecess**.
- The angles subtended by these arcs at O1 are called **angle of approach** and **angleof recess**respectively.

Length of the arc ofcontact GPH $= (GP + PH)$

= Arc of approach+ Arc of recess

$$= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi}$$

$$= \frac{KP + PL}{\cos\cos ø}$$

$$= \frac{\text{Length of path of contact}}{\cos ø}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

- The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circularpitch.

  Mathematically,
  Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{\text{Circular pitch}}$$

$$= \frac{\text{Length of arc of contact}}{\pi \text{ m}}$$

Note:

- For continuous transmission of the motion, at least one tooth of any one wheel must be in contact with another tooth of second wheel so ‘n’ must be greater thanunity.
- If ‘n’ lies between 1& 2, no. of teeth in contact at any time will not be less than one and will never matetwo.
- If ‘n’ lies between 2 & 3,it is never less than two pair of teeth and not more than three pairs and soon.
- If ‘n’ is 1.6, one pair of teeth are always in contact where as two pair of teeth are in contact for 60% of thetime

5.8 Interference in Involute Gears

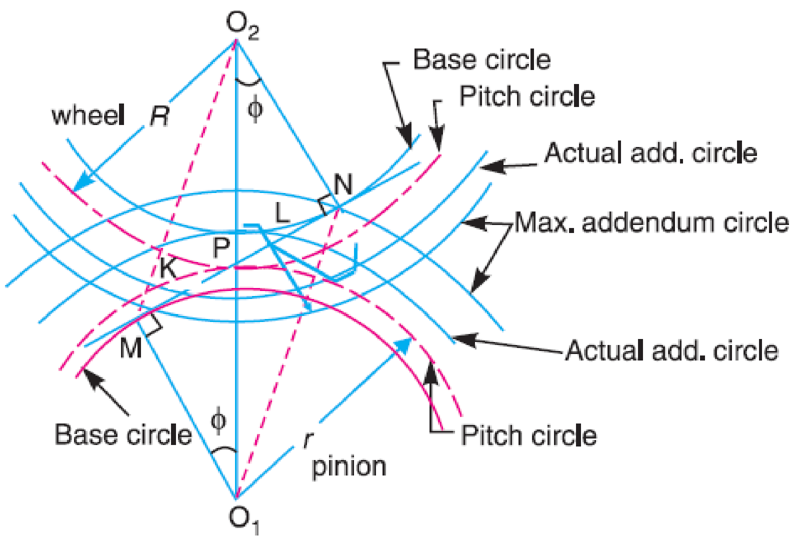


Fig.5.8 Interference in involute gears

- Fig. shows a pinion with centerO_1, in mesh with wheel or gear with centreO_2. MN isthe common tangent to the base circles and KL is the path of contact between the two matingteeth.
- A little consideration will show that if the radius of the addendum circle of pinion isincreased to O_1N, the point of contact L will move from L to N. When this radius isfurther increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as **interference,** and occurs when the teeth are being cut. In brief, the phenomenon when the tip of tooth undercuts the root on its mating gear is known asinterference.

- Similarly, if the radius of the addendum circles of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth onpinion.
- The points *M* and *N* are called **interference points.** Interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel isO_2M.

How to avoid interference?

- The interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both theteeth.

OR

- Interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points oftangency.

– When interference is just avoided, the maximum length of path of contact is*MN*

$$\text{Maximum length of path of contact} = MN = MP + PN = r\sin\phi + R\sin\phi = (r+R)\sin\phi$$

$$\text{Maximum length of arc of contact} = \frac{(r+R)\sin\phi}{\cos\phi}$$

- Path of approach, $KP = \frac{1}{2}$

$$\sqrt{r_A \quad\quad} \Big) - \frac{}{2}$$

- Path of recess, $PL = \frac{1}{2} PN$

$$\sqrt{r_A^2 - \left(r\cos\phi\right)^2} - r\sin\phi = \frac{1}{2} R\sin\phi$$

- Length of the path of contact $= KP + PL$

$$= \frac{1}{2} MP + \frac{1}{2} P$$

$$= \frac{(r + R)\sin\phi}{2}$$

5.9 Minimum Number of Teeth on the Pinion in Order to Avoid Interference

- In order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency.
- The limiting condition reaches, when the addendum circles of pinion and wheel pass through points *N* and *M* (see Fig.) respectively.

Let
t = Number of teeth on the pinion,
T = Number of teeth on the wheel,
m = Module of the teeth,
r = Pitch circle radius of pinion = $mt/2$
G = Gear ratio = $T/t = R/r$
ϕ = Pressure angle or angle of obliquity.

From O_1NP,

$$O N^2 = O P^2 + PN^2 - 2OP \times PN\cos\left(O PN\right)$$

$$\therefore O_1N^2 = r^2 + (R\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_1N^2 = r^2 + (R\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_1N^2 = r^2 + R^2\sin^2\phi + 2rR\sin^2\phi$$

$$\therefore O_1N^2 = r^2\left[1+\frac{R^2\sin^2\phi}{r^2}+\frac{2R\sin^2\phi}{r}\right]$$

$$\therefore O_1N^2 = r^2\left[1+\frac{R^2\sin^2\phi}{r^2}+\frac{2R\sin^2\phi}{r}\right]$$

$$\therefore O_1N^2 = r^2\left[1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi\right]$$

$$\therefore O_1N = r\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi}$$

$$\therefore O_1N = \frac{mt}{2}\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi}$$

Let $A_p \cdot m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

Addendum of the pinion = $O_1N - O_1P$

$$A_P.m = \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - \frac{mt}{2}$$

$$\therefore A_P.m = \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - \frac{mt}{2}$$

$$\therefore A_P.m = \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}-1\right]$$

$$\therefore A_P.m = \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]$$

$$\therefore A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]$$

$$\therefore A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]$$

$$\therefore A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]$$

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]}$$

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+G(G+2)\sin^2 ø}-1\right]}$$

Note:

– If the pinion and wheel have equal teeth, then $G = 1$.

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+3\sin^2 ø}-1\right]}$$

Min. no of teeth on pinion

Sr. no	System of gear teeth	Min. no of teeth on pinion
1	Composite	12
2	Full depth involute	32
3	Full depth involute	18
4	Stub involute	14

5.10 Minimum Number of Teeth on the Wheel in Order to Avoid Interference

Let T = Minimum number of teeth required on the wheel in order to avoid interference,

$A_w \cdot m$ = Addendum of the wheel, where A_W is a fraction by which the standard Addendum for the wheel should be multiplied.

From O_2MP

$$O_2M^2 = O_2P^2 + PM^2 - 2O_2P \times PM\cos(O_2PM)$$

$$\therefore O_2M^2 = R^2 + (r\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_2M^2 = R^2 + r^2\sin^2\phi + 2rR\sin^2\phi$$

$$\therefore O_2M^2 = R^2\left[1 + \frac{r^2\sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right]$$

$$\therefore O_2M^2 = R^2\left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi\right]$$

$$\therefore O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi}$$

$$\therefore O_2M = \frac{mT}{2}\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi}$$

Addendum of the wheel $= O_2M - O_2P$

$$A_w m = \frac{mT}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - \frac{mT}{2}$$

$$\therefore A_w m = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_w m = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1\right]$$

$$\therefore A_w = \frac{T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1\right]$$

$$\therefore T = \frac{2A_w}{\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1\right]}$$

$$\therefore T = \frac{2A_w}{\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1\right]}$$

Note:

- From the above equation, we may also obtain the minimum number of teeth on pinion. Multiplying both sides by t/T,

$$T \times \frac{t}{T} = \frac{2A_w \times \frac{t}{T}}{\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1\right]}$$

$$\therefore t = \frac{2A_w}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1\right]}$$

- If wheel and pinion have equal teeth, then $G = 1$,

$$\therefore T = \frac{2A_w}{\left[\sqrt{1 + 3\sin^2 \phi} - 1\right]}$$

5.11 Minimum Number of Teeth on a Pinion for Involute Rack in Order toAvoid Interference

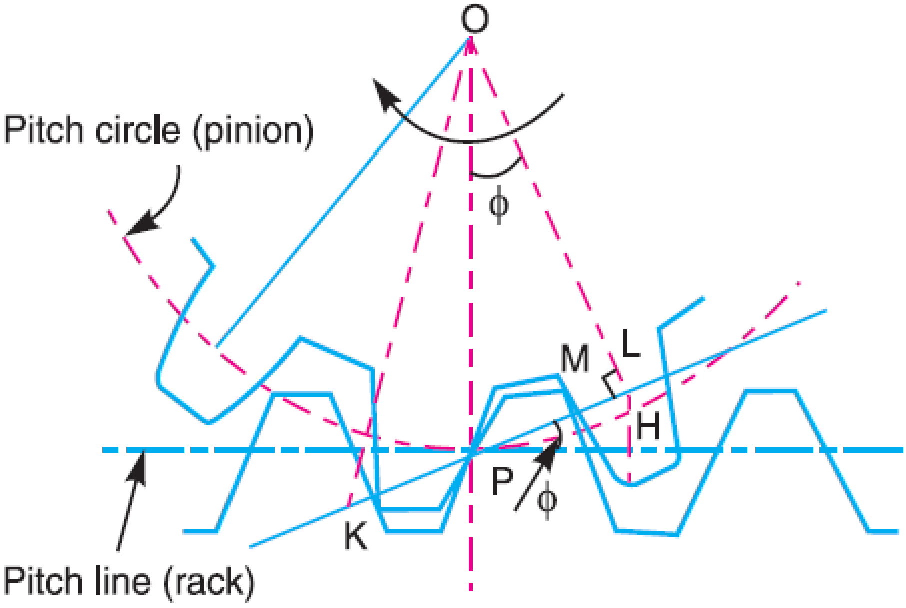


Fig.5.11 Rack and pinion in mesh

Let t = Minimum number of teeth on thepinion,

r = Pitch circle radius of the pinion = $\frac{m \cdot t}{2}$ and

ø= Pressure angle or angle of obliquity, and

$A_R \cdot$m= Addendum for rack, where A_R is the fraction by which the standard addendum of one module for the rack is to be multiplied.

Addendumforrack, $A_R \cdot m = LH$

$\therefore A_R \cdot m = PL \sin \phi$

$\therefore A_R \cdot m = r \sin \varphi \times \sin \varphi$

$\therefore A \cdot m = r \sin^2 \varphi$

$\therefore A_R \cdot m = \frac{mt \sin^2 \varphi}{2}$

$$\therefore t = \frac{2A_R}{\sin^2\varphi}$$

Note:

- In case of pinion, max. value of addendum radius to avoid interference ifAF

$$= O\,M^2 + AF^2$$

$$= (r\cos ø)^2 + (R\sin ø + r\sin ø)^2$$

- Max value of addendum of pinionis

$$A_{max} = r\left[\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2 ø} - 1\right]$$

$$= \frac{mt}{2}\left[\sqrt{1 + G(G+2)\sin^2 ø} - 1\right]$$

5.12 Comparison of Cycloidal and Involute toothforms

Cycloidal teeth	Involute teeth
Pressure angle varies from maximum at the beginning of engagement, reduce to zero at the pitch point and again increase to maximum at the end of the engagement resulting in smooth running of gears.	Pressure angle is constant throughout the engagement of teeth. This result in smooth running of the gears.
It involves double curves for the teeth, epicycloid and hypocycloid. This complicates the manufacturer.	It involves the single curves for the teeth resulting in simplicity of manufacturing and of tool
Owing to difficulty of manufacturer, these are costlier	These are simple to manufacture and thus are cheaper.
Exact center distance is required to transmit a constant velocity ratio.	A little variation in a centre distance does not affect the velocity ratio.
Phenomenon of interference does not occur at all.	Interference can occur if the condition of minimum no. of teeth on a gear is not followed.
The teeth have spreading flanks and thus are stronger.	The teeth have radial flanks and thus are weaker as compared to the Cycloidal form for the same pitch.
In this a convex flank always has contact with a concave face resulting in less wear.	Two convex surfaces are in contact and thus there is more wear.

5.13 HELICAL AND SPIRALGEARS

- In helical and spiral gears, the teeth are inclined to the axis of a gear. They can be right handed or left-handed, depending upon the direction in which the helix slopes away from the viewer when a gear is viewed parallel to the axis of thegear.

