12223

3 Hours / 70 Marks   Seat No. ☐☐☐☐☐☐☐☐

Instructions –   (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

(7) Programmable Pocket Calculator is not allowed.

Marks

1.   **Attempt any FIVE of the following:**   **10**

a) Define Odd and Even functions.

b) If $f(x) = 3x + a$ and $f(1) = 7$, find a and $f(4)$.

c) If $y = x^{10} + 10^x + e^x + \log x$ find $\dfrac{dy}{dx}$.

d) Evaluate $\int x \log x \, dx$

e) Evaluate $\int \dfrac{1}{1 - \cos 2x} \, dx$

f) Using integration, find the area of the region bounded by the lines $2y + x = 8$, x-axis and the lines $x = 2$ and $x = 4$.

g) State the Simpson's one-third rule of numerical integration.

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2.   **Attempt any THREE of the following:**   12

a)   Find $\dfrac{dy}{dx}$ if $y = \log[\operatorname{cosec} x - \cot x]$.

b)   Find $\dfrac{dy}{dx}$ if $x = \sec^2\theta$, $y = \tan^3\theta$ at $\theta = \dfrac{\pi}{3}$.

c)   A beam is bent in the form of the curve $y = 2\sin x - \sin 2x$. Find the radius of curvature at $x = \dfrac{\pi}{2}$.

d)   A telegraph wire hangs in the form of a curve $y = a \log \sec\left(\dfrac{x}{a}\right)$ where a is constant show that the curvature at any point is $\dfrac{1}{a} \cos\left(\dfrac{x}{a}\right)$.

3.   **Attempt any THREE of the following:**   12

a)   Find the equation of the tangent and normal to the curve $13x^3 + 2x^2y + y^3 = 1$ at $(1, -2)$.

b)   Find $\dfrac{dy}{dx}$ if $y = x^{\sin x} + (\tan x)^x$.

c)   If $y = \log(x\sin 2x)$ find $\dfrac{dy}{dx}$.

d)   Evaluate $\displaystyle\int \dfrac{e^x(x+1)}{\sin^2(xe^x)}\, dx$.

4.   **Attempt any THREE of the following:**   12

a)   Evaluate $\displaystyle\int \dfrac{1}{9x^2 + 6x + 10}\, dx$

b)   Evaluate $\displaystyle\int \dfrac{1}{3 + 2\sin x + \cos x}\, dx$

c)   Evaluate $\displaystyle\int x^2 - e^{3x}\, dx$

d)   Evaluate $\displaystyle\int \dfrac{\cos x}{(4 + \sin x)(3 + \sin x)}\, dx$

e)   Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sqrt[3]{\sec x}}{\sqrt[3]{\sec x} + \sqrt[3]{\operatorname{cosec} x}}\, dx$

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5. **Attempt any TWO of the following:** 12

a) Find the area cut off from the parabola $4y = 3x^2$ with the line $2y = 3x + 12$.

b) Attempt the following

    i) Form the differential equation from the relation $y = Ae^{3x} + Be^{-3x}$ where A & B are arbitrary constant's.

    ii) Solve $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$.

c) A right circular cone has height 9 cm and radius of the base 5 cm. It is inverted and water is poured into it. If at any instant the water level rises at the rate of $\left(\frac{\pi}{A}\right)$ cm/sec, where A is the area of the water surface at that instant, show that the vessel will be full in 75 seconds.

6. **Attempt any TWO of the following:** 12

a) Attempt the following

    i) Evaluate $\int\limits_{3}^{8} \log_e x \, dx$ using Trapezoidal rule from the following data

x	3	4	5	6	7	8
$\log_e x$	1.0986	1.3863	1.6094	1.7918	1.9459	2.0794

    ii) Apply Simpson's one-third rule to find $\int\limits_{0}^{2} f(x) \, dx$ using the following data.

x	0	1.0	1.5	2.0
$f(x)$	1.1	2.4	5.7	8.1

b) Evaluate $\int\limits_{0}^{\frac{\pi}{2}} \cos x \, dx$ using Simpson's $\frac{3}{8}$ rule with n = 8.

c) Evaluate $\int\limits_{1}^{5} \frac{1}{x+2} \, dx$ using Simpson's one third rule. Divide the interval [1, 5] into 4 equal subintervals.