- In Fig. Gear1 is a right-handed helical gear whereas 2 are left handed. The two mating gears have parallel axes and equal helix angle α OR ψ. The contact between two teeth on the two gears is first made at one end which extends through the width of the wheel with the rotation of thegears.
- Figure (a) shows the same two gears when looking from above. Now, if the helix angle of the gear 2 is reduced by a few degrees so that the helix angle of the gear 1is ψ_1 , and that of gear 2 is ψ_2 and it is desired that the teeth of the two gears still mesh with each other tangentially, it is essential to rotate the axis of gear 2 through some angle as shown in Fig. (b).

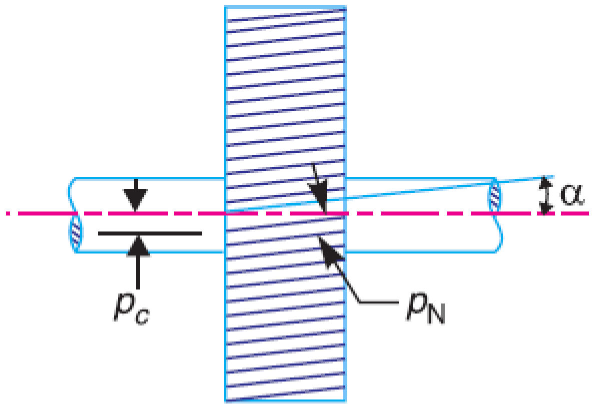


Fig.5.13(a) Helical Gear

- The following definitions may be clearly understood in connection with a helical gear as shown inFig.

1. Normal pitch. It is the distance between similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth. It is denoted byp_N.

2. Axial pitch. It is the distance measured parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by pc. If α is the helix angle,then

$$\text{Circular pitch}, p_c = \frac{p_N}{\cos\alpha}$$

Note: The **helix angle** is also known as **spiral angle** of the teeth.

Efficiency of Spiral Gears

- A pair of spiral gears 1 and 2 in mesh is shown in Fig. .Let the gear 1 be the driver and the gear 2 the driven. The forces acting on each of a pair of teeth in contact are shown inFig.
- The forces are assumed to act at the center of the width of each teeth and in the plane tangential to the pitchcylinders

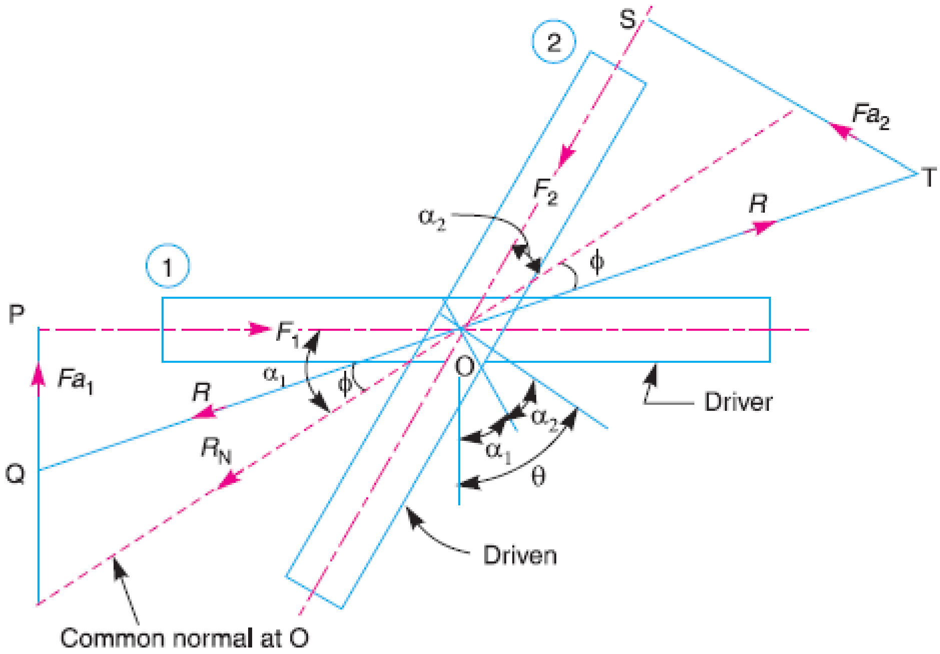


Fig.5.13 (b)

Let F_1 = Force applied tangentially on the driver,

F_2 = Resisting force acting tangentially on the driven, F_{a1} = Axial or end thrust on thedriver,

F_{a2} = Axial or end thrust on the driven,

R_N = Normal reaction at the point of contact

ϕ = Angle of friction,

R = Resultant reaction at the point of contact, and

θ = Shaft angle $= \alpha_1 + \alpha_2$

...($\because$ Both gears are of the samehand)

FromtriangleOPQ, $F1 = R\cos(\alpha_1 - \phi)$

$\therefore$ Workinputtothedriver$= F_1 \times \pi d_1 \cdot N_1 = R\cos(\alpha_1 - \phi) \times \pi d_1 \cdot N_1$

FromtriangleOST, $F_2 = R\cos(\alpha_2 + \phi)$

$\therefore$ Workoutputofthedriven$=F_2\times \pi d_2\cdot N_2=R\cos(\alpha_2+\phi)\times \pi d_2\cdot N_2$

$\therefore$ Efficiency of spiral gears,

$$\eta=\frac{\text{Workoutput}}{\text{Workinput}}=\frac{R\cos(\alpha_2+\phi)\times\pi d_2\cdot N_2}{R\cos(\alpha_1-\phi)\times\pi d_1\cdot N_1}$$

$$=\frac{\cos(\alpha_2+\phi)\times d_2\cdot N_2}{\cos(\alpha_1-\phi)\times d_1\cdot N_1}$$

Pitch circle diameter of gear1,

$$d_1=\frac{p_{c1}\times T_1}{\pi}=\frac{p_N}{\text{Cos}\alpha}\times\frac{T_1}{\pi}$$

Pitch circle diameter of gear 2,

$$d_2=\frac{p_{c2}\times T_2}{\pi}=\frac{p_N}{\text{Cos}\alpha}\times\frac{T_2}{\pi}$$

$$\therefore \frac{d_2}{d_1}=\frac{T_2\text{Cos}\alpha_1}{T_1\text{Cos}\alpha_2} \quad \cdots\cdots\cdots(2)$$

$$\frac{N_2}{N_1}=\frac{T_1}{T_2} \quad \cdots\cdots\cdots(3)$$

Multiplying equation (2) and (3) we get

$$\frac{d_2N_2}{d_1N_1}=\frac{\cos\alpha_1}{\cos\alpha_2}$$

Substituting this value in equation (1)

$$\eta=\frac{\cos(\alpha_2+\phi)\times\cos\alpha_1}{\cos(\alpha_1-\phi)\times\cos\alpha_2} \quad \cdots\cdots(4)$$

$$=\frac{\cos(\alpha_1+\alpha_2+\phi)+\cos(\alpha_1-\alpha_2-\phi)}{\cos(\alpha_1+\alpha_2-\phi)+\cos(\alpha_1-\alpha_2+\phi)}$$

$$\left(\because \cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]\right)$$

$$= \frac{\cos(\theta+\phi)+\cos(\alpha_1-\alpha_2-\phi)}{\cos(\theta-\phi)+\cos(\alpha_1-\alpha_2+\phi)} \quad \cdots\cdots\cdots(5)$$

$$(\theta = \alpha_1 + \alpha_2)$$

Since the angle θ and ϕ are constants, therefore the efficiency will be maximum, when $\cos(\alpha_1-\alpha_2+\phi)$ is maximum *i.e.*

$$\cos(\alpha_1-\alpha_2+\phi)=1$$

$$\therefore \alpha_1 - \alpha_2 + \phi = 0$$

$\therefore \alpha_1 = \alpha_2 + \phi$ and $\alpha = \alpha - \phi$

Since $\alpha_1 + \alpha_2 = \theta$ therefore

$\alpha_1 = \theta - \alpha_2 = \theta - \alpha_1 + \phi$ OR $\alpha_1 = \dfrac{\theta+\phi}{2}$

Similarly $\alpha_2 = \dfrac{\theta-\phi}{2}$

Substituting $\alpha_1 = \alpha_2 + \phi$ and $\alpha_2 = \alpha_1 - \phi$ in equation (5) we get

$$\eta_{max} = \frac{\cos(\theta+\phi)+1}{\cos(\theta-\phi)+1}$$

$$\therefore \frac{1}{3} = \frac{300}{N_1} = \frac{T_1}{72}$$

$$\therefore T_1 = 24 \;\&\; N_1 = 900 \text{ rpm}$$

Pitch line velocity

$V_P = r_1\omega_1 = r_2\omega_2$

$$= \frac{2\pi N_1}{60} \times \frac{d_1}{2}$$

$$= \frac{2\pi N_1}{60} \times \frac{mT_1}{2}$$

$$= \frac{2\pi \times 900}{60} \times \frac{8 \times 24}{2}$$

$= 9047.78$ mm / sec

Example 5.2: The number of teeth of a spur gear is 30 and it rotates at 200 rpm. What will be its circular pitch and the pitch line velocity if it has a module of 2 mm?

Solution:

Givendata

$T = 30$

$N = 200rpm$

$m = 2mm$

Find:

$P_c = ?$

$V_p = ?$

Circular pitch $P_c = \pi \cdot m$

$= \pi \cdot 2$

$= 6.28mm$

Pitch line velocity $V_p = \omega \cdot r$

$= \frac{2\pi N}{0} \times \frac{d6}{2}$

$= \frac{2\pi \times 200}{} \times \frac{2 \times 3060}{2}$

$= 628.3mm/ s$

Example 5.3: The following data relate to two meshing gears velocity ratio = 1/3, module = 1mm, Pressure angle 20°, center distance= 200 mm. Determine the number of teeth and the base circle radius of the gear wheel.

Solution:

Givendata

$VR = 1/3$

$Ø = 20°$

$C = 200mm$

$= ?$ $m = 4mm$

Find:

$T_1 = ?$

$T_2 = ?$

Base circle radius of gear wheel

(1) $VR = \frac{N_2}{} = \frac{1}{3} = \frac{T_1}{T_2} \frac{1}{} N_1$

Example 5.5: Two involute gears in mesh have 20° pressure angle. The gear ratio is 3 and the number of teeth on the pinion is 24. The teeth have a module of 6 mm. The pitch line velocity is 1.5 m/s and the addendum equal to one module. Determine the angle of action of pinion (the angle turned by the pinion when one pair of teeth is in the mesh) and the maximum velocity ofsliding.

Solution:

Givendata

$\text{Ø} = 20°$

$G = T / t = 3$

$t = 24$

$m = 6$ mm

$V_p = 1.5$ m / s Addendum = 1 module

Find:

Angle of action of the pinion =?

Max.velocityof sliding =?

$$r = \frac{mt}{2} = \frac{6 \times 24}{2} = 72\text{mm}$$

$$(\because T = 24 \times 3 = 72)$$

$$R = \frac{mT}{2} = \frac{6 \times 72}{2} = 216\text{mm}$$

$$r_a = r + \text{Add.} = 72 + (1 \times 6) = 78\text{mm}$$

$$R_A = R + \text{Add.} = 216 + (1 \times 6) = 222\text{mm}$$

Let the length of path of contact KL =KP+PL

$$KP = \left(\sqrt{R_A^2 - (R\cos\text{ø})^2} - R\sin\text{ø}\right)$$

$$\left(\sqrt{222^2-\left(216\cos 20\right)^2}-216\sin 20^\circ\right)$$

$$=16.04\text{mm}$$

$$PL=\left(\sqrt{r_A^2-\left(r\cos\phi\right)^2}-r\sin\phi\right)$$

$$\left(\sqrt{78^2-\left(72\cos 20\right)^2}-72\sin 20^\circ\right)$$

$$=14.18\text{mm}$$

$$\text{Arc of contact}=\frac{\text{Path of contact}}{\cos\varphi}$$

$$=\frac{16.04+14.18}{\cos 20^\circ}$$

$$=32.16\text{mm}$$

$$\text{Angle turned through by pinion}(\theta)=\frac{\text{Length of arc of contact}\times 360^\circ}{\text{circumference of pinion}}$$

$$=\frac{32.16\times 360^\circ}{2\pi\times 72}$$

$$=25.59^\circ$$

$$\text{Max. velocity of sliding}=(\omega_p+\omega_g)\times KP$$

$$=\left(\frac{V}{r}+\frac{V}{R}\right)\times KP \qquad (\because V=r\omega)$$

$$=\left(\frac{1500}{72}+\frac{1500}{216}\right)\times 16.04$$

$$=445.6\text{mm/sec}$$

Example 5.6: Two involute gears in a mesh have a module of 8mm and pressure angle of 20°. The larger gear has 57 while the pinion has 23 teeth. If the addendum on pinion and gear wheels are equal to one module,Determine

i. Contact ratio(No. of pairs of teeth in contact)
ii. Angle of action of pinion and gearwheel

iii. Ratio of sliding to rolling velocity atthe
   a. Beginning of thecontact.
   b. Pitchpoint.
   c. End of thecontact.

Solution:

Givendata

$\varnothing = 20°$

m =8mm

=57

t =23

Addendum =1module

=8mm

Find:

1. Contact ratio =?
2. Angle of action of pinion and gear =?T
3. Ratio of sliding to rolling velocity at the
   a. Beginning ofcontact
   b. Pitchpoint
   c. End ofcontact

i. Let the length of path of contact KL =KP+PL

$$KP = \left(\sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi\right)$$

$$\left(\sqrt{236^2 - (228\cos 20^\circ)^2} - 228\sin 20\right)$$

= 20.97mm

$$PL = \left(\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi\right)$$

$$\left(\sqrt{100^2 - (92\cos 20)^2} - 92\sin 20\right)$$

=18.79mm

$$\text{Arcofcontact} = \frac{\text{Pathofcontact}}{\cos\varphi}$$

$$= \frac{KP + PL}{\cos\varphi}$$

$$= \frac{20.97 + 18.79}{\cos 20^\circ}$$

= 42.29mm

$$\text{Contactratio} = \frac{\text{Lengthofarcofcontact}}{P_c}$$

$$= \frac{42.21}{\pi m} = 1.68 \quad \text{say2}$$

ii.

$$\text{Angleofactionofpinion}(\delta_p) = \frac{\text{Lengthofarcofcontact} \times 360^\circ}{\text{circumferenceofpinion}}$$

$$= \frac{42.31 \times 360^\circ}{2\pi \times 92}$$

$$= 26.34^\circ$$

$$\text{Angleofactionofpinion}(\delta_g) = \frac{\text{Lengthofarcofcontact} \times 360^\circ}{\text{circumferenceof gear}}$$

$$= \frac{42.31 \times 360^\circ}{2\pi \times 228}$$

$$= 10.63$$

i. Ratio of sliding to rollingvelocity:

a. Beginning ofcontact

$$\frac{\text{Slidingvelocity}}{\text{ollingvelocity}} = \frac{(\omega_p + \omega_g) KP_R}{\omega_p r}$$

$$= \frac{\left(\omega_p + \frac{92}{228}\omega\right) \times 20.97}{\omega_p \times 92}$$

$$= 0.32$$

b. Pitchpoint

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p+\omega_g)KP}{\omega_p r}$$

$$= \frac{(\omega_p+\omega_g)\times 0}{\omega_p r}$$

$$=0$$

c. End of contact

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p+\omega_g)PL}{\omega_p r}$$

$$= \frac{\left(\omega_p+\frac{92}{228}\omega_p\right)\times 18.79}{\omega_p \times 92}$$

$$=0.287$$

Example 5.7: Two 20° gears have a module pitch of 4 mm. The number of teeth on gears 1and 2 are 40 and 24 respectively. If the gear 2 rotates at 600 rpm, determine the velocity of sliding when the contact is at the tip of the tooth of gear 2. Take addendum equal to one module. Also, find the maximum velocity of sliding.

Solution:

Given data	Find:
$\emptyset = 20°$	Velocity of sliding =?
$m = 4mm$	Max. velocity of sliding = ?
$N_p = 600rpm$	
$T = 40$	
$t = 24$	
Addendum = 1 module = 4 mm	

$$r = \frac{mt}{2} = \frac{4\times 24}{2} = 48mm$$

$$R = \frac{mT}{2} = \frac{4\times 40}{2} = 80mm$$

$$r_a = r + Add. = 48 + (1\times 4) = 52mm$$

$$R_A = R + Add. = 80 + (1\times 4) = 84mm$$

(Note: The tip of driving wheel is in contact with a tooth of driving wheel at the end of engagement. So it is required to find path of recess.)

Path of recess

$$PL = \left(\sqrt{r^2 - (r\cos\phi)^2} - r\sin\phi\right)$$

$$\left(\sqrt{52^2 - (48\cos 20)^2} - 48\sin 20\right)$$

$$= 9.458\,mm$$

Velocity of sliding

$$= (\omega_p + \omega_g) \times PL$$

$$= \frac{2\pi}{60}(600 + 360) \times 9.458$$

$$\left(\frac{N_g}{N_P} = \frac{t}{T} \Rightarrow N_g = 600 \times \frac{24}{40} = 360\,rpm\right)$$

$$= 956.82\,mm/sec$$

Path of recess

$$\therefore KP = \left(\sqrt{R^2 - (R\cos\phi)^2} - R\sin\phi\right)$$

$$\left(\sqrt{84^2 - (80\cos 20)^2} - 80\sin 20\right)$$

$$= 10.108\,mm$$

Max. Velocity of sliding

$$= (\omega_p + \omega_g) \times KP$$

$$\frac{\pi}{60}(600 + 360) \times 10.108$$

$$= 1016.16\,mm/sec$$

Example 5.8: Two 20° involute spur gears mesh externally and give a velocity ratio of 3.The module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at the 120 rpm,determine

I. Minimum no of teeth on each wheel to avoidinterference

II. Contactratio

Solution:

Givendata

$\phi = 20°$

VR=3

m=3

$N_P = 120$

Addendum = 1.1 module

Find:

t_{min} & T_{min} =?

Contact ratio =?

I.

$$T = \frac{2A_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$$\therefore T = \frac{2\times 1.1}{\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20°}-1}$$

$\therefore$ T=49.44teeth

$\therefore$ T=51teeth And $t = \frac{T}{3} = \frac{51}{3} = 17$teeth

II.

$$r = \frac{mt}{2} = \frac{3\times 17}{2} = 25.5\text{mm} \qquad R = \frac{mT}{2} = \frac{3\times 51}{2} = 76.5\text{mm}$$

$r_a = r$ + Add. = 25.5 +(1.1×3) = 28.8mm $R_A = R$ + Add. = 76.5 + (1.1×3) = 28.8mm

$$\text{Contactratio} = \frac{\text{Lengthofpathofcontact}}{\cos\varphi \times P_c}$$

$$= \frac{\left(\sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi\right) + \left(\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi\right)}{\cos 20^\circ \times \pi \times 3}$$

$$= \frac{\left(\sqrt{79.8^2 - (76.5\cos 20^\circ)^2} - 76.5\sin 20^\circ\right) + \left(\sqrt{28.8^2 - (25.5\cos 20^\circ)^2} - 25.5\sin 20^\circ\right)}{\cos 20^\circ \times \pi \times 3}$$

=1.78

Thus 1 pair of teeth will always remain in contact whereas for 78 % of the time, 2 pairs of teeth will be in contact.

Example 5.9: Two involute gears in a mesh have a velocity ratio of 3. The arc of approach is not to be less than the circular pitch when the pinion is the driver The pressure angle of the involute teeth is 20°.Determine the least no of teeth on the each gear. Also find the addendum of the wheel in terms of module.

Solution:

Givendata

$\phi = 20^\circ$

VR =3

Find:

least no of teeth on the each gear =?

Addendum=?

Arc of approach = circular pitch

$= \pi \cdot m$

$\therefore$ Pathofapproach= Arcofapproach×cos20 °

$= \pi \cdot m \cdot \cos 20^\circ$

$= 2.952m \quad \cdots\cdots(1)$

Let the max lengthofpathofapproach=rsinφ

$= \frac{mt}{2}\sin 20^\circ$

$= 0.171mt \quad \cdots\cdots(2)$

From eq. 1.And 2.

$\therefore 0.171mt = 0.2952m$

$\therefore t = 17.26 \cong 18$teeth

$$T = 18\times3 = 54 \text{ teeth}$$

Max. Addendum of the wheel

$$A_{w\max} = \frac{mT}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1\right]$$

$$= \frac{m\times54}{2}\left[\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20^\circ}-1\right]$$

$$= 1.2m$$

Example 5.10: Two 20° involute spar gears have a module of 10 mm. The addendum is equalto one module. The larger gear has 40 teeth while the pinion has 20 teeth will the gear interfere with thepinion?

Solution:

Givendata

$Ø = 20°$

$m = 10mm$

Addendum = 1 module

$= 1\times10$

$= 10mm$

Find:

Interference or not?

Let the pinion is the driver

$t = 20$ teeth

$T = 40$ teeth

$$r = \frac{mt}{2} = \frac{10\times20}{2} = 100mm \qquad R = \frac{mT}{2} = \frac{10\times40}{2} = 200mm$$

$r_a = r + Add. = 100 + 10 = 110mm$ $\qquad R_A = R + Add. = 200 + 10 = 210mm$

Pathofapproach $= \left(\sqrt{R_A^2-(R\cos ø)^2}-R\sin ø\right)$

$$\left(\sqrt{210^2-(200\cos20^\circ)^2}-200\sin20^\circ\right)$$

$= 25.29mm$

To avoid theinterference…….

Maxlengthofpathofapproach=rSinφ

$= 100\times \text{Sin}20^0$

$=34.20\text{mm}>25.29\text{mm}$

So Interference will **notoccur.**

Example 5.11: Two 20° involute spur gears have a module of 10 mm. The addendum is one module. The larger gear has 50 teeth and the pinion has l3 teeth. Does interference occur? If itoccurs, to what value should the pressure angle be change to eliminateinterference?

Solution:

Given data

Ø=20°
m=10 mm
Addendum = 1 module =10 mm T =50 and t =13

$r= \frac{mt}{2} = \frac{10\times13}{2} =65\text{mm}$ $\qquad R= \frac{mT}{2} = \frac{10\times50}{2} =250\text{mm}$

$r_a=r + \text{Add.} = 65 + 10 = 75\text{mm}$ $R_A =R + \text{Add.} = 250 + 10 = 260\text{mm}$

$R_{amax}= \sqrt{(R\text{Cos}\varphi)^2+(R\text{Sin}\varphi+r\text{Sin}\varphi)^2}$

$= \sqrt{(250\text{Cos}20^\circ)^2+(250\text{Sin}20^\circ+65\text{Sin}20^\circ)^2}$

$= 258.45\text{mm}$

Here actual addendum radius R_a (260 mm) > $R_{a\ max}$ value

So interference will occur. The new value of ø can be found by comparing

$R_{a\ max} = R_a$

$\therefore R_a = R_{a\ max}$

$$\therefore R_a = \sqrt{(R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2}$$

$$\therefore 260 = \sqrt{(250\cos\varphi)^2 + (250\sin\varphi + 65\sin\varphi)^2}$$

$$\therefore 260^2 = (250\cos\varphi)^2 + (250\sin\varphi + 65\sin\varphi)^2$$

$$\therefore \cos^2\varphi = 0.861$$

$$\therefore \varphi = 21.88^\circ$$

Note: If pressure angle is increased to 21.88° interference can be avoided

Example 5.12: The following data related to meshing involute gears:
No. of teeth on gear wheel = 60
Pressure angle = 20°
Gear ratio =1.5
Speed of gear wheel = 100 rpm
Module = 8 mm

The addendum on each wheel is such that the path of approach and path of recess on each side are 40 % of the maximum possible length each. Determine the addendum for the pinion and the gear and the length of arc of contact.

Solution:

Givendata
- T=60
- Ø= 20°
- G=1.5
- N_g=100
- rpm m=8
- mm

Find:
- Addendum for gear andpinion=?
- Length of arc ofcontact=?

Let pinion is driver…

Max. Possible length of path of approach =rsinϕ

∴ Actual length of path of approach =0.4rsinϕ (Given in data)

Same way…

Actual length of path of recess = 0.4 $R\sin\phi$ (Given in data)

$$\therefore 0.4 r\sin\phi = \left(\sqrt{R_A^2-(R\cos\phi)^2}-R\sin\phi\right)$$

$$\therefore 0.4\times160\sin20 = \left(\sqrt{R_A^2-(240\cos20)^2}-240\sin20\right)$$

$$\therefore R_a = 248.33 \text{ mm}$$

$$\therefore \text{Addendum of wheel} = 248.3 - 240 = 8.3 \text{mm}$$

Also

$$0.4\,R\sin\phi = \sqrt{r_A^2-(r\cos\phi)^2}-r\sin\phi$$

$$\therefore 0.4\times240\times\sin20 = \sqrt{r_A^2-(160\cos20)^2}-160\sin20$$

$$\therefore r_a = 173.98 = 174 \text{ mm}$$

$$\therefore \text{Addendum of pinion} = 174 - 160 = 14 \text{ mm}$$

$$\text{Length of Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$

$$= \frac{(r\sin\varphi + R\sin\phi)\times 0.4}{\cos\phi}$$

$$= \frac{(160+240)\times\sin20\times0.4}{\cos20}$$

$$= 58.2 \text{ mm}$$

Gear Train

Introduction

Definition

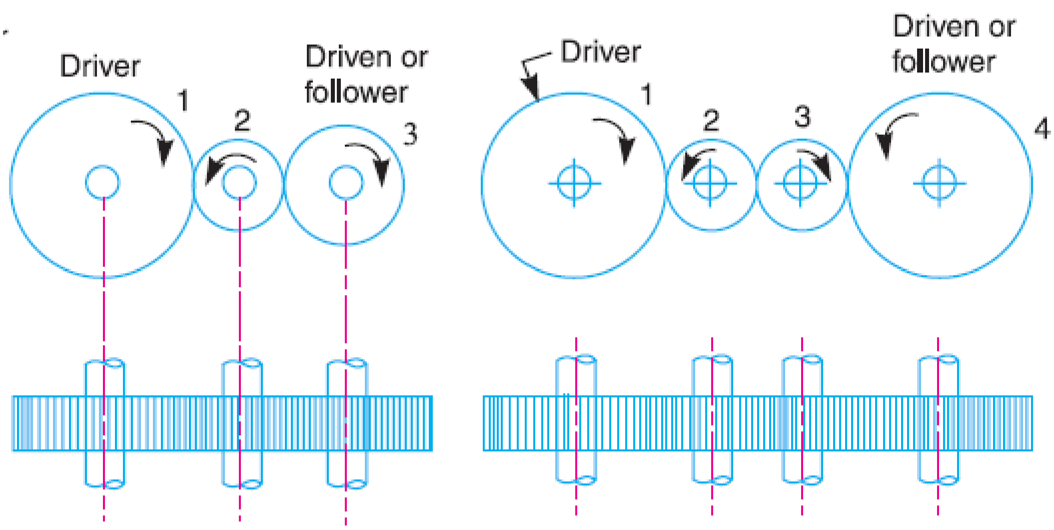
- When two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.
- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiralgears.

Types of Gear Trains

1. Simple gear train
2. Compound geartrain
3. Reverted geartrain
4. Epicyclic geartrain
5. Compound epicyclic geartrain

Simple gear train.

- When there is only one gear on each shaft, as shown in Fig. , it is known as **simple gear train.** The gears are represented by their pitchcircles.
- When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown inFig.
- Since the gear 1 drives the gear 2, therefore gear 1 is called the **driver** and the gear 2 is called the **driven** or **follower.** It may be noted that the motion of the driven gear is



opposite to the motion of drivinggear.

(a) (b)

Fig.5.2.1 Simple gear train

Let

N_1 = Speed of driver rpm
N_2 = Speed of intermediate wheel rpm
N_3 = Speed of follower rpm
T_1 = Number of teeth on driver
T_2 = Number of teeth on intermediate wheel
T_3 = Number of teeth on follower

- Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \cdots\cdots\cdots(1)$$

- Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \qquad \cdots\cdots\cdots(2)$$

- The speed ratio of the gear train as shown in Fig. (a) Is obtained by multiplying the equations (1) and (2).

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$

$$\therefore \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

- Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods:

1. By providing the large sized gear, or
   - A little consideration will show that this method (i.e. providing large sized gears) is very inconvenient and uneconomical method.

2. By providing one or more intermediate gears.
   - This method (i.e. providing one or more intermediate gear) is very convenient and economical.

- It may be noted that when the number of intermediate gears are **odd**, the motion of both the gears (i.e. driver and driven or follower) is like as shown in Fig.(a).
- If the numbers of intermediate gears are **even**, the motion of the driven or follower will be in the **opposite direction** of the driver as shown in Fig(b).

- **speed ratio** (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number ofteeth.

$$\text{Speedratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

- **Train value** of the gear train is the ratio of the speed of the driven or follower to the speed of thedriver.

$$\text{Trainvalue} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

Compound Gear Train

– When there is more than one gear on a shaft, as shown in Fig., it is calleda **compound train of gear.**

– The idle gears, in a simple train of gears do not affect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

– But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediateshafts.

– In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown inFig.

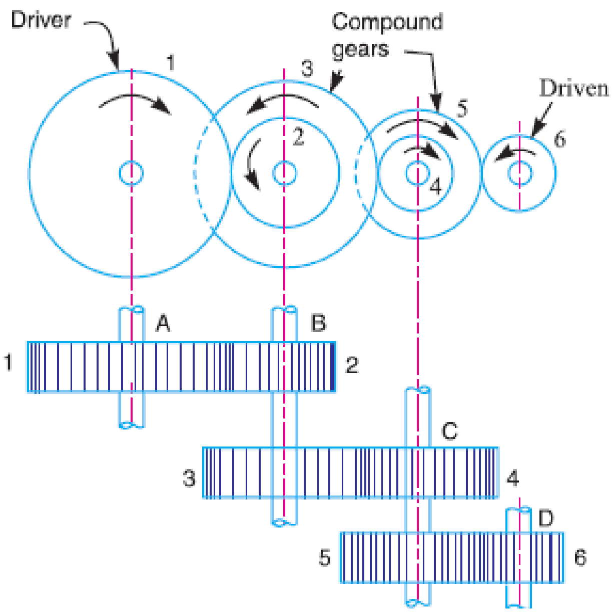


Fig. 5.2.2 compound gear train

- In a compound train of gears, as shown in Fig., the gear 1 is the driving gear mounted on shaft A; gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaftD.

Let

N1 = Speed of driving gear 1,
T1 = Number of teeth on driving gear 1,
N2 ,N3 ..., N6 = Speed of respective gears in r.p.m.,
and T2 ,T3..., T6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \cdots\cdots\cdots(1)$$

Similarly, for gears 3 and 4, speed ratiois

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \cdots\cdots\cdots(2)$$

And for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \cdots\cdots\cdots(3)$$

The speed ratio of compound gear train is obtained by multiplying the equations (1), (2) and (3),

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

- The **advantage** of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with smallgears.
- If a simple gear train is used to give a large speed reduction, the last gear has to be verylarge.
- Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing isemployed.

Reverted Gear Train

- When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as **reverted geartrain.**
- Gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is**like.**

Let

T1 = Number of teeth on gear1,

r1 = Pitch circle radius of gear 1,and

N1 = Speed of gear 1 in r.p.m.

Similarly,

T2, T3, T4 = Number of teeth on respective gears,

r2, r3, r4 = Pitch circle radii of respective gears, and

N2, N3, N4 = Speed of respective gears in r.p.m.

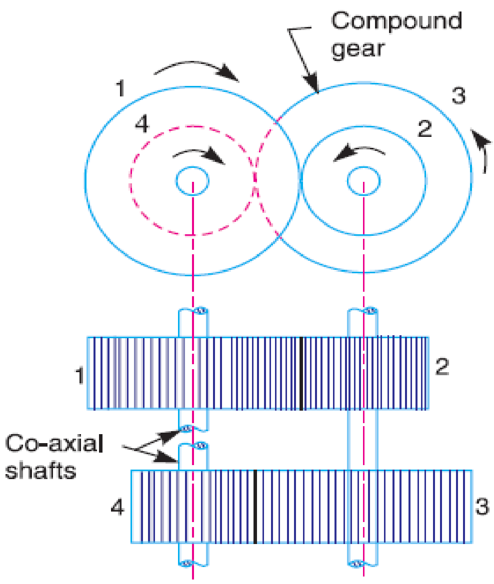


Fig. 5.2.3 Reverted gear train

- Since the distance between the centers of the shafts of gears 1 and 2 as well as gears3 and 4 is same,therefore

$$r_1 + r_2 = r_3 + r_4$$

– Also, the circular pitch or module of all the gears is assumed to be same; therefore number of teeth on each gear is directly proportional to its circumference orradius.

$$T_1 + T_2 = T_3 + T_4$$

$$\text{Speedratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

Application

– The reverted gear trains are used in automotive transmissions**,** lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

Epicyclic GearTrain

– In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. where a gear A and the arm C have a common axis at O1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate.

– If the arm is fixed, the gear train is simple and gear A can drive gear B or **vice- versa**, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate **upon** and **around** gear A. Such a motion is called **epicyclic**and the gear trains arranged in such a manner that one or more of their members move upon and around another member is known as **epicyclic gear trains** (**epi.** means upon and **cyclic** means around). The epicyclic gear trains may be **simple** or**compound.**

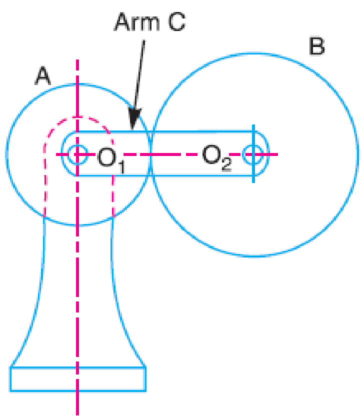


Fig. 5.2.4 Epicyclic gear train

Sr. No.	Condition of motion	Revolution of element		
		Arm C	Gear A	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through + x revolutions	0	$+x$	$-x\frac{T_A}{T_B}$
3	Add + y revolutions to all elements	+ y	+ y	+ y
4	Total motion	+ y	$x+y$	y- $x\frac{T_A}{T_B}$

Application

- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Compound Epicyclic Gear Train—Sun and Planet Gear

- A compound epicyclic gear train is shown in Fig. It consists of two co-axial shafts S1 and S2, an annulus gear A which is fixed, the compound gear (or planet gear) B-C, the sun gear D and the arm H. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm H. The sun gear is co-axial with the annulus gear and the arm but independent ofthem.
- The annulus gear A meshes with the gear B and the sun gear D meshes with thegear

  C. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

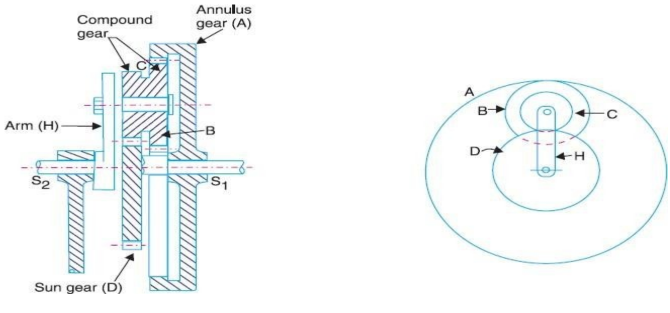


Fig. 5.2.5 Compound epicyclic gear train.

Note: The gear at the center is called the **sun gear** and the gears whose axes move are called **planet gears.**

Let T_A, T_B, T_C, and T_D be the teeth and N_A, N_B, N_C and N_D be the speeds for the gears *A*, *B*, *C* and *D* respectively. A little consideration will show that when the arm is fixed and the sun gear *D* is turned anticlockwise, then the compound gear *B*-*C* and the annulus gear A will rotate in the clockwisedirection.

The motion of rotations of the various elements is shown in the table below.

Table of motions

Sr. No.	**Condition of motion**	**Revolution of motion**			
		Arm	**Gear D**	**Compound Gear (B-C)**	**Gear A**
1	Arm fixe, gear D rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
2	Arm fixed gear D rotates through + *x* revolutions	0	+x	$-x\frac{T_D}{T_C}$	$-x\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
3	Add + *y* revolutions to all elements	+y	+y	+y	+y
4	Total motion	+y	x + y	$y - x\frac{T_D}{T_C}$	$y - x\frac{T_D}{T_C}\times\frac{T_B}{T_A}$

EXAMPLES

Example 5.1.The gearing of a machine tool is shown in Fig.2.1. The motor shaft is connected to gear A and rotates at 975 rpm. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F? The number of teeth on each gear is as givenbelow:

Gear	A	B	C	D	E	F
No. of teeth	20	50	25	75	26	65

Solution:

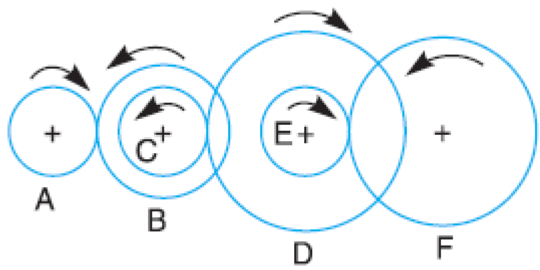


Fig. 6.1

Given data

$T_A = 20$ $\quad$ $N_F = ?$

$T_B = 50$

$T_C = 25$

$T_D = 75$

$T_E = 26$

$T_F = 65$

$N_A = 975$ rpm

$$N_F = \frac{T_A}{T_B} \times \frac{T_C}{T_D} \times \frac{T_E}{T_F} N_A$$

$$\therefore \frac{N_F}{975} = \frac{20}{50} \times \frac{20}{75} \times \frac{26}{65}$$

$\therefore N_F = 52$

Example 5.2 In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 rpm in the anticlockwise direction about the center of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed makes 300 rpm in the clockwise direction, what will be thespeed of gear B?

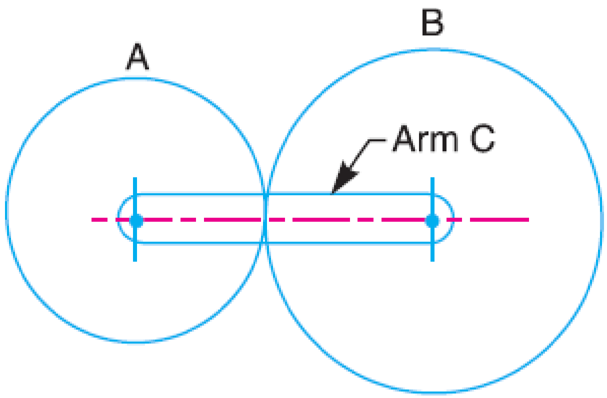


Fig.6.2

Solution :

Givendata Find

T_A=36 GearAfixed⇒N_B=?

T_B=45 N_A=−300(Clockwise) ⇒N_B=

? N_C= 150(Anticlockwise)

Sr. No.	Condition of motion	Revolution of element		
		Arm C	Gear A	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through + x revolutions	0	$+x$	$-x\frac{T_A}{T_B}$
3	Add + y revolutions to all elements	+ y	+ y	+ y
4	Total motion	+ y	$x + y$	$y - x\frac{T_A}{T_B}$

1. Speed of gear B (N_B) when gear A is fixed

Here, gear Afixed

$$\Rightarrow x + y = 0$$
$$\Rightarrow x + 150 = 0$$
$$\Rightarrow x = -150$$

Speed of gear B (N_B) $= y - x\dfrac{T_A}{T_B}$

$= y - (-150)\dfrac{36}{45}$

$\boxed{= +270 \text{rpm (Anticlockwise)}}$

2. Speed of gear B (N_B) when gear N_A = -300 (Clockwise)

Here given

$$x + y = -300$$
$$\therefore x + 150 = -300$$
$$\therefore x = -450 \text{ rpm}$$

Speed of gear B (N_B)

$$= y - x\frac{T_A}{T_B}$$
$$= 150 - (-450)\frac{36}{45}$$
$$\boxed{= +510 \text{ rpm(Anti clockwise)}}$$

Example 5.3 In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed anddirectionofgearCwhengearBisfixedandthearmAmakes100rpmclockwise.

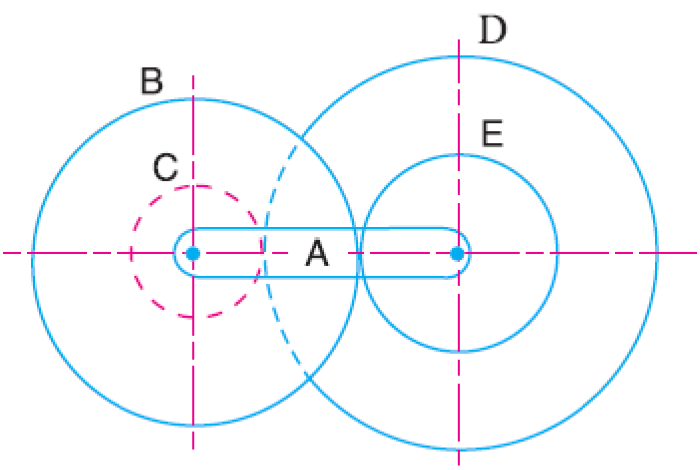


Fig. 6.3

Solution Given data find

$T_B = 75$ Gear B fixed $\Rightarrow N_C = ?$

$T_C = 30$ $N_A = -100 \Rightarrow N_C = ?$

$T_D = 90$

$N_A = -100$ (Clockwise)

Let $d_C + d_D = d_B + d_E$ $(r_C + r_D = r_B + r_E)$

$\therefore T_C + T_D = T_B + T_E$

$\therefore 30 + 90 = 75 + T_E$

$\therefore T_E = 45$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$-x\frac{T_E}{T_B}$	$-x\frac{T_D}{T_C}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_E}{T_B}$	$y-x\frac{T_D}{T_C}$

Gear B is fixed $\Rightarrow y - x\frac{T_E}{T_B} = 0$

$\Rightarrow -100 - x\frac{45}{75} = 0$

$\Rightarrow x = -166.67$

Speed of gear C $(N_C) = y - x\frac{T_D}{T_C}$

$= -100 - (-166.67)\times\frac{90}{30}$

$= +400$ rpm (Anti clockwise)

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_B}{T_E}$	$+\frac{T_B}{T_E}\times\frac{T_D}{T_C}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$x\frac{T_B}{T_E}$	$+x\frac{T_B}{T_E}\times\frac{T_D}{T_C}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_B}{T_E}$	$y+x\frac{T_B}{T_E}\times\frac{T_D}{T_C}$

Fromfig $(r_C+r_D=r_B+r_E)$

$\therefore T_C+T_D=T_B+T_E$

$\therefore T_E=90+30-75$

$\therefore T_E=45$

When gear B is fixed

$\therefore x+y=0$

$\therefore x+(-100)=0$

$\therefore x=100$

Now N_C $=y+x\frac{T_B}{T_E}\times\frac{T_D}{T_C}$

$=-100+100\times\frac{75}{45}\times\frac{90}{30}$

$\boxed{N_C=400\text{ rpm (Anticlockwise)}}$

Example 5.4 Anepicyclic gear consists of three gears A, B and C as shown in Fig. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 rpm. If the gear A is fixed, determine the speed of gears B andC.

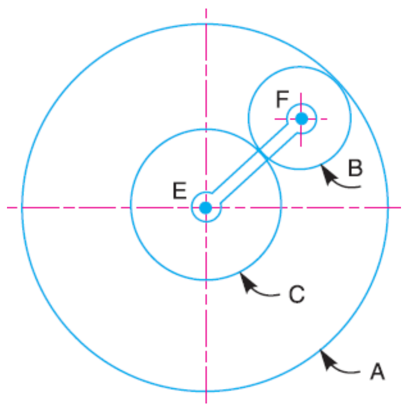


Fig. 6.4

Solution:

T_B=72(Internal) GearAfixed⇒N_B=?

T_C = 32 ⇒N_C = ?

(External) Arm

EF = 18rpm

From the geometry of fig.

$r_A = r_C + 2r_B$

$\therefore T_A = T_C + 2T_B$

$\therefore T_B = 20$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_A}\times\frac{T_B}{T_A}=-\frac{T_C}{T_B}$
2	Arm fixed gear A rotates through + *x* revolutions	0	+x	$-x\frac{T_C}{T_B}$	$-x\frac{T_C}{T_A}$
3	Add + *y* revolutions to all elements	+y	+y	+y	+y
4	Total motion	y	x+y	$y-x\frac{T_C}{T_B}$	$y-x\frac{T_C}{T_A}$

1. Speed of gear C(N_c)

$$\text{Gear A is fixed} \Rightarrow y - x\frac{T_C}{T_A} = 0$$

$$\Rightarrow -18 - x\frac{32}{72} = 0$$

$$\Rightarrow x = -40.5$$

$$\text{Speed of gear C } (N_C) = x + y$$

$$= 40.5 + 18$$

2. Speed of gear B(N_B) = 58.5 rpm(inthedirectionof

$$\text{Speed of gear B} = y - x\frac{T_C}{T_B}$$

$$= -18 - 40.5\times\frac{32}{20}$$

$$= -46.8 \text{ rpm}$$

= 46.8 rpm (intheopposite direction of

Example 5.5 Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 rpm, find the speed of shaft B.

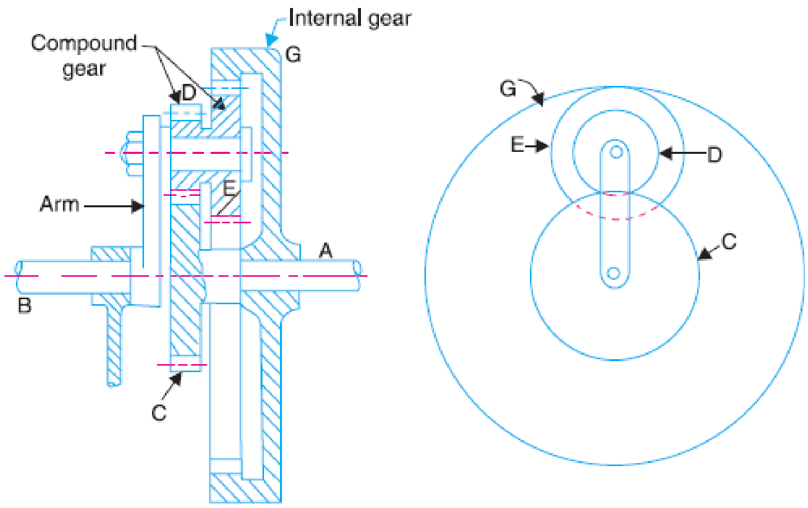


Fig 6.5

Solution:

$T_C = 50$ No.ofteethon internal gear =?

$T_D = 20$ Speed ofshaft B =?

$T_E = 35$

$N_C = 110$ (Rotationofshaft)

From the geometry of fig.

$$\frac{d_G}{2} = \frac{d_C}{2} + \frac{d_D}{2} + \frac{d_E}{2}$$

$$\therefore d_G = d_C + d_D + d_E$$

$$\therefore T_G = T_C + T_D + T_E$$

$$\therefore T_G = 50 + 20 + 35$$

$$\therefore T_G = 105$$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear C (Shaft A)	Compound Gear (D-E)	Gear G
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	+x	$-x\frac{T_C}{T_D}$	$-x\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
3	Add $+y$ revolutions to all elements	+y	+y	+y	+y
4	Total motion	y	x+y	$y-x\frac{T_C}{T_D}$	$y-x\frac{T_C}{T_D}\times\frac{T_E}{T_G}$

Speed of shaft B

Here given gear G is fixed

$$\therefore y - \frac{T_C}{T_D}\times\frac{T_E}{T_G} = 0$$

$$\therefore y - x\frac{50}{20}\times\frac{35}{105} = 0$$

$$\therefore y - x\times\frac{5}{6} = 0 \qquad \ldots\ldots\ldots\ldots(1)$$

Also given gear C is rigidly mounted on shaft A

$$\therefore x + y = 110 \qquad \ldots\ldots\ldots\ldots(2)$$

Solving eq. (1) & (2)

x=60

y=50

Speed of shaft B = Speed of arm = + y = 50 rpm

Example 6.6: In an epicyclic gear train, as shown in Fig.13.33, the number of teeth on wheels *A*, *B* and *C* are 48, 24 and 50 respectively. If the arm rotates at 400 rpm, clockwise,

Find: 1. Speed of wheel *C* when *A* is fixed, and

2. Speed of wheel *A* when *C* is fixed

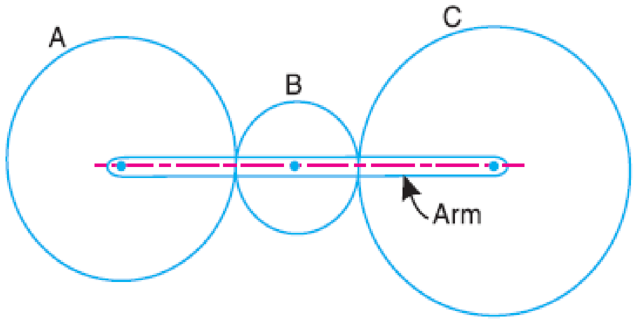


Fig. 6.6

Solution:

$T_A = 48$ Gear A fixed $\Rightarrow N_C = ?$

$T_B = 24$ Gear C fixed $\Rightarrow N_A = ?$

$T_C = 50$ $y = -400$ rpm (Arm rotation clockwise)

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$	$\left(-\frac{T_A}{T_B}\right)\times\left(-\frac{T_B}{T_C}\right)=+\frac{T_A}{T_C}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$-x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_C}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-x\frac{T_A}{T_B}$	$y+x\frac{T_A}{T_C}$

1. Speed of wheel C when A isfixed

When A is fixed

$\Rightarrow x + y = 0$

$\Rightarrow x - 400 = 0$

$\Rightarrow x = 0$

$N_C = y + x\frac{T_A}{T_C}$

$= -400 + 400 \times \frac{48}{50}$

$= -16$ rpm

2. Speed wheel A when C isfixed

When C is fixed

$\therefore N_C = 0$

$\therefore y + x\frac{T_A}{T_C} = 0$

$\therefore -400 + x\frac{48}{50} = 0$

$\therefore x = 416.67$

$N_A = x + y$

$= 416.67 - 400$

Example 5.7: An epicyclic gear train, as shown in Fig. 13.37, has a sun wheel *S* of 30 teeth and two planet wheels *P-P* of 50 teeth. The planet wheels mesh with the internal teeth of a fixed annulus *A*. The driving shaft carrying the sunwheel transmits 4 kW at 300 rpm. The driven shaft is connected to an arm which carries the planet wheels. Determine the speed of the driven shaft and the torque transmitted, if the overall efficiency is 95%.

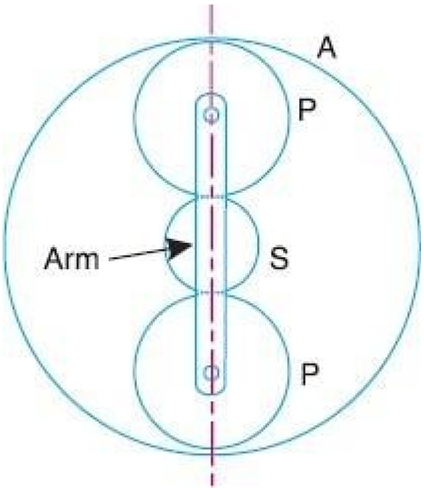


Fig.6.7

Solution

$T_S = 30 \quad T_P = 50 \quad T_A = 130$

$N_S = 300 \text{ rpm} \quad P = 4 \text{ KW}$

From the geometry of fig.

$r_A = 2r_P + r_S$

$\therefore T_A = 2T_P + T_S$

$= 2\times 50 + 30$

$= 130$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$\left(-\frac{T_S}{T_P}\right)\times\left(\frac{T_P}{T_A}\right) = +\frac{T_S}{T_A}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_A}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-x\frac{T_S}{T_P}$	$y-x\frac{T_S}{T_A}$

Here,

$$N_S = 300 rpm$$

$$\therefore x + y = 300 \qquad \ldots\ldots\ldots(1)$$

Also, Annular gear A is fixed

$$\therefore y - x\frac{T_S}{T_A} = 0$$

$$\therefore y - x \times \frac{30}{130} = 0$$

$$\therefore y = 0.23x \qquad \ldots\ldots\ldots(2)$$

Solving equation eq. (1) & (2)

$$x = 243.75$$

$$y = 56.25$$

Speed of Arm = Speed of driven shaft = y = 56.25 rpm

Here, P = 4 KW $\quad \eta = 95\%$

&

$$\therefore \eta = \frac{P_{out}}{P_{in}}$$

$$\therefore P_{out} = \eta \times P_{in}$$

$$= \frac{95}{100} \times 4$$

$$= 3.8 \text{ KW}$$

Also,

$$P_{out} = \frac{2\pi N T}{60}$$

$$\therefore 3.8 \times 10^3 = \frac{2\pi \times 56.30 T}{60}$$

$$\therefore T = 644.5 \text{ N}\cdot\text{m}$$

Example 6.8 Anepicyclic gear train is shown In fig. Find out the rpm of pinion D if arm A rotate at 60 rpm in anticlockwise direction. No of teeth on wheels are given below.

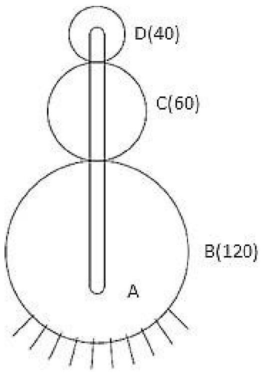


Fig.6.8

Solution:

T_D=40 N_D=?

T_C= 60

T_B= 120

N_A =+60 rpm(Anticlockwise)

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear B	Gear C	Gear D
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_B}{T_C}$	$\frac{T_B}{T_C}\times\frac{T_C}{T_D}=+\frac{T_B}{T_D}$
2	Arm fixed gear A rotates through + *x* revolutions	0	+*x*	$-x\frac{T_B}{T_C}$	$+x\frac{T_B}{T_D}$
3	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4	Total motion	*y*	*x* +*y*	$y-x\frac{T_B}{T_C}$	$y+x\frac{T_B}{T_D}$

From fig. Gear B is fixed

$\therefore$ x + y = 0

$\therefore$ x +60=0 (∵rpmofarmA= 60 =y)

$\therefore$ x =–60

Now motion of gear D

$=y+x\frac{T_B}{T_D}$

$= 60 - 60 \times \frac{120}{40}$

=–120 rpm

D rotates 120 rpm in clockwise direction.

Note: By fixing any gear C OR B this problem can be solved

Example 6.9 An epicyclic gear train for an electric motor is shown in Fig. The wheel *S* has 15 teeth and is fixed to the motor shaft rotating at 1450 rpm. The planet *P* has 45 teeth, gears with fixed annulus *A* and rotates on a spindle carried by an arm which is fixed to the output shaft. The planet *P* also gears with the sun wheel *S*. Find the speed of the output shaft. If the motor is transmitting 1.5 kW, find the torque required to fix the annulus*A*.

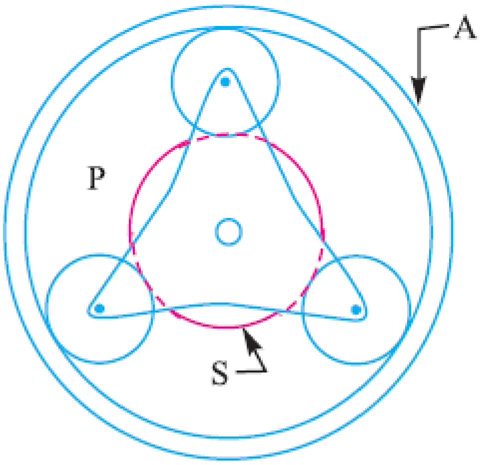


Fig. 6.9

Solution:

$T_S = 15$ Speedofoutputshaft=?

$T_P = 45$ Torque=?

From fig.

$r_A = r_s + 2r_P$

$\therefore T_A = T_S + 2T_P$

$\therefore T_A = 105$

Sr. No.	Condition of motion	Revolution of element			
		Spindle	Gear S	Gear P	Gear A
1	Sector/Spindle fixed, gear S rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_A}\times\frac{T_P}{T_A}=-\frac{T_S}{T_P}$
2	Spindle fixed gear S rotates through + *x* revolutions	0	+*x*	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_A}$
3	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4	Total motion	*y*	*x* +*y*	$y - x\frac{T_S}{T_P}$	$y - x\frac{T_S}{T_A}$

Motor shaft is fixed with gear S

$$\therefore x + y = 1450 \qquad \ldots\ldots\ldots(1)$$

And Annular A is fixed

$$\therefore y - x\frac{T_S}{T_A} = 0$$

$$\therefore y - x\frac{15}{105} = 0$$

$$\therefore y = x\frac{15}{105} \qquad \ldots\ldots\ldots(2)$$

By solving equation (1) & (2)

x= 1268.76

y= 181.25

Speed of output shaft y = 181.25 rpm

- Torque on sun wheel (S) (inputtorque)

$$P = \frac{2\pi N T_i}{60}$$

$$\therefore T_i = \frac{P\times 60}{2\pi N} = \left(\frac{2\times10^3}{1.35}\right)\times\frac{60}{2\pi\times1450} \qquad \left(\because 1.35\,HP = 1\,KW \Rightarrow 2\,HP = \frac{2}{1.35}\,KW\right)$$

$$= 9.75\,N\cdot m$$

- Torque on output shaft (with 100% mechanicalefficiency)

$$\therefore T_o = \frac{P\times 60}{2\pi N} = \left(\frac{2\times10^3}{1.35}\right)\times\frac{60}{2\pi\times181.25}$$

$$= 78.05\,N\cdot m$$

- Fixingtorque

$$= T_o - T_i$$

$$= 78.05 - 9.75$$

$= 68.3\ \text{N·m}$

Example 6.10: **If wheel D of gear train as shown in fig. is fixed and the arm A makes 140 revolutions in a clockwise direction. Find the speed and direction of rotation of B & E. C is a compoundwheel.**

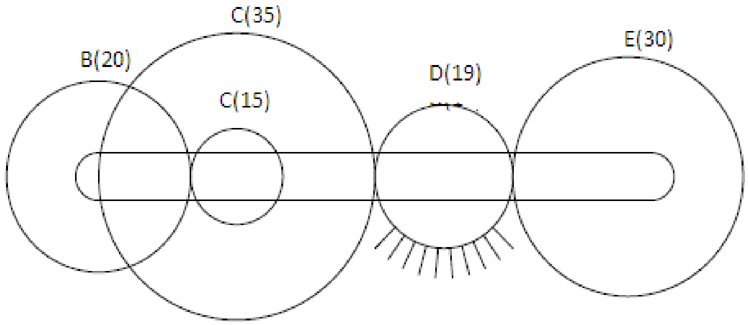


Fig.6.10

Solution:

$T_B = 30$ $T_C = 35$ $T_D = 19$ $T_E = 30$

Sr. No.	Condition of motion	Revolution of element			
		Spindle	Gear S	Gear P	Gear A
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{20}{15}$	$\left(-\frac{20}{15}\right)\times\left(-\frac{35}{19}\right)\times\left(-\frac{19}{30}\right)$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$-1.33x$	$-1.555x$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-1.33x$	$y-1.555x$

- When gear D isfixed

$y + 2.456x = 0$

$\therefore -140 + 2.456x = 0$ $\quad$ ($\because y = -140$ rpm given)

$\therefore x = 57$

- Speed of gearB

$N_B = x + y$

$= +57 - 140$

$= -83$ rpm(Clockwise)

- Speed of gearE

$N_E = y - 1.555x$

$= -140 - 1.555(57)$

$= -228.63$ rpm (Clockwise)

Example 6.11: **The epicyclic train as shown in fig. is composed of a fixed annular wheel A having 150 teeth. Meshing with A is a wheel b which drives wheel D through an idle wheel C,D being concentric with A. Wheel B and C are carried on an arm which revolve clockwise at 100 rpm about the axis of A or D. If the wheels B and D are having 25 teeth and 40 teeth respectively, Find the no. of teeth on C and speed and sense of rotation ofC.**

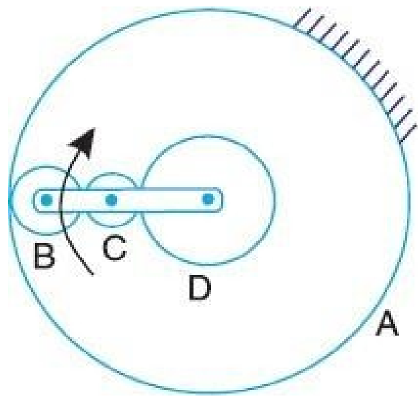


Fig. 6.11

Solution:

From the geometry of fig.

$$r_A = 2r_B + 2r_C + r_D$$

$$\therefore T_A = 2T_B + 2T_C + T_D$$

$$\therefore 150 = 50 + 2T_C + 40$$

$$\therefore T_C = 30$$

Sr. No.	Condition of motion	Revolution of element				
		Arm	Gear D	Gear C	Gear B	Gear A
1	Arm fixe, gear D rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_D}{T_C}$	$+\frac{T_D}{T_B}$	$+\frac{T_D}{T_A}$
2	Arm fixed gear D rotates through + x revolutions	0	$+x$	$-x\frac{T_D}{T_C}$	$+x\frac{T_D}{T_B}$	$+x\frac{T_D}{T_A}$
3	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_D}{T_C}$	$y+x\frac{T_D}{T_B}$	$y+x\frac{T_D}{T_A}$

Now

$$N_A = 0$$

$$\therefore y + x\frac{T_D}{T_A} = 0$$

$$\therefore -100 + x \times \frac{40}{150} = 0$$

$$\therefore x = 375$$

Let

$$N_C = y - x\frac{T_D}{T_C}$$

$$= -100 - 375 \times \frac{40}{30}$$

$$= -600 rpm$$

Example 6.12: Fig. 13.24 shows a differential gear used in a motor car. The pinion A on the propeller shaft has 12 teeth and gears with the crown gear B which has 60 teeth. The shafts P and Q form the rear axles to which the road wheels are attached. If the propeller shaft rotates at 1000 rpm and the road wheel attached to axle Q has a speed of 210 rpm. while taking a turn, find the speed of road wheel attached to axle P.

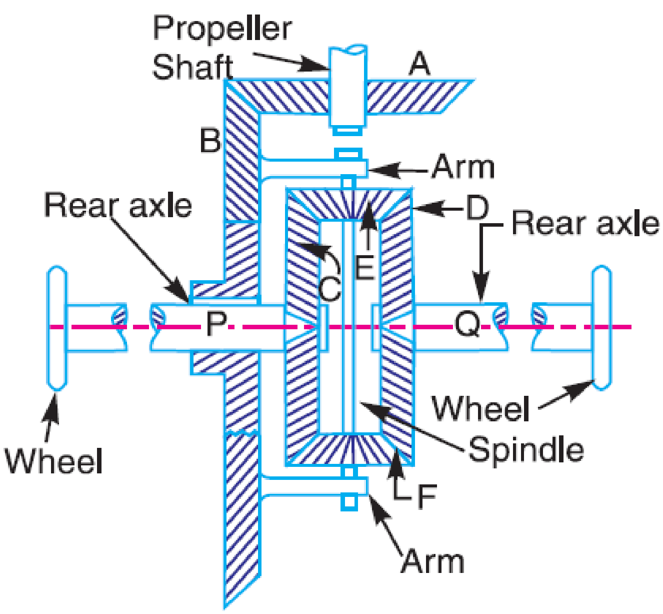


Fig. 6.12

Solution:

$T_A = 12$

$T_B = 60$

$N_Q = N_D = 210 rpm$

$N_A = 1000 rpm$

Let

$$N_A \times T_A = N_B T_B$$
$$\therefore N_B = N_A \times \frac{T_A}{T_B}$$
$$= 1000 \times \frac{12}{60}$$
$$= 200 \text{rpm}$$

Sr. No.	Condition of motion	Revolution of element			
		Gear B	Gear C	Gear E	Gear D
1	Gear B is fixed, gear C rotates +1 revolution(anticlockwise)	0	+1	$+\frac{T_C}{T_E}$	-1
2	Gear B is fixed gear C rotates through $+x$ revolutions	0	$+x$	$+x\frac{T_C}{T_E}$	$-x$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y+x\frac{T_C}{T_E}$	$y-x$

Let here speed of gear B is 200 rpm

$$N_B = 200 = y$$

From table

$$N_D = y - x = 210$$
$$\therefore x = y - 210$$
$$\therefore x = 200 - 210$$
$$\therefore x = -10 \text{rpm}$$

Let speed of road wheel attached to the axle P = Speed of gear C

$$= x + y$$
$$= -10 + 200$$
$$= 180 \text{rpm}$$

Example 6.13: Two bevel gears A and B (having 40 teeth and 30 teeth) are rigidly mounted on two co-axial shafts X and Y. A bevel gear C (having 50 teeth) meshes with A and B and rotates freely on one end of an arm. At the other end of the arm is welded a sleeve and the sleeve is riding freely loose on the axes of the shafts X and Y. Sketch the arrangement. If the shaft X rotates at 100 rpm. clockwise and arm rotates at 100 rpm. anticlockwise, find the speed of shaftY.

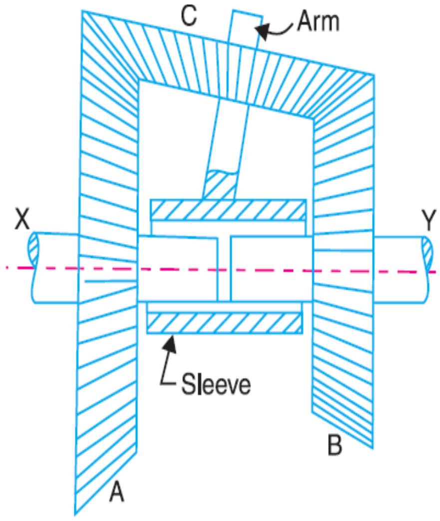


Fig. 6.13

Solution :

$T_A = 40 \quad T_C = 50 \quad T_B = 30$

$N_X = N_A = -100$ rpm(Clockwise)

Speed of arm = 100rpm

Sr. No.	Condition of motion	Revolution of element			
		Arm	Gear A	Gear C	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$\pm\frac{T_A}{T_C}$	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$\pm x\frac{T_A}{T_C}$	$-x\frac{T_A}{T_B}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y \pm x\frac{T_A}{T_C}$	$y - x\frac{T_A}{T_B}$

Here speed of arm = y = +100 rpm (given)

Also given $N_A = N_X = -100$rpm

$\therefore N_A = x + y$

$\therefore -100 = x + 100$

$\therefore x = -200$

Speed of shaft Y = N_B

$= y - x\frac{T_A}{T_B}$

$= 100 + 200 \times \frac{40}{30}$

$= +366.7$rpm (Anticlockwise)

Example 6.14. An epicyclic train of gears is arranged as shown in Fig. How many revolutions does the arm, to which the pinions B and C are attached, make: 1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and 2. when A makes one revolution clockwise and D is stationary? The number of teeth on the gears A and D are 40 and 90 respectively.

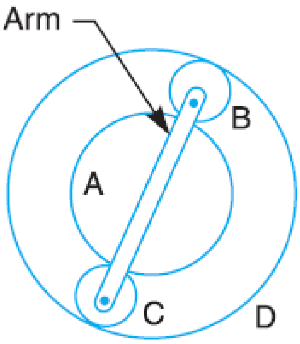


Fig. 6.14

Solution:

$T_A = 40$

$T_D = 90$

First of all, let us find the number of teeth on gear B and C (i.e. T_B and T_c). Let d_A , d_B, d_C, $d_D$be the pitch circle diameter of gears A, B, C, and D respectively. Therefore from the geometryoffig, $d_A + d_B + d_C = d_D$ or $d_A + 2d_B = d_D$ ...($d_B = d_C$)

Since the number of teeth are proportional to their pitch circle diameters,therefore,

$T_A + 2\,T_B = T_D$ or $40 + 2T_B = 90$

$\therefore$ $T_B = 25$, and $T_C = 25$ ...($T_B = T_C$)

Sr. No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound Gear B-C	Gear D
1	Arm fixe, gear Arotates -1revolution(clockwise)	0	-1	$+\frac{T_A}{T_B}$	$\left(+\frac{T_A}{T_B}\right)\times\left(+\frac{T_B}{T_D}\right)=+\frac{T_A}{T_D}$
2	Arm fixed gear A rotates through - x revolutions	0	$-x$	$+x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_D}$
3	Add - y revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4	Total motion	$-y$	$-x-y$	$x\frac{T_A}{T_B}-y$	$x\frac{T_A}{T_C}-y$

1. **Speed of arm when A makes 1 revolution clockwise and D makes halfrevolution anticlockwise**

   Since the gear A makes 1 revolution clockwise, therefore from the fourth row ofthe table, $-x - y = -1$ or $x + y = 1$ ...(1)

Also, the gear D makes half revolution anticlockwise, therefore

$$x \times \frac{T_A}{T_D} - y = \frac{1}{2}$$

$$\therefore x \times \frac{40}{90} - y = \frac{1}{2}$$

$$\therefore 40x - 90y = 45$$

$$\therefore x - 2.25y = 1.125 \quad \text{...(2)}$$

From equations (1) and (2),

$x = 1.04$ and $y = -0.04$

Speed of arm $= -y = -(-0.04) = +0.04$

0.04

2. Speed of arm when A makes 1 revolution clockwise and D is stationary

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1$$

$$\therefore x + y = 1 \quad \text{...(3)}$$

Also the gear *D* is stationary, therefore

$$x \times \frac{T_A}{T_D} - y = 0$$

$$\therefore x \times \frac{40}{90} - y = 0$$

$$\therefore 40x - 90y = 0$$

$$\therefore x - 2.25y = 0 \quad \text{...(4)}$$

From equations (3) and (4),

$\therefore$ Speed of arm $= -y = -0.308$

$\therefore$ Speed of arm = 0.308 revolution

Example 6.15. In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth is: TC = 28; TD = 26; TE = TF = 18. 1. Sketch the arrangement; 2. Find the number of teeth on A and B; 3. If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B; and 4. If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise; find the speed of Wheel B.

Solution:

Given: $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$

1. **Sketch thearrangement**

The arrangement is shown in Fig.

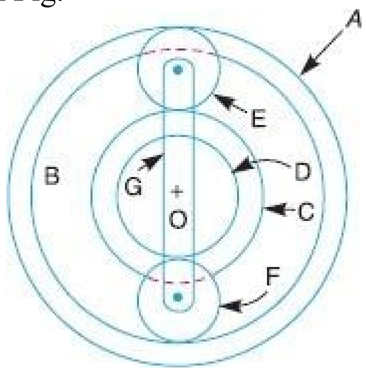


Fig. 6.15

2. **Number of teeth on wheels A andB**

T_A = Number of teeth on wheel *A*, and

T_B = Number of teeth on wheel *B*.

If d_A , d_B , d_C , d_D, d_E and d_F are the pitch circle diameters of wheels *A*, *B*, *C*, *D*, *E* and *F* respectively, then from the geometry of Fig.

$$d_A = d_C + 2\, d_E$$

And $$d_B = d_D + 2 d_F$$

Since the number of teeth are proportional to their pitch circle diameters, for the same module, therefore

$$T_A = T_C + 2\, T_E = 28 + 2 = 64$$

And $$T_B = T_D + 2\, T_F = 26 + 2 = 62$$

3. **Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A isfixed**

First of all, the table of motions is drawn as given below:

Sr. No	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1	Arm fixe, A rotates +1 revolution (Anti clockwise)	0	+1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C} = -\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B} = +\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2	Arm fixed A rotates through + *x* revolutions	0	$+x$	$+x\frac{T_A}{T_E}$	$-x\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$x\frac{T_A}{T_E}+y$	$y-x\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

Since the arm *G* makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$\mathbf{y = -100}$$

Also, the wheel *A* is fixed, therefore from the fourth row of the table,

$$\mathbf{x + y = 0 \quad or \quad x = -y = 100}$$

$$\text{Speed of wheel B} = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$$

$$= -100 + 100 \times \frac{64}{28} \times \frac{26}{62}$$

$$= -100 + 95.8 \text{ r.p.m.} = -4.2 \text{ r.p.m}$$

Speed of wheel B = -4.2 r.p.m

4. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise

Since the arm *G* makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$y = -100 \qquad \text{...(3)}$$

Also the wheel *A* makes 10 r.p.m. counter clockwise, therefore from the fourth row of the table,

$$x + y = 10$$

$$\therefore x = 10 - y$$

$$\therefore x = 10 + 100$$

$$\therefore x = 110 \qquad \text{...(4)}$$

$$\therefore \text{Speed of wheel B} = + y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$$

$$= -100 + 110 \times \frac{64}{28} \times \frac{26}{62}$$

$$= -100 + 105.4 \text{ r.p.m}$$

$$= +5.4 \text{ r.p.m}$$

$\therefore$ Speed of wheel B = 5.4 r.p.m

Example 6.16. Fig. shows diagrammatically a compound epicyclic gear train. Wheels A , D and E are free to rotate independently on spindle O, while B and C are compound and rotate together on spindle P, on the end of arm OP. All the teeth on different wheels have the same module. A has 12 teeth, B has 30 teeth and C has 14 teeth cut externally. Find the number of teeth on wheels D and E which are cut internally. If the wheel A is driven clockwise at 1 r.p.s. while D is driven counter clockwise at 5 r.p.s., determine the magnitude and direction of the angular velocities of arm OP and wheel E.

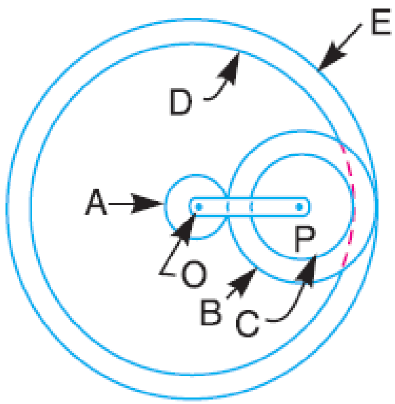


Fig. 6.16

Solution:

Given: $T_A=12$; $T_B=30$; $T_C=14$; $N_A=1$ r.p.s.; $N_D=5$r.p.s

Number of teeth on wheels D and E

Let T_D and T_E be the number of teeth on wheels D and E respectively. Let d_A , d_B , d_C , $d_D$and d_E be the pitch circle diameters of wheels A, B, C, D and E respectively. From the geometry of the figure,

$$d_E= d_A +2d_B \quad \text{and} \quad d_D= d_E-(d_B-d_C)$$

Since the number of teeth are proportional to their pitch circle diameters for the same module, therefore

$$T_E=T_A+2T_B \qquad T_D= T_E-(T_B- T_C)$$

$$\therefore T_E =12+2\times30 \qquad \therefore T_D =72-(30-14)$$

$$\boxed{\therefore T_E =72} \qquad \boxed{\therefore T_D = 56}$$

Magnitude and direction of angular velocities of arm OP and wheel

The table of motions is drawn as follows:

Sr. No.	Condition of motion	Revolutions of elements				
		Arm	Wheel A	Compound wheel B-C	Wheel D	Wheel E
1	Arm fixe, gear A rotates -1 revolution(clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$+\frac{T_A}{T_E}\times\frac{T_B}{T_E}=+\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through - x revolutions	0	$-x$	$+x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$+x\frac{T_A}{T_E}$
3	Add - y revolutions to all elements	$-y$	$-y$	$-y$	$-y$	$-y$
4	Total motion	$-y$	$-x-y$	$x\frac{T_A}{T_B}-y$	$x\frac{T_A}{T_B}\times\frac{T_C}{T_D}-y$	$x\frac{T_A}{T_E}-y$

Since the wheel *A* makes 1 r.p.s. clockwise, therefore from the fourth row of the table,

$$-x - y = -1$$

$$\therefore x + y = 1 \qquad (1)$$

Also, the wheel *D* makes 5 r.p.s. counter clockwise, therefore

$$x \frac{T_A}{T_B} \times \frac{T_C}{T_D} - y = 5$$

$$\therefore x \frac{T_A}{T_B} \times \frac{T_C}{T_D} - y = 5$$

$$\therefore x \frac{12}{30} \times \frac{14}{56} - y = 5$$

$$\therefore 0.1x - y = 5 \qquad (2)$$

From equations (1) and (2),

$$x = 5.45 \qquad \text{and} \qquad y = -4.45$$

Angular velocity of arm OP

$$= -y = -(-4.45) = 4.45 \text{ r.p.s}$$

$$= 4.45 \times 2\pi = 27.964 \text{ rad/sec (Anti clockwise)}$$

And angular velocity of wheel E

$$= x \frac{T_A}{T_E} - y$$

$$= 5.45 \times \frac{12}{72} - (-4.45)$$

$$= 5.36 \text{ r.p.s}$$

$$= 5.36 \times 2\pi$$

$$= 33.68 \text{ rad / sec (Anti}$$

fixed wheel E. Pinion C has 15 teeth and is integral with B (B, C being a compound gear wheel). Gear C meshes with annular wheel D, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B,
Q. If the motor runs at 1000 r.p.m., find the speed of the machine shaft. Find the torque exerted on the machine shaft, if the motor develops a torque of 100N-m.

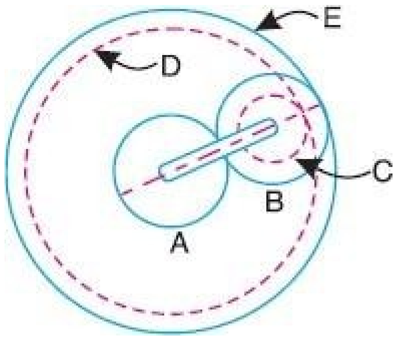


Fig. 6.18

Solution:

Given : $T_A = 15$; $T_B = 20$; $T_C = 15$; $N_A = 1000$ r.p.m.;
Torque developed by motor (or pinion A) = 100 N-m

1. Speed of the machineshaft

The table of motions is given below:

Sr. No.	Condition of motion	Revolution of element				
		Arm	Pinion A	Compound wheel D-C	Wheel D	Wheel E
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-\frac{T_A}{T_E}\times\frac{T_B}{T_E}=-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$-x\frac{T_A}{T_B}$	$-x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-x\frac{T_A}{T_E}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_A}{T_B}$	$y-x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$y-x\frac{T_A}{T_E}$

First of all, let us find the number of teeth on wheels D and E. Let T_D and T_E bethe number of teeth on wheels D and E respectively. Let d_A, d_B, d_C, $d_D$and d_E be the pitch circle diameters of wheels A, B, C, D and E respectively. From the geometry of thefigure,

$d_E = d_A + 2d_B$ and $d_D = d_E - (d_B - d_C)$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_E = T_A + 2\,T_B = 15 + 2\times 20 = 55$$

$$T_D = T_E - (T_B - T_C) = 55 - (20 - 15) = 50$$

We know that the speed of the motor or the speed of the pinion *A* is 1000 r.p.m.

Therefore

$$x + y = 1000 \qquad ...(1)$$

Also, the annular wheel *E* is fixed, therefore

$$y - x\frac{T_A}{T_E} = 0$$

$$\therefore y = x\frac{T_A}{T_E}$$

$$\therefore y = x\frac{15}{55}$$

$$\therefore y = 0.273x \qquad ...(2)$$

From equations (1) and (2),

$$x = 786 \qquad \text{and} \qquad y = 214$$

$\therefore$ Speed of machine shaft = Speed of wheel D

$$N_D = y - x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$$

$$= 214 - 786\times\frac{15}{20}\times\frac{15}{50}$$

$$= +\,37.15 \text{ r.p.m.}$$

$\therefore N_D = 37.15$

Torque exerted on the machine shaft

We know that

Torque developed by motor × Angular speed of motor

= Torque exerted on machine shaft × Angular speed of machine shaft

$\therefore 100 \times \omega_A$ = Torque exerted on machine shaft $\times\,\omega_D$

$\therefore$ Torque exerted on machine shaft $= 100 \times \frac{\omega_A}{\omega_D}$

$$= 100\times\frac{N_A}{N_D} = 100\times\frac{1000}{37.5}$$

$\therefore$ Torque exerted on machine shaft = 2692 N-m

Example 6.19. An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star- shaped planet carrier C. The sizes of different toothed wheels are such that the planet carrier C rotates at 1/5th of the speed of the sun wheel S. The minimum number of teeth on any wheel is 16. The driving torqueon

the sun wheel is 100 N-m. Determine: 1. Number of teeth on different wheels of the train, and 2. torque necessary to keep the internal gear stationary.
Solution:

Given $N_C = \frac{N_S}{5}$

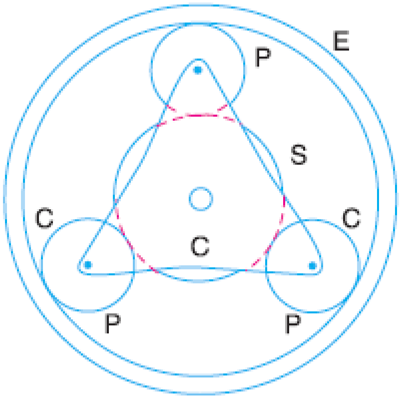


Fig. 6.19

1. Number of teeth on differentwheels

The arrangement of the epicyclic gear train is shown in Fig.. Let T_S and T_E be the number of teeth on the sun wheel S and the internal gear E respectively. The table of motions is given below:

Sr. No.	Conditions of motion	Revolutions of elements			
		Plant carrier C	Sun wheel S	Planet Wheel P	Internal Gear E
1	Planet carrier *C* fixed, sun wheel *S* rotates through + 1 revolution (anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_E} \times \frac{T_P}{T_E} = -\frac{T_S}{T_P}$
2	Planet carrier *C* fixed, sun wheel *S* rotates through + *x* revolutions	0	$+x$	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_E}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y - x\frac{T_S}{T_P}$	$y - x\frac{T_S}{T_E}$

We know that when the sun wheel *S* makes 5 revolutions, the planet carrier *C* makes 1 revolution. Therefore from the fourth row of the table,

$$y = 1, \quad \text{and} \quad x + y = 5$$

$$\therefore x = 4$$

Since the gear E is stationary, therefore from the fourth row of the table,

$$y - x\frac{T_S}{T_E} = 0$$

$$\therefore 1 - 4\frac{T_S}{T_E} = 0$$

$$\therefore T_E = 4T_S$$

Since the minimum number of teeth on any wheel is 16, therefore let us take the number of teeth on sun wheel,

$$T_S = 16$$

$$\boxed{\therefore T_E = 4 \times 16 = 64}$$

Let d_S, d_P and d_E be the pitch circle diameters of wheels S, P and E respectively. Now from the geometry of Fig

$$d_S + 2\,d_P = d_E$$

Assuming the module of all the gears to be same, the number of teeth are proportional to their pitch circle diameters.

$$T_S + 2\,T_P = T_E$$

$$\therefore 16 + 2T_P = 64$$

$$\boxed{\therefore T_P = 24}$$

2. Torque necessary to keep the internal gearstationary

We know that

Torque on S× Angular speed of S = Torque on C× Angular speed of C

$$100 \times \omega_S = \text{Torque on C} \times \omega_C$$

$$\therefore \text{Torque on C} = 100 \times \frac{\omega}{\omega_C}$$

$$= 100 \times \frac{N_S}{N_C}$$

$$= 100 \times 5$$

$$\therefore \text{Torque on C} = 500 \text{ N·m}$$

$\therefore$ Torque necessary to keep the internal gear stationary

$$= 500 - 100$$

$$= 400$$

TUTORIAL QUESTIONS

1. a)Make a comparison of cycloidal and involute tooth forms.b) Two 200 pressure angle involute gears in mesh have a module of 10mm. Addendumis
1 module. Large gear has 50 teeth and the pinion has 13 teeth. Does interference occur?If it occurs, to what value should the pressure angle be changed to eliminateinterference?

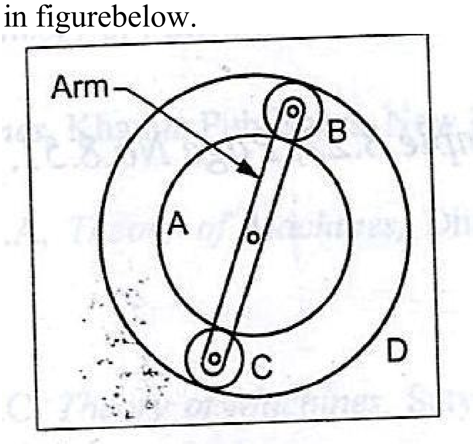
2. Sketch two teeth of a gear and show the following: face, flank, top land, bottom land, addendum, dedendum, tooth thickness, space width, face width and circularpitch.
(b) Derive a relation for minimum number of teeth on the gear wheel and the pinion to avoid interference

3. Two gears in mesh have a module of 10 mm and a pressure angle of 250. The pinion has 20 teeth and the gear has 52. The addendum on both the gears is equal to onemodule.
Determine (i) The number of pairs of teeth in contact (ii) The angles of action of the pinion and the wheel (iii) The ratio of the sliding velocity to the rolling velocity at the pitch point and at the beginning and end of engagement.

4. What is a worm and worm wheel? Where is itused?
(b) Two 200 involute spur gears mesh externally and give a velocity ratio of 3. Module is 3 mm and the addendum is equal to 1.1 module. If the pinton rotates at 120 r.p.m. find: (i) The minimum number of teeth on each wheel to avoid interference. (ii) The number of pairs of teeth in contact

5. Two involute gears of 20° pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5 mm, and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find (i) the angle turned through by pinion when one pair of teeth is in mesh; and (ii) the maximum velocity ofsliding

6. An epicyclic gear train shown in figurebelow.



The internal gear D has 90 teeth and the sun gear A has 40 teeth. The two planet gears B & C are identical and they are attached to an arm as shown. How many revolutions does the arm makes, (i) When'A' makes one revolution in clockwise and 'D' , makes one revolution in clockwise and 'D' makes ½ revolutions in opposite sense.
(ii) When 'A' makes one revolution in clockwise and 'D' remains stationary